Machine Dreams: The Early History and Mathematics of Computer Redistricting, 1950–1980

John Hessler
Applied Mathematician and Geographic Information Systems Scientist
Relic {Code} Lab
jhessle1@jhu.edu

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"A computer may grind out district lines which can totally frustrate the popular will on an overwhelming number of critical issues."

— Justice John M. Harlan II, dissenting,
Wells v. Rockefeller, 394 U.S. 542 (1969)

Abstract

Between 1961 and 1980, a small community of economists, operations researchers, political scientists and engineers tried to turn congressional and legislative districting into a problem a computer could solve. This paper traces that effort from its 1950s preconditions to the eve of the 1981 redistricting round. It reconstructs the mathematics behind each major approach.

Three families of method emerged. The first was randomized or mechanical growth procedures, following Vickrey (1961). The second was location–allocation models that minimize a population "moment of inertia", following Weaver and Hess (1963) and Hess et al. (1965). The third was local-search heuristics that trade units between districts, following Nagel (1965). A fourth line, exact optimization by implicit enumeration and set partitioning (Garfinkel and Nemhauser 1970), showed what optimality would cost.

The paper argues that the project failed on its own terms but succeeded in another sense. No state handed its map to a neutral algorithm. Yet the work defined the vocabulary of modern districting: plans as partitions of a graph, compactness as a measurable quantity, and population equality as a hard numerical constraint. It also showed early that the combinatorics of the problem defeat exhaustive search. By 1980 the computer had become a tabulation and evaluation tool in the hands of map-drawers rather than a replacement for them. The paper opens and closes by connecting that combinatorial barrier to current research. There, NP-hardness results have pushed the field from searching for one optimal map toward sampling large ensembles of valid plans.

1. Introduction

Redistricting is a partition problem. A state is cut into a fixed number of districts, each a union of smaller geographic units such as counties, census tracts, enumeration districts or precincts.

Every possible map reflects choices about which units belong together. For most of American history, those choices were made by legislators with paper maps, colored pencils and adding machines.

The idea that a machine could make those choices instead appeared in the early 1960s, almost as soon as two conditions were met. First, the Supreme Court made unequal district populations a constitutional question, starting with *Baker v. Carr* (1962). Second, digital computers and the new discipline of operations research offered tools for large assignment and allocation problems. The early advocates' central claim was simple. If the rules were fixed in advance and a computer applied them, there would be no room for gerrymandering.

This paper follows that claim from its roots in the 1950s to 1980. It has two aims. The historical aim is to identify who built the first redistricting algorithms, why, and with what result. The mathematical aim is to state each method precisely enough that its strengths and blind spots are visible. That includes its objective function, its constraints and its search procedure.

The argument runs as follows. Section 2 describes the preconditions of the 1950s. Section 3 covers the legal catalyst of 1962–1969. Section 4 sets out the formal model that all later work shares. Sections 5 to 8 examine the four main technical lines: Vickrey's procedural proposal, the Weaver–Hess location–allocation model, Nagel's local search and Garfinkel and Nemhauser's exact method. Section 9 surveys the 1970s. Section 10 explains why fully automated districting had stalled by 1980, and what computers were actually doing when the 1981 round began.

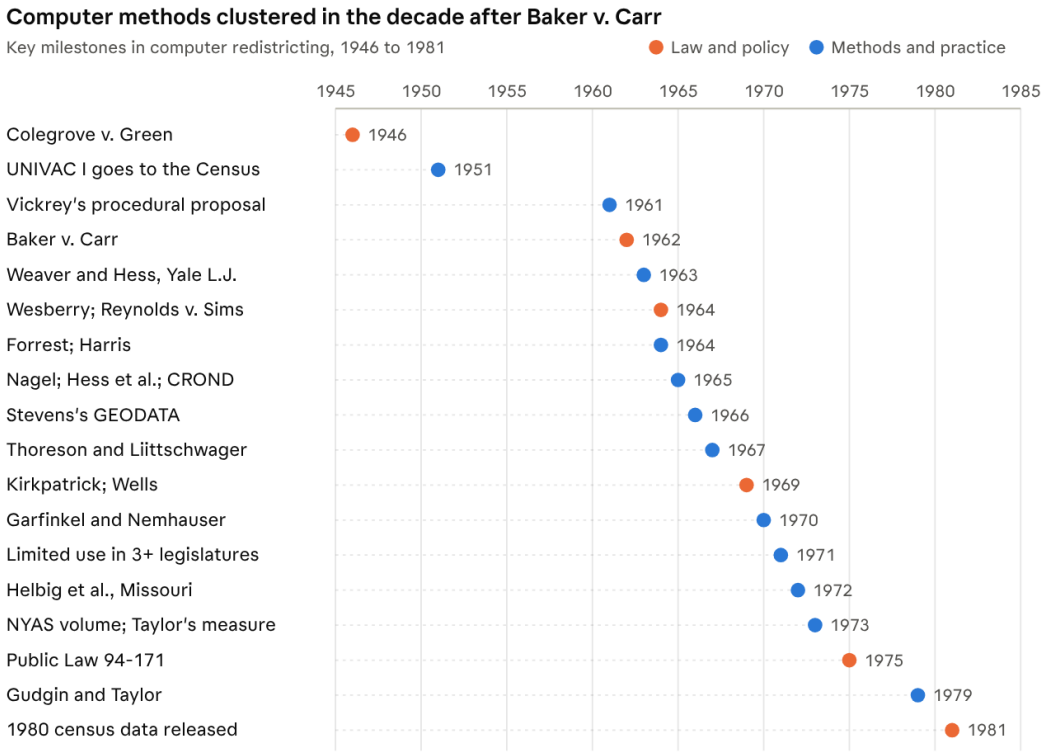

Figure 1. Apart from Vickrey's 1961 proposal, the main computer methods appeared in the decade after Baker v. Carr, as the courts tightened the equality standard. Use in practice lagged until the 1970s, and even then it was limited.

1.1 Then and now: a hard problem

The obstacle the pioneers kept running into now has a precise name. Partitioning a graph into a fixed number of connected parts of equal size is NP-complete, even for planar graphs (Dyer and Frieze 1985). Standard formulations of automated redistricting inherit that hardness (Altman 1997). Exhaustive search therefore fails for a structural reason, not only because the computers of the 1960s were slow.

Current research responds in two ways. The first improves optimization. Validi, Buchanan and Lykhovyd (2022) added exact contiguity constraints to the Hess et al. (1965) model, found that this did not make it harder to solve, and computed optimally compact tract-level plans for 21 states. The second changes the question. Instead of searching for one best map, researchers sample large ensembles of valid maps and ask whether an enacted plan is an outlier among them (Chikina, Frieze and Pegden 2017; Herschlag et al. 2020; McCartan and Imai 2023).

Moon Duchin and the MGGG Redistricting Lab have been central to this second approach. Their Recombination (ReCom) Markov chain merges two adjacent districts and splits them again along a random spanning tree. It mixes far more efficiently than "flip" chains, which reassign one unit at a time (DeFord, Duchin and Solomon 2021). Sampling has a complexity barrier of its own: drawing connected partitions uniformly at random is intractable in the worst case, even for the planar graphs that arise in redistricting (Najt, DeFord and Solomon 2019). This paper traces where these ideas began, and Section 11 returns to them.

2. Prelude: the 1950s

The secondary literature records no proposal for computer districting before 1961. The 1950s matter because they supplied four of its preconditions: a problem, a habit of measurement, the machines and the mathematics.

2.1 The problem: malapportionment

By mid-century many state legislatures had not redrawn their districts in decades. Rural-to-urban migration had left wide population gaps. Tennessee had not reapportioned its General Assembly since 1901. Moore County, with 2,340 residents, sent two representatives, as did Rutherford County with 25,316 (Steingart 2022). Congressional districts within states also varied widely in size.

The federal courts declined to intervene. In *Colegrove v. Green* (1946), Justice Frankfurter warned courts away from the "political thicket" of apportionment. Through the 1950s, malapportionment was a political grievance without a legal remedy. No one needed an algorithm to produce equal districts, because no court required them.

2.2 The habit of measurement

Political scientists nonetheless began to quantify the problem. Dauer and Kelsay (1955) ranked state legislatures by the smallest share of the population that could, in principle, elect a majority of a chamber. Indices like this turned "unfairness" into a number that could be compared across states. That move set up the later idea that a district plan could be scored.

Work on district shape also predates computers. In their 1963 review, Weaver and Hess cite several pre-computer proposals for measuring compactness. These include Kallenbach's ratio of east–west to north–south extent, and a minimum-variance-of-population criterion proposed by Pray and Maner and put to an unsuccessful referendum in Oklahoma. Reock's (1961) circle-based measure appeared the same year as Vickrey's paper.

One mathematical question had already been settled: apportionment among the states. The method of equal proportions (Huntington–Hill) had been fixed in federal law in 1941. It allocates seats as divisor-method rounding, with no geography involved. Weaver and Hess later used the same method to divide Delaware's 35 House seats among its three counties (24, 5 and 6). Districting, by contrast, had no accepted method at all.

2.3 The machines

The Census Bureau took delivery of UNIVAC I in 1951. Over the decade, digital computers proved themselves in data processing and scientific calculation, though the market remained small (Steingart 2022). By the early 1960s, large firms and universities had machines such as the IBM 7090. Nagel used one at Stanford in 1964–65. Census counts for small units were becoming available on machine-readable media, which made district-level tabulation practical.

2.4 The mathematics: operations research

The most important inheritance was operations research. It grew out of wartime logistics, and the Operations Research Society of America was founded in 1952. Linear programming, the simplex method and the transportation problem were standard industrial tools by the late 1950s. The transportation problem is the problem of shipping goods from warehouses to customers at least cost. Firms used it to assign customer orders to depots.

The early computer districters were mostly practitioners of this discipline. Many worked in industry with access to company computers (Steingart 2022). Weaver and Hess came from Atlas Chemical Industries, and their collaborators from DuPont, both in Wilmington, Delaware (Hess et al. 1965). Their key insight was that districting resembles locating warehouses and assigning customers to them. Related "location–allocation" models were being formalized in the same years (Cooper 1963; Hakimi 1964). Lloyd's alternating centroid algorithm, now called k-means, was circulating at Bell Labs from 1957, although it was published only in 1982.

3. The legal catalyst, 1962–1969

Within seven years, the Supreme Court turned population equality from an aspiration into a near-absolute numerical constraint. That shift created both the demand for computer methods and the mathematical form they took.

Year	Case	Holding relevant to computation
1962	*Baker v. Carr*	Malapportionment claims are justiciable; no standard set
1963	*Gray v. Sanders*	Introduces the phrase "one person, one vote"
1964	*Wesberry v. Sanders*	Congressional districts must be equal "as nearly as practicable"
1964	*Reynolds v. Sims*	Both houses of state legislatures must be substantially equal; mathematical exactness not required
1969	*Kirkpatrick v. Preisler*	Missouri congressional plan (about 6% total spread) struck; no de minimis deviation allowed
1969	*Wells v. Rockefeller*	New York plan struck; Harlan's dissent warns that equality does not prevent gerrymandering

Baker opened the courthouse door without saying what a fair plan was. *Wesberry* and *Reynolds* supplied the standard: districts of equal population. For congressional districts, the requirement tightened sharply in *Kirkpatrick*. There the Court required a good-faith effort to achieve precise equality and rejected any fixed tolerance. By the end of the decade, congressional plans were typically held to within about 1%, and state legislative plans to about 10% (Steingart 2022).

This legal history shaped the mathematics in three ways. First, equality became a quantity a computer could check exactly, and a constraint an optimizer could enforce. Second, tight tolerances forced map-drawers onto smaller building blocks. Counties were too coarse, so plans had to be assembled from census tracts, enumeration districts or precincts. Smaller units mean many more units, and the number of possible plans grows explosively with that number (Section 4).

Third, the decisions created a new institutional client. Courts that struck down a plan often had to draw or choose a replacement. Weaver and Hess (1963) wrote explicitly for judges who wanted a remedy that did not require them to make political choices. Hess et al. (1965) reported that federal courts had received a computer plan for Delaware and had asked for computer districts in Connecticut.

Justice Harlan saw the flaw early. Dissenting in *Wells v. Rockefeller*, he argued that strict equality was compatible with severe gerrymandering. A computer, he noted, could generate many equal-population plans with very different political effects (Steingart 2022). The rest of this paper can be read as a test of that observation.

4. Formalizing the districting problem

All the methods in this paper share one underlying model, though no single early author wrote it out in full. Stating it first makes the differences between them clear.

4.1 Units, adjacency and plans

Let the state consist of n indivisible units $V = \{1, \dots, n\}$. Each unit i has a population p_i and a representative point x_i, usually its geographic or population center. Two units are adjacent if they share a boundary. Adjacency defines a graph $G = (V, E)$. Practitioners built this graph by hand as a "touch list" of which units border which (Brace 2011). Let k be the number of seats and $\bar{P}$ the ideal district population:

$$\bar{P} = \frac{1}{k} \sum_{i=1}^{n} p_i$$

A districting plan is a partition of V into k nonempty sets $D_1, \dots, D_k$. Contiguity requires each D_j to induce a connected subgraph of G. Redistricting is therefore a problem of partitioning a graph into connected parts.

4.2 The criteria

Population equality is usually written as a tolerance τ around the ideal:

$$(1-\tau)\,\bar{P} \;\le\; P(D_j) = \sum_{i \in D_j} p_i \;\le\; (1+\tau)\,\bar{P}, \qquad j = 1, \dots, k$$

It can also appear as an objective, such as minimizing the largest deviation $\max_j |P(D_j) - \bar{P}|$.

Compactness had no single definition. The early literature used at least four families of measure:

- **Dispersion (moment of inertia).** Sum, over the district's population, of squared distances to the district's population center. This was the measure of Weaver and Hess (1963).
- **Circumscribing circle.** District area divided by the area of the smallest enclosing circle (Reock 1961).
- **Perimeter.** District perimeter compared with the circumference of a circle of equal area (Schwartzberg 1966).
- **Diameter.** The squared distance between the two farthest units, divided by area. This is the test Garfinkel and Nemhauser (1970) used to screen candidate districts.

The dispersion measure is central to this history and deserves a formula. With c_j the population-weighted center of district j:

$$I(D_j) = \sum_{i \in D_j} p_i \, \lVert x_i - c_j \rVert^2, \qquad c_j = \frac{\sum_{i \in D_j} p_i x_i}{\sum_{i \in D_j} p_i}$$

Other criteria could be added: following county or town lines, keeping communities of interest together, or the partisan balance implied by past vote counts. Most early work excluded partisan data on principle. Nagel (1965) was the conspicuous exception.

4.3 Why exhaustive search fails

The number of ways to partition n units into k nonempty groups is the Stirling number of the second kind, $S(n,k)$. When n is much larger than k, this is roughly $k^n/k!$. The figures grow very quickly:

Units (n)	Districts (k)	Partitions S(n, k)
30	6	about 3.0×10^{20}
50	5	about 7.4×10^{32}

Units (n)	Districts (k)	Partitions S(n, k)
100	10	about 2.8×10^{93}

Contiguity and population limits rule out most of these partitions, but the feasible remainder is still far too large to list for realistic problems. A single state might have hundreds of tracts or thousands of precincts. This is why every practical method in the period was a search procedure that examined a tiny fraction of the plan space. Weaver and Hess (1963) acknowledged that no known technique could produce a single best answer. They expected better integer-programming solvers to arrive within a few years.

That expectation did not account for computational complexity, a theory still being formulated in the early 1970s (Karp 1972). Later analysis showed that standard formulations of automated redistricting are NP-hard. Exact solutions for large instances are therefore not expected from faster computers alone (Altman 1997).

4.4 Three formulations

Early authors posed the problem in one of three ways:

1. **Feasibility.** Find any plan that satisfies contiguity, population and compactness thresholds, by a procedure fixed in advance (Vickrey).
2. **Single-objective optimization.** Minimize one measure, such as total moment of inertia, subject to the population constraint (Weaver and Hess; Garfinkel and Nemhauser, who minimized population deviation subject to compactness).
3. **Weighted multi-criteria optimization.** Minimize a weighted sum of several penalties, with weights chosen by the user (Nagel).

The choice of formulation is a choice of values. Choosing the objective, the tolerance or the weights is already a political decision. That point was made forcefully by critics of the period (Section 10).

5. Vickrey's procedural proposal (1961)

The first call for automated districting came from an economist, not a computer scientist. William Vickrey of Columbia University, a future Nobel laureate, published "On the Prevention of Gerrymandering" in *Political Science Quarterly* in March 1961, a year before *Baker v. Carr*. Steingart (2022) identifies it as the first proposal for automated redistricting. Vickrey described an algorithm in principle but did not run one on a computer.

5.1 The argument: procedural fairness

Vickrey's starting point was skeptical. He saw no agreed criterion of substantive fairness by which a plan could be judged. He therefore sought procedural fairness instead (Becker and Solomon 2020). The districting process should be entirely mechanical, so that once it was set up no human choice remained. The choice of procedure should also be far enough removed from its results that no one could predict, in detail, which plan it would produce. Gerrymandering requires foresight, and Vickrey's idea was to remove the foresight.

The substantive requirements were minimal: equal population and geographic compactness (Altman 1997). Vickrey judged that building such a neutral procedure would be "not at all difficult" (Vickrey 1961, 110). That judgment proved too optimistic.

5.2 The mechanism: growing districts

Later surveys classify Vickrey's procedure as an early "flood fill" method (Becker and Solomon 2020). In this type of algorithm, districts grow outward from seed units:

1. Choose a starting unit by a rule fixed in advance, such as a random draw.
2. Add adjacent unassigned units, preferring those that keep the district compact, until the district reaches its population target.
3. Start the next district from an unassigned unit on the boundary of those already built.
4. If the remaining units cannot form a valid district, discard the attempt and restart.

The output of such a procedure is a random variable: a draw from some probability distribution over valid plans. That reframing is the proposal's lasting contribution. It anticipates modern ensemble and sampling methods, which judge a plan by comparing it with many neutrally generated alternatives.

5.3 Limitations

The procedure has two limitations that later work made explicit. First, it can get stuck. Growing districts one at a time can leave stranded pockets of units that cannot be combined into a district of the right size, so restarts are needed. Second, the distribution of plans it produces is unknown and generally not uniform. The procedure is "neutral" only in the sense that no one chose the result. It is not neutral in the sense of giving every valid plan an equal chance, so a procedure chosen in advance can still systematically favor some kinds of plan.

Vickrey's design was taken up directly in the Midwest. Thoreson and Liittschwager (1967) built a computer simulation that grew districts by attaching adjacent units, and Liittschwager (1973) later applied a Vickrey-style method to Iowa (Altman 1997; Becker and Solomon 2020).

Figure 2 shows a frame from Animation 1, a reconstruction of this growth procedure on a synthetic state. The animation includes a failed attempt and the spread of outcomes across random starts (Appendix A).

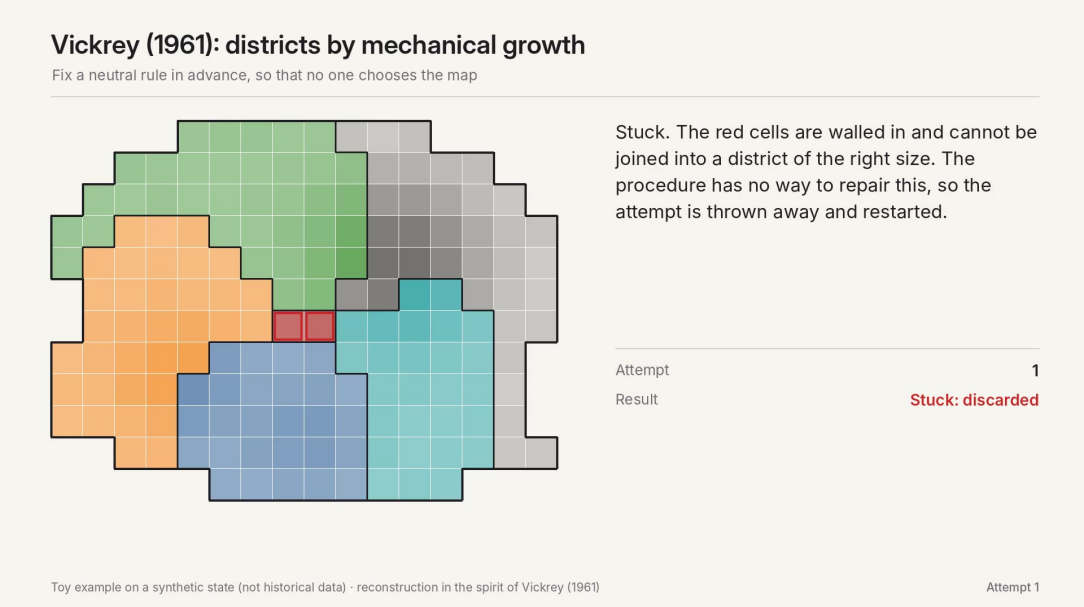

Figure 2. Still from Animation 1, Vickrey-style growth on a synthetic state. The first attempt is stuck: the red cells are walled in and cannot form a district of the right size. Watch the animations.

6. Weaver, Hess and CROND: the location–allocation model

The first working computer method came from Wilmington, Delaware, and it was an application of industrial operations research. James B. Weaver was an engineer at Atlas Chemical Industries and a member of Delaware's civic Committee of 39. Sidney W. Hess was a recent operations research PhD from Case who managed Atlas's operations research section (Steingart 2022). Their "A Procedure for Nonpartisan Districting: Development of Computer Techniques" appeared in the *Yale Law Journal* in 1963. A fuller treatment, written with three DuPont operations researchers, appeared in *Operations Research* in 1965.

6.1 Compactness as moment of inertia

Weaver and Hess defined the compactness of a plan as the sum, over its districts, of each district's moment of inertia about its own population center (Section 4.2). The plan with the smallest total is the most compact. The measure is borrowed from mechanics. Each unit's population acts like a mass, and the measure penalizes population that sits far from the district's center.

The choice has two consequences. Because the measure weights by population, it is not purely geometric. Districts tend to form around concentrations of people, which the authors presented as a benefit for communities of interest. And because distances are squared, elongated districts are penalized heavily.

6.2 The integer program

In modern notation, the model of Hess et al. (1965) chooses k units as district centers and assigns every unit to one center. Let $x_{ij} = 1$ if unit i is assigned to the district centered at unit j, and $x_{jj} = 1$ if unit j is a center. Let d_{ij} be the distance between units i and j, and let $a < 1 < b$ be population tolerances. Then:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2\, p_i\, x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \qquad \text{(each unit in one district)} \\
& \sum_{j=1}^{n} x_{jj} = k \qquad \text{(exactly } k \text{ centers)} \\
& a\,\bar{P}\,x_{jj} \le \sum_{i=1}^{n} p_i\, x_{ij} \le b\,\bar{P}\,x_{jj} \quad \forall j \qquad \text{(population)} \\
& x_{ij} \in \{0,1\}
\end{aligned}
$$

The population constraint also forces units to be assigned only to open centers. Contiguity does not appear at all. Compact solutions tend to be contiguous, but nothing guarantees it, and Weaver and Hess checked contiguity by hand.

6.3 The "warehouse" heuristic

In 1963 no solver could handle this integer program at realistic scale. Weaver and Hess therefore used a heuristic, which the 1965 paper describes as a "warehouse-location" method. Units play the part of customers, district centers the part of warehouses, and squared distance times population the part of freight cost. The procedure alternates two steps:

1. **Choose trial centers** (initial guesses).
2. **Assign.** With the centers fixed, assign units to centers so as to minimize total moment of inertia, subject to exactly equal district populations. This is a transportation problem, which linear programming solves efficiently. The solution may split a few units between districts.
3. **Round.** Give each split unit wholly to the district that holds its largest share. This reintroduces a small population deviation.
4. **Re-center.** Compute each district's actual population center and its moment of inertia.
5. **Repeat** steps 2–4 with the new centers until assignments stop changing.
6. **Restart** from different initial guesses. Discard plans that are worse on both deviation and moment than another plan, then check the remaining plans for contiguity.

The transportation step explains why only a few units are split. A basic optimal solution of a transportation problem with n sources and k destinations has at most $n + k - 1$ positive entries. Every unit needs at least one, so at most $k - 1$ units can be divided. Weaver and Hess cite the corresponding bound from Garvin's linear programming textbook.

6.4 A balanced k-means

In later terminology, this procedure is a population-balanced version of Lloyd's k-means algorithm. With centers fixed, the assignment step is optimal. With assignments fixed, the population centroid minimizes the sum of weighted squared distances. Before rounding, then, each iteration cannot increase the objective. The method converges, but to a local optimum that depends on the starting centers. The rounding step can also break this monotone improvement. Hence

the authors' reliance on many restarts, and their admission that results depend on the initial guesses.

6.5 The Sussex County test

Weaver and Hess tested the method on Sussex County, Delaware, not the whole state. In 1960 the county had 73,195 residents and six House seats, an ideal of 12,199 per district. The building blocks were census enumeration districts averaging about 700 people, ranging from 0 to 2,200. The authors examined about 20 sets of starting centers. The first trial gave a maximum deviation of 5% and a moment of 143,774 (in map units). After re-centering, the best plan reached 1% deviation and a moment of 133,923, about 7% lower. Further iterations produced little or no change.

6.6 CROND and the Gerry Index

To put the method into practice, Weaver, Hess and DuPont colleagues founded Computer Research on Nonpartisan Redistricting, Inc. (CROND). It received a $96,000 Ford Foundation grant through the National Municipal League. CROND drew maps for Delaware's legislature and consulted with Pennsylvania, New York and Nevada (Steingart 2022). Hess et al. (1965) reported that federal courts had received a computer plan for Delaware and asked for computer districts in Connecticut.

The CROND network also produced an early interactive system. Chandler Harrison Stevens, an MIT economist and independent member of the Massachusetts House from 1965 to 1968, built GEODATA in 1966. It was a "man-machine" display system that let users move towns between districts on a screen. Stevens renamed the moment-of-inertia measure the "Gerry Index". He used it to show that the Massachusetts legislative committee's congressional plan was the least compact of the alternatives. His results also showed that plans within 1% population deviation could still differ widely in compactness. In other words, room for traditional gerrymandering remained even under strict equality (Steingart 2022).

Stevens himself called districting an art as well as a science. Weaver later conceded that the CROND technique worked by trial and error (Altman 1997). He also said the parties later turned computers to gerrymandering (Steingart 2022).

Figure 3 shows a frame from Animation 2, which steps through the assign, round and re-centre cycle on a synthetic state. The animation includes a contiguity failure and a restart that converges to a different plan (Appendix A).

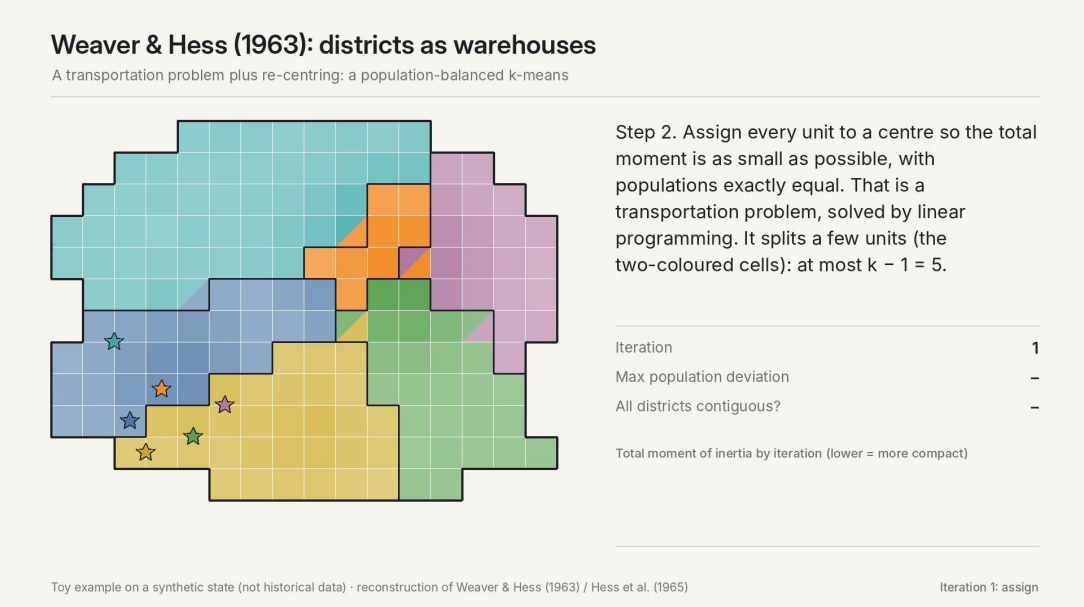

Figure 3. Still from Animation 2, the Weaver–Hess model. In the transportation step, two-coloured cells are units split between districts; there are at most $k - 1 = 5$. Watch the animations.

7. Nagel's local search and the other 1960s programs

The second major method treated districting as improvement rather than construction. It started from an existing map and changed it one step at a time. Its author, the political scientist Stuart S. Nagel, was also the first to say openly that a program could serve partisan goals.

7.1 "Simplified Bipartisan Computer Redistricting"

Nagel earned his PhD at Northwestern in 1961 and taught at the University of Illinois. As a fellow at Stanford's Center for Advanced Study in the Behavioral Sciences in 1964–65, he programmed an IBM 7090. The result appeared in the *Stanford Law Review* in 1965 (Steingart 2022). His program let the user weight three considerations: population equality among districts, contiguity and compactness, and the effect of the plan on the partisan balance of power (ICPSR 7244). Partisan effects were estimated from past voting returns.

The search worked on a starting plan, usually the existing map, using two kinds of change:

- **Move:** shift one unit from its district to an adjacent district.
- **Trade:** swap two units on either side of a shared boundary. A trade can improve compactness while leaving populations nearly unchanged (Becker and Solomon 2020).

A change was kept if it improved the weighted score, and the program stopped when no single move or trade helped.

7.2 The mathematics of local search

In formal terms, Nagel's method is descent on a weighted objective. For a plan $\pi = (D_1, \ldots, D_k)$:

$$F(\pi) = w_1\,\Delta(\pi) + w_2\,C(\pi) + w_3\,B(\pi)$$

Here Δ is a population-deviation penalty, C a compactness penalty and B a partisan term, which can reward either balance or advantage. The neighborhood of a plan is the set of contiguous plans reachable by one move or one trade. The algorithm replaces the current plan with an improving neighbor until none exists.

Three properties follow. First, the result is a local optimum only, and Nagel acknowledged that his algorithm did not guarantee the lowest possible score. He argued that it would be low enough to satisfy political and judicial authorities (Becker and Solomon 2020). Second, the result depends heavily on the starting plan. Starting from the existing map tends to produce plans close to it, which quietly protects incumbents. Third, the weights w are the user's choice, and so is the sign of the partisan term. The same program could produce a bipartisan plan or a gerrymander.

Nagel was candid about this. He criticized Forrest's claim that computers could not gerrymander. He also released his program freely, while Forrest kept his proprietary (Steingart 2022). The program was later archived at ICPSR.

7.3 Nagel's program in practice

The local-search approach had the longest practical life of any 1960s method. In Illinois, Nagel's program was tested but did not move the legislature. Deadlock there led to an at-large election. In California in November 1965, William Below, a staff analyst for the Assembly Committee on Elections and Reapportionment, adapted Nagel's program. Below described it as preserving the political status quo and avoided questions about its criteria (Steingart 2022).

The code also outlived the decade. Jack Moshman of Moshman Associates maintained a COBOL program derived from Nagel's work. In the late 1970s, a team including Kimball Brace used it on Massachusetts's 359 towns. The state was expected to lose a congressional seat after the 1980 census. Nearly 100 runs with varied criteria all pointed to the same district for elimination. The legislature eliminated a different one (Brace 2011). Brace recalls that the program tended to produce snake-like districts. It was more useful for testing human-drawn plans and producing reports than for drawing maps.

7.4 Forrest, Harris and Kaiser: the neutrality claim

Other 1960s authors made stronger claims for neutrality. Edward Forrest, of the early software firm Computer Applications, Inc., published "Apportionment by Computer" in *The American Behavioral Scientist* in 1964. He argued that a computer does not know how to gerrymander, so its maps would be unbiased (Becker and Solomon 2020). His program's details were never published. New York's advisory council on reapportionment hired him in 1965 and concluded that human judgment still did better on some problems (Steingart 2022).

Harris (1964) and Kaiser (1966) argued, in *Behavioral Science* and the *Midwest Journal of Political Science* respectively, that gerrymandering could be eliminated by requiring maximally compact districts (Altman 1997). Their "objective" methods rested on the same assumption as Weaver and Hess: that one geometric criterion could stand in for fairness.

7.5 Iowa: from simulation to statute

The most durable institutional use began in Iowa. John Liittschwager of the University of Iowa College of Engineering and his students developed a sequence of programs. Thoreson and Liittschwager (1967) described districting by computer simulation, growing districts from adjacent units. Gearhart and Liittschwager (1969) described a fuller system, including supporting programs for the entire redistricting process, developed through Iowa subdistricting work. Liittschwager (1973) summarized it as the Iowa Redistricting System.

These programs were tied to the university's computer center. A 1981 memo from the Iowa Legislative Service Bureau notes that the Bureau planned to use the Iowa Redistricting System again. Because the programs could be run only in Iowa City, staff spent considerable time driving there from Des Moines (Iowa Legislative Service Bureau 1981). Iowa's nonpartisan, staff-drawn process remains the clearest case of computational districting surviving into practice.

7.6 Beyond the United States

The problem also attracted operations researchers abroad. Mills (1967) addressed local government electoral boundaries in Britain in the *Operational Research Quarterly*. British geographers later produced the most thorough spatial analysis of the period (Gudgin and Taylor 1979).

Figure 4 shows a frame from Animation 3, which runs this move-and-trade search on a synthetic state and then reruns it with only the partisan weight changed (Appendix A).

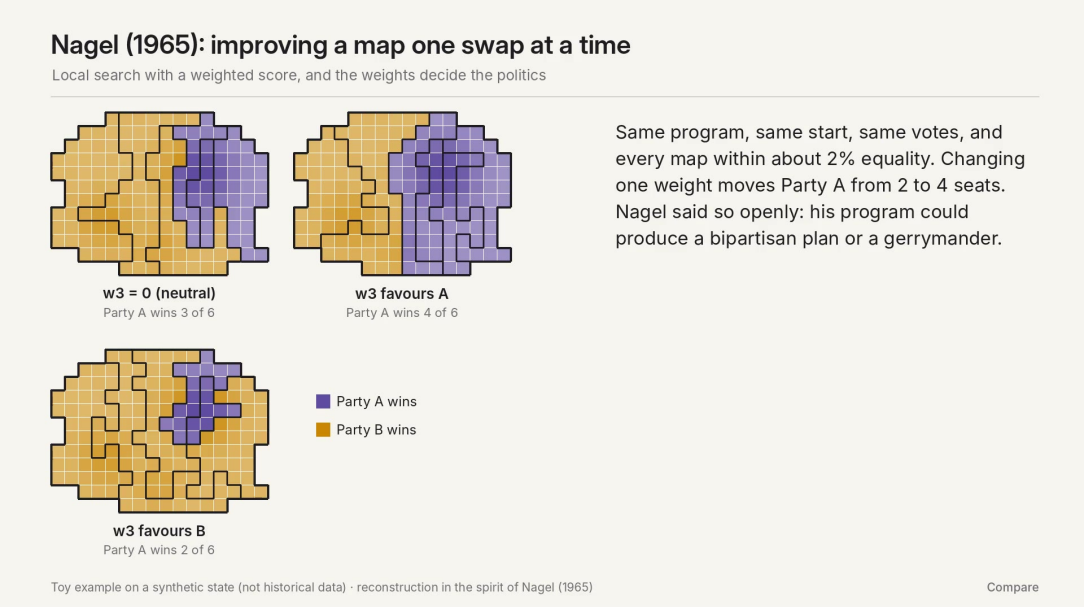

Figure 4. Still from Animation 3, Nagel-style local search. With the same program, starting map and votes, changing one weight gives Party A 3, 4 or 2 of 6 seats. Watch the animations.

8. Exact optimization: Garfinkel and Nemhauser (1970)

By 1970, integer programming had advanced enough for two operations researchers to try what Weaver and Hess had called premature: finding provably optimal plans. Robert S. Garfinkel of the University of Rochester and George L. Nemhauser, then of Cornell and the Center for Operations Research and Econometrics in Louvain, published "Optimal Political Districting by

Implicit Enumeration Techniques" in *Management Science* in April 1970. Their algorithm finds all optimal solutions for a given set of criteria.

8.1 Two phases

The method separates the problem into generation and selection.

Phase 1: enumerate feasible districts. Treat units as indivisible. Generate every subset of units that could serve as a district, meaning it is contiguous, compact and within the allowed population deviation. Compactness was imposed as a screening test based on the district's diameter (the distance between its two farthest units) relative to its area. The enumeration is implicit. A tree search builds districts unit by unit and prunes any branch that can no longer satisfy the tests, so most candidates are never examined.

Phase 2: select a partition. Choose exactly M of the feasible districts so that every unit is covered exactly once. Among those choices, minimize the largest deviation of any district's population from the mean.

8.2 The set-partitioning formulation

Let J index the feasible districts from Phase 1, with $a_{ij} = 1$ if unit i belongs to district j. Let P_j be its population and $\delta_j = |P_j - \bar{P}|$ its deviation. With binary $y_j = 1$ if district j is selected:

$$
\begin{aligned}
\min \quad & z \\
\text{s.t.} \quad & \sum_{j \in J} a_{ij}\, y_j = 1 \quad \forall i \qquad \text{(each unit covered exactly once)} \\
& \sum_{j \in J} y_j = M \\
& z \geq \delta_j\, y_j \quad \forall j \in J \\
& y_j \in \{0, 1\}
\end{aligned}
$$

This is a set-partitioning problem with a bottleneck (min–max) objective. Garfinkel and Nemhauser solved it by a second implicit enumeration over the columns, which yields every optimal plan, not just one.

8.3 What the method showed

The paper was a mathematical success and a practical warning. Its two-phase design reversed the roles of the criteria relative to Weaver and Hess. Compactness became a constraint and population equality the objective, which matches the legal priority set by *Kirkpatrick*. It also made contiguity exact, which the warehouse heuristic never did.

The cost was scale. The number of feasible districts in Phase 1 grows exponentially with the number of units, and Phase 2 is itself a hard combinatorial problem. Exact methods of this period could be applied only to very small instances (Altman 1997). Real states with hundreds or thousands of building blocks were out of reach.

The structure nonetheless proved durable. Generating candidate districts and then choosing a partition of them is the template for later column-generation methods for districting, including

the work of Mehrotra, Johnson and Nemhauser (1998). Other enumeration approaches followed in the 1970s and early 1980s, such as Shepherd and Jenkins's enumeration of "amalgamations", Papayanopoulos (1973), and Rossiter and Johnston's program GROUP (1981). All of them ran into the same combinatorial wall.

Figure 5 shows a frame from Animation 4, which runs both phases of the method on a 5×5 grid and compares how the number of possible plans grows with problem size (Appendix A).

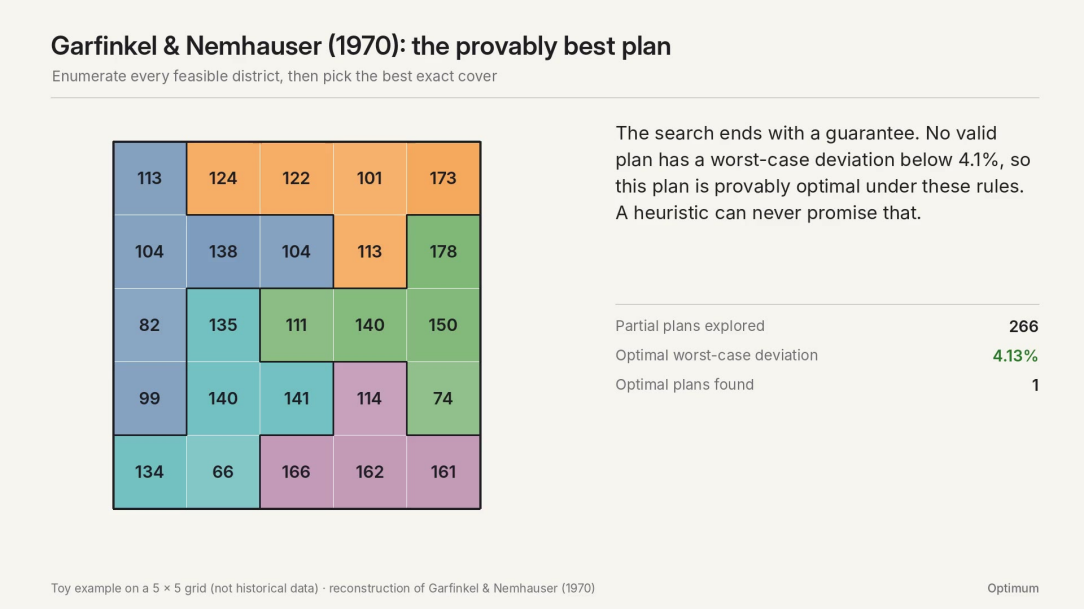

Figure 5. Still from Animation 4, the Garfinkel–Nemhauser method: the provably optimal plan for the 5×5 toy instance. Watch the animations.

9. The 1970s: refinement, measurement and limited practice

The 1970s produced few new paradigms. They brought refinements of the 1960s methods, a large literature on measuring districts, and the first routine but limited use of computers by legislatures.

9.1 The 1971 round

Computer methods met real practice for the first time after the 1970 census. Software capable of automated redistricting was deployed in a limited way in at least three state legislatures in 1971. Because automation proved elusive, these systems were generally used only to tabulate data (Altman, Mac Donald and McDonald 2005). Most states still worked with paper maps and adding machines, and districts tended to follow counties and townships (Brace 2011).

The idea still had public appeal. On November 17, 1970, the *St. Louis Post-Dispatch* urged that Missouri's redistricting be done by computer rather than by partisan bargaining. Students at Washington University in St. Louis took up the challenge (Helbig, Orr and Roediger 1972). Their paper in *Communications of the ACM* considered only nonpartisan criteria: population equality, compactness, contiguity and preservation of natural or political boundaries. Using 1970 census data, they reported plans for Missouri's ten congressional districts and the seven St. Louis County Council seats. They stressed that computer redistricting involved much more

than simply feeding population figures into a machine.

9.2 New algorithms

Much of the decade's technical work appeared in a 1973 volume of the *Annals of the New York Academy of Sciences* on quantitative methods for representation and apportionment, edited by Papayanopoulos. The methods covered most of the design space later surveys recognize (Altman 1997):

- **Block-trading hill climbing** (Moshman and Kokiko 1973), extending Nagel's move-and-trade search.
- **Clustering** (Bodin 1973), which builds districts by adding tracts iteratively.
- **Vickrey-style growth applied to Iowa** (Liittschwager 1973).
- **Cartographic transformation** (Tobler 1973), which remaps a state so that area is proportional to population. Districting the distorted map with simple geometric rules then yields equal-population districts.
- **Enumeration and review** (Papayanopoulos 1973).

The Rose Institute at Claremont described a tract-adding method in 1980 (Altman 1997).

9.3 Measuring compactness

As automatic drawing stalled, attention shifted to measuring plans drawn by people. Compactness measures multiplied. Reock (1961) compared a district's area with its smallest enclosing circle. Schwartzberg (1966) compared perimeters. Taylor (1973) proposed a shape measure in the *American Political Science Review* based on the indentations of a district's boundary. Each captures a different aspect of irregularity, and no consensus emerged. This was a quiet blow to the 1960s project. If experts could not agree on what compactness meant, maximizing "the" compactness could not settle disputes.

9.4 Geography and partisan outcomes

The most important conceptual advance of the decade came from British geographers. In *Seats, Votes, and the Spatial Organisation of Elections*, Gudgin and Taylor (1979) framed districting explicitly as a set-partitioning problem (Altman 1997). They studied how the division of space into districts shapes how votes turn into seats. Their analysis showed that the seat outcome depends on the districting even when every plan obeys the same neutral rules. Districting has partisan consequences whether or not anyone intends them. This confirmed Harlan's intuition with spatial analysis. It also pointed toward the later practice of judging a plan against a set of alternatives.

9.5 The legal backdrop

The Supreme Court relaxed equality for state legislatures in the 1970s. In cases such as *Mahan v. Howell* and *Gaffney v. Cummings* (both 1973), it allowed modest deviations, leading to a rule of thumb of about 10%. For congressional districts, the strict *Kirkpatrick* standard held. The Court reaffirmed it in *Karcher v. Daggett* (1983), which struck down a New Jersey plan with

a total deviation of under 1%. Congressional districting thus remained the domain in which precise arithmetic mattered most, and the one in which computer tabulation was most valuable.

One statutory change mattered for computing. Public Law 94-171 (1975) set up a voluntary program under which states could supply their own geography, such as precincts, to the Census Bureau for tabulation (Brace 2011). This prepared the way for the 1980 redistricting data.

10. Why automated districting stalled, and the landscape in 1980

By 1980, no state had handed its districting to a neutral algorithm. Iowa came closest, with nonpartisan staff working with university-built software. The four approaches had complementary strengths and a shared set of weaknesses.

Approach	Key work	Objective	Contiguity	Optimality	Main weakness
Procedural growth	Vickrey 1961; Thoreson and Liittschwager 1967	None; feasibility by a fixed rule	Built in	None claimed	Can get stuck; unknown distribution over plans
Location–allocation	Weaver and Hess 1963; Hess et al. 1965	Minimize total moment of inertia	Not enforced; checked by hand	Local optimum; depends on seeds	Splits units; compactness stands in for fairness
Local search	Nagel 1965; Moshman and Kokiko 1973	Weighted sum of population, compactness, partisan terms	Enforced by moves	Local optimum; depends on starting plan	Weights are political choices; snake-like districts
Exact enumeration	Garfinkel and Nemhauser 1970	Minimize maximum population deviation	Exact	Global, all optima	Only very small instances

10.1 Five reasons

1. Combinatorics. The plan space is astronomically large (Section 4.3). Exact methods worked only on toy problems. Heuristics returned local optima that depended on arbitrary starting points. Every method that reached real scale was, as Weaver admitted of CROND's, a form of trial and error.

2. The criteria are values. Each method needed an objective, a tolerance or a set of weights. Critics argued from the start that these choices embed political values (Dixon 1968; Altman 1997). Alfred de Grazia (1963) went further. He attacked the equal-population doctrine itself as a "number obsession" that treated citizens as interchangeable. To make the point, he sketched an algorithm that could generate millions of maps, and argued that choosing among them exposed the problem (Steingart 2022). The proliferation of compactness measures (Section 9.3) showed that even the most "technical" criterion had no agreed definition.

3. Many acceptable plans. Every method produced a set of acceptable plans, not a single answer. Weaver and Hess conceded that some discretion might be needed to choose among them

(Steingart 2022). Stevens showed that plans within 1% of equality could differ widely in shape. Gudgin and Taylor showed that they could differ in partisan outcome. Choosing from the set put politics back into the process, as Harlan had predicted.

4. Institutions. Legislators had little reason to give up control of their own districts. Illinois deadlocked despite Nagel's program, and New York's advisory council concluded that human judgment still beat the machine on some problems. California's staff used Nagel's code to preserve the status quo. In Massachusetts, a hundred computer runs did not decide which seat the legislature cut (Steingart 2022; Brace 2011). Even Ronald Reagan, as governor of California, publicly endorsed computer redistricting with political registration excluded (Goff 1972, quoted in Altman 1997). Endorsements like his did not translate into delegation.

5. Data and infrastructure. There were no digital maps. Adjacency "touch lists" were compiled by hand, and work ran on mainframes using punch cards and keypunch sheets. Personal computers did not arrive until about 1983–84, and geographic information systems were in their infancy (Brace 2011). Redistricting software cost $60,000–75,000 in 1980 (Brace 2011).

10.2 The computer in 1980

What computers actually did on the eve of the 1981 round was tabulate. By 1981 only a handful of states used redistricting computer systems, mainly to tabulate population and race by district (Altman, Mac Donald and McDonald 2005).

The 1980 census made that tabulation newly feasible. Under Public Law 94-171, the Census Bureau asked states to freeze precinct boundaries by April 1977. Fifteen states used the program's first option and 29 an alternative approach. Five contracted for statewide block data: Georgia, Mississippi, New York, Rhode Island and Virginia. Six did not take part: Colorado, Hawaii, Massachusetts, New Mexico, Vermont and Wyoming. The redistricting file was released in February and March 1981, with total population and racial breakdowns but no voting-age population. That came later in 1981 (Brace 2011).

The work was physical. In Illinois, a consulting team for the House minority leader matched about 11,000 precincts to roughly 100,000 census blocks on punch cards. Students spent the summer of 1980 tracing precincts onto 3-by-4-foot census maps. Computer time came from a Springfield bank, usually after 9 p.m., and printouts came back by 4–5 a.m. (Brace 2011). In Michigan, the same firm combined census data with election returns for the Democratic Party.

These tools served map-drawers who had political goals. Steingart (2022) draws the period's irony. Computers made strict numerical equality the central criterion, and equality checked by computer shielded legislators from challenges on other grounds. It also gave them more room to shape districts within the law. A technology first promoted as an antidote to gerrymandering was, by 1981, becoming one of its instruments. The full shift came after this paper's endpoint. Personal computers and GIS spread in the mid-1980s, TIGER files arrived in 1990, and by 1991 almost every state used computers to redistrict (Altman, Mac Donald and McDonald 2005).

11. Conclusion

The first two decades of computer redistricting failed to deliver what their advocates promised: a neutral machine that would make gerrymandering impossible. Yet they built almost all of the

intellectual equipment the field still uses.

Four legacies stand out. First, the formal model: a plan as a partition of a weighted adjacency graph into connected parts, with population equality as a tolerance constraint. Second, Weaver and Hess's moment of inertia, whose balanced k-means logic survives in modern centroid and power-diagram methods. Third, Vickrey's procedural idea, that a fixed random process yields a distribution over plans. It resurfaced decades later in ensemble and Markov chain methods for evaluating maps. Fourth, Garfinkel and Nemhauser's generate-then-select structure, the ancestor of column-generation approaches.

The period's deepest lesson was negative, and its pioneers learned it quickly. An algorithm can enforce criteria but cannot choose them. Choosing an objective, a tolerance, a compactness measure, a set of weights or even a starting map is a political act. Because the plan space is vast, any tractable method returns one of many acceptable plans for reasons that are partly arbitrary. Harlan's 1969 warning, Stevens's compactness results and Gudgin and Taylor's spatial analysis all point to the same conclusion. Population equality, the one criterion the computer could enforce perfectly, left most of the politically relevant freedom intact.

The complexity barrier those pioneers met has not been broken. It has been worked around. Each modern line of research has an ancestor in this period. Exact integer programs that now enforce contiguity at the scale of whole states extend the Hess et al. model (Validi, Buchanan and Lykhovyd 2022). Generate-then-select column generation descends from Garfinkel and Nemhauser (Mehrotra, Johnson and Nemhauser 1998). Single-unit "flip" chains are random versions of Nagel's moves. Recombination and sequential Monte Carlo samplers are disciplined descendants of Vickrey's growth procedure (DeFord, Duchin and Solomon 2021; McCartan and Imai 2023).

What changed is the computer's role. In the 1960s it was offered as the author of a neutral map. Today it more often supplies the baseline: a large set of lawful alternatives against which a human-drawn map is judged. Ensemble analyses have been used as evidence in redistricting litigation (Herschlag et al. 2020). In *Allen v. Milligan* (2023), the Supreme Court affirmed a ruling in which illustrative plans drawn by Duchin for the plaintiffs had been found to perform generally better on compactness, on average, than the state's plan. Sampling is itself provably hard in the worst case (Najt, DeFord and Solomon 2019), so these methods trade proofs of optimality for statistical evidence. That trade began with Vickrey.

By 1980, the computer had settled into the role it would hold through the next decade, as a tabulating and evaluating assistant to people with interests. The ambition of the 1960s did not disappear. It waited for the cheap computing, digital geography and statistical frameworks that made it possible, after 2000, to ask a different question. Not "what is the fair map?" but "how unusual is this map among all the maps that could have been drawn?" The tool for answering it, a neutral random process that generates plans, was first sketched by Vickrey in 1961.

Appendix A. Supplementary animations

Four animations accompany this paper. Each runs a working reconstruction of one method on a synthetic test case, with on-screen commentary. They show how the methods behave. They are not historical maps, and the code is not the original authors' code. All videos are 1080p

MP4. Stills appear as Figures 2–5, and all four videos are available here: Watch the animations (`arcg.is/r80vq2`).

A.1 Vickrey's mechanical growth (Section 5, Figure 2)

A 163-unit invented state with one city and two towns is cut into six districts within ±8% of the ideal population. Districts grow from random seeds by adding the neighbouring unit nearest their centre. The first attempt strands two units and is discarded. On this map only 3 of 60 random starts finish, and each finished run gives a different plan.

Video: Animation 1 (2 min 02 s).

A.2 Weaver and Hess's location–allocation model (Section 6, Figure 3)

From deliberately poor starting centres, the animation alternates the transportation-problem assignment, rounding of split units and re-centring until the assignment stops changing. One iteration produces a district in two pieces, which the method does not detect. A restart from different centres converges to a different local optimum.

Video: Animation 2 (2 min 20 s).

A.3 Nagel's local search (Section 7, Figure 4)

From an inherited map of six equal-area strips (maximum population deviation 74%), moves and trades lower a weighted score to a local optimum within 0.3% equality. Rerunning from the same start with only the partisan weight changed gives Party A 2, 3 or 4 of 6 seats on a 51% statewide vote.

Video: Animation 3 (2 min 36 s).

A.4 Garfinkel and Nemhauser's exact method (Section 8, Figure 5)

On a 5×5 grid, Phase 1 examines 2,799 connected sets of units and keeps 609 feasible districts. Phase 2 runs a branch-and-bound search for an exact cover and finds a unique optimal plan with a worst-case population deviation of 4.13%. A closing panel shows how the number of possible partitions grows with problem size.

Video: Animation 4 (1 min 57 s).

References

Links are given for sources consulted directly in preparing this paper. Works marked "cited in" are described here from the secondary sources named, and their details should be checked against the originals before citation.

Cases

- *Colegrove v. Green*, 328 U.S. 549 (1946)
- *Baker v. Carr*, 369 U.S. 186 (1962)
- *Gray v. Sanders*, 372 U.S. 368 (1963)

- *Wesberry v. Sanders*, 376 U.S. 1 (1964)
- *Reynolds v. Sims*, 377 U.S. 533 (1964)
- *Kirkpatrick v. Preisler*, 394 U.S. 526 (1969)
- *Wells v. Rockefeller*, 394 U.S. 542 (1969)
- *Mahan v. Howell*, 410 U.S. 315 (1973)
- *Gaffney v. Cummings*, 412 U.S. 735 (1973)
- *Karcher v. Daggett*, 462 U.S. 725 (1983)

Primary sources, 1955–1983

- Bodin, L. D. 1973. "A Districting Experiment with a Clustering Algorithm." *Annals of the New York Academy of Sciences* 219. Cited in Altman 1997.
- Cooper, L. 1963. "Location-Allocation Problems." *Operations Research* 11(3): 331–343.
- Dauer, M. J., and R. G. Kelsay. 1955. "Unrepresentative States." *National Municipal Review* 44(11).
- de Grazia, A. 1963. *Apportionment and Representative Government*. New York: Praeger.
- Dixon, R. G. 1968. *Democratic Representation: Reapportionment in Law and Politics*. New York: Oxford University Press.
- Forrest, E. 1964. "Apportionment by Computer." *The American Behavioral Scientist* 7(4). Cited in Steingart 2022.
- Garfinkel, R. S., and G. L. Nemhauser. 1970. "Optimal Political Districting by Implicit Enumeration Techniques." *Management Science* 16(8): B495–B508.
- Garvin, W. W. 1960. *Introduction to Linear Programming*. New York: McGraw-Hill.
- Gearhart, B. C., and J. M. Liittschwager. 1969. "Legislative Districting by Computer." *Behavioral Science* 14(5).
- Goff, 1972, quoted in Altman 1997 (Reagan remark on computer reapportionment).
- Gudgin, G., and P. J. Taylor. 1979. *Seats, Votes, and the Spatial Organisation of Elections*. London: Pion.
- Hakimi, S. L. 1964. "Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph." *Operations Research* 12(3): 450–459.
- Harris, C. C. 1964. "A Scientific Method of Districting." *Behavioral Science* 9(3): 219–225.
- Helbig, R. E., P. K. Orr, and R. R. Roediger. 1972. "Political Redistricting by Computer." *Communications of the ACM* 15(8): 735–741.
- Hess, S. W., J. B. Weaver, H. J. Siegfeldt, J. N. Whelan, and P. A. Zitlau. 1965. "Nonpartisan Political Redistricting by Computer." *Operations Research* 13(6): 998–1006.
- Iowa Legislative Service Bureau. 1981. Memorandum on legislative redistricting.
- Jenkins, M. A., and J. W. Shepherd. 1972. "Decentralizing High School Administration in Detroit: An Evaluation of Alternative Strategies of Political Control." *Economic Geography* 48(1). Cited in Altman 1997 and Becker and Solomon 2020.
- Kaiser, H. F. 1966. "An Objective Method for Establishing Legislative Districts." *Midwest Journal of Political Science* 10(2): 200–213.
- Karp, R. M. 1972. "Reducibility among Combinatorial Problems." In R. E. Miller and J. W. Thatcher (eds.), *Complexity of Computer Computations*. New York: Plenum.
- Liittschwager, J. M. 1973. "The Iowa Redistricting System." *Annals of the New York Academy of Sciences* 219.

- Lloyd, S. P. 1982. "Least Squares Quantization in PCM." *IEEE Transactions on Information Theory* 28(2): 129–137 (Bell Labs memorandum, 1957).
- Mills, G. 1967. "The Determination of Local Government Electoral Boundaries." *Operational Research Quarterly* 18(3): 243–255.
- Moshman, J., and G. Kokiko. 1973. Block-trading redistricting algorithm. *Annals of the New York Academy of Sciences* 219. Cited in Altman 1997.
- Nagel, S. S. 1965. "Simplified Bipartisan Computer Redistricting." *Stanford Law Review* 17: 863–899.
- Nagel, S. S. 1972. "Computers and the Law and Politics of Redistricting." *Polity* 5(1).
- Nagel, S. S. Congressional Redistricting Computer Program (ICPSR 7244).
- Papayanopoulos, L. (ed.). 1973. *Democratic Representation and Apportionment: Quantitative Methods, Measures, and Criteria. Annals of the New York Academy of Sciences* 219.
- Reock, E. C. 1961. "A Note: Measuring Compactness as a Requirement of Legislative Apportionment." *Midwest Journal of Political Science* 5(1): 70–74.
- Rossiter, D. J., and R. J. Johnston. 1981. "Program GROUP: The Identification of All Possible Solutions to a Constituency-Delimitation Problem." *Environment and Planning A* 13. Cited in Becker and Solomon 2020.
- Schwartzberg, J. E. 1966. "Reapportionment, Gerrymanders, and the Notion of Compactness." *Minnesota Law Review* 50: 443–452.
- Taylor, P. J. 1973. "A New Shape Measure for Evaluating Electoral District Patterns." *American Political Science Review* 67(3): 947–950.
- Thoreson, J. D., and J. M. Liittschwager. 1967. "Computers in Behavioral Science: Legislative Districting by Computer Simulation." *Behavioral Science* 12(3): 237–247.
- Tobler, W. R. 1973. "A Continuous Transformation Useful for Districting." *Annals of the New York Academy of Sciences* 219.
- Vickrey, W. 1961. "On the Prevention of Gerrymandering." *Political Science Quarterly* 76(1): 105–110.
- Weaver, J. B., and S. W. Hess. 1963. "A Procedure for Nonpartisan Districting: Development of Computer Techniques." *Yale Law Journal* 73(2): 288–308.

Secondary sources

- Altman, M. 1997. "The Computational Complexity of Automated Redistricting: Is Automation the Answer?" *Rutgers Computer and Technology Law Journal* 23(1): 81–142. Also chapter 5 of Altman, *Districting Principles and Democratic Representation*, PhD diss., Caltech, 1998.
- Altman, M., K. Mac Donald, and M. P. McDonald. 2005. "From Crayons to Computers: The Evolution of Computer Use in Redistricting." *Social Science Computer Review* 23(3).
- Altman, M., K. Mac Donald, and M. P. McDonald. 2005. "Pushbutton Gerrymanders? How Computing Has Changed Redistricting." In T. E. Mann and B. E. Cain (eds.), *Party Lines: Competition, Partisanship, and Congressional Redistricting.* Washington, DC: Brookings.
- Becker, A., and J. Solomon. 2020. "Redistricting Algorithms." arXiv:2011.09504. Published in Duchin and Walch (eds.), *Political Geometry* (2022).

- Brace, K. W. c. 2011. "Technology and Redistricting: A Personal Perspective on the Use of Technology in Redistricting over the Past Thirty Years." Paper prepared for a Brookings Institution conference.
- Mehrotra, A., E. L. Johnson, and G. L. Nemhauser. 1998. "An Optimization Based Heuristic for Political Districting." *Management Science* 44(8): 1100–1114.
- Ricca, F., A. Scozzari, and B. Simeone. 2013. "Political Districting: From Classical Models to Recent Approaches." *Annals of Operations Research* 204: 271–299.
- Steingart, A. 2022. "Law, Computing and Redistricting in the 1960s." In M. Duchin and O. Walch (eds.), *Political Geometry: Rethinking Redistricting in the US with Math, Law, and Everything In Between.* Cham: Birkhäuser.

Recent work on complexity, optimization and ensembles

- *Allen v. Milligan*, 599 U.S. 1 (2023).
- Chikina, M., A. Frieze, and W. Pegden. 2017. "Assessing Significance in a Markov Chain without Mixing." *Proceedings of the National Academy of Sciences* 114(11): 2860–2864.
- DeFord, D., M. Duchin, and J. Solomon. 2021. "Recombination: A Family of Markov Chains for Redistricting." *Harvard Data Science Review* 3(1).
- Duchin, M., and O. Walch (eds.). 2022. *Political Geometry: Rethinking Redistricting in the US with Math, Law, and Everything In Between.* Cham: Birkhäuser.
- Dyer, M. E., and A. M. Frieze. 1985. "On the Complexity of Partitioning Graphs into Connected Subgraphs." *Discrete Applied Mathematics* 10(2): 139–153.
- Herschlag, G., H. S. Kang, J. Luo, C. V. Graves, S. Bangia, R. Ravier, and J. C. Mattingly. 2020. "Quantifying Gerrymandering in North Carolina." *Statistics and Public Policy* 7(1): 30–38.
- McCartan, C., and K. Imai. 2023. "Sequential Monte Carlo for Sampling Balanced and Compact Redistricting Plans." *Annals of Applied Statistics* 17(4): 3300–3323.
- Najt, E., D. DeFord, and J. Solomon. 2019. "Complexity and Geometry of Sampling Connected Graph Partitions." arXiv:1908.08881.
- Validi, H., A. Buchanan, and E. Lykhovyd. 2022. "Imposing Contiguity Constraints in Political Districting Models." *Operations Research* 70(2): 867–892.