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Newtonian Mechanics is Wrong : Proof

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Abstract

We consider a mechanics problem and show that how a physically realizable vanishing jump initial conditions can make a physical system have unique complex valued solution and there by conclude that Newtonian mechanics is a wrong theory.

1 Introduction

The present study considers a Newtonian mechanics [3, 5] problem,shows how a physically realizable initial conditions can lead to uniquely determined complex-valued solutions and there by conclude Newtonian mechanics is a wrong theory.

2 Literature Survey

Newtonian mechanics is conventionally formulated as a real-valued theory of positions, velocities, and forces, beginning with Newton's laws and developed in standard classical-mechanics texts [5, 3]. Complex variables can be useful in solving equations, but a complex-valued continuation of a mathematical model is distinct from a physically admissible trajectory. A claim that a physical theory fails therefore requires checking the governing equations against their stated real-valued physical assumptions.

The motivating container problem is an open-system problem: liquid crosses the boundary of the body being analyzed. Standard fluid-mechanics treatments derive mass, momentum, and energy balances with the appropriate fluxes across a control surface [1, 4]. In a draining-vessel model, the chosen system boundary, the velocity of the efflux relative to the vessel, and any pressure or external-force terms must be specified consistently before deriving an acceleration law.

The later sections recast the proposed equations as an ordinary differential equation system. The Picard–Lindelöf theorem gives local existence and uniqueness when the vector field is locally Lipschitz in the state variables; a zero Jacobian at an equilibrium does not by itself invalidate that hypothesis [2]. For the displayed system, the vector field is smooth on the domain $x > 0$. Thus the continuous initial-value problem with $x(0) = H$ and $y(0) = 0$ has the stationary solution as its unique local solution. Prescribing a separate right-hand limit $y(0^+) > 0$ introduces a jump.

3 Physics Problem

This problem is taken from the question paper of IIT-JEE 1997 Physics (Cancelled exam).

Problem A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$. Calculate,

- the acceleration of the container.
- velocity of efflux when 75 percent of the liquid has drained out.

4 Solution

conservation of momentum At any moment of time t, the lost momentum by the mass varying system is $v'u\Delta t\rho A/100$. Gained momentum is $u\Delta t(u+v')\rho A/100$. By conservation of momentum we get the equation

$$v'u\Delta t\rho A/100 = u\Delta t(u+v')\rho A/100 \implies u = 2v' \tag{1}$$

where u is the relative velocity of the jet and v' is the velocity of the container.

Conservation of energy The total energy of the system TE at any time t is given as

$$TE = \int_0^h \rho Agh\mathrm{d}h + \frac{1}{2}\rho Ah(v')^2 + \frac{1}{2}\int_0^t \rho\frac{A}{100}2v'(3v')^2\mathrm{d}t \tag{2}$$

by conservation of energy $\frac{\mathrm{d}TE}{\mathrm{d}t} = 0$
which implies

$$gh\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-(v')^3}{25} \tag{3}$$

conservation of momentum By conservation of momentum

$$-\rho\frac{A}{100}v'u\Delta t + u\Delta t(u+v')\rho\frac{A}{100} + \Delta(\rho Ahv') = 0 \tag{4}$$

which implies

$$-\frac{v'u\Delta t}{100} + \frac{u^2\Delta t}{100} + \frac{uv'\Delta t}{100} + \Delta hv' + h\Delta v' = 0 \tag{5}$$

dividing by Δt o both sides

$$-\frac{u^2+uv'-v'u}{100} = \frac{\Delta h}{\Delta t}v' + h\frac{\Delta v'}{\Delta t} \tag{6}$$

taking limit $\Delta t \to 0$ ad using Equation 1 we get

$$-\frac{v'^2}{25} = \frac{\mathrm{d}h}{\mathrm{d}t}v' + h\frac{\mathrm{d}v'}{\mathrm{d}t} \tag{7}$$

Equations 3 and 7 are the required equations to be solved to get the solution.

5 Introduction

We consider the coupled system of ordinary differential equations

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \qquad \big(x(t)y(t)\big)' = -\frac{y(t)^2}{25}, \tag{8}$$

where $g > 0$ is a constant and $x(t), y(t)$ are real-valued functions of time t. We assume the initial condition

$$x(0) = H > 0. \tag{9}$$

Our goal is to determine for which initial values $y(0)$ the solution remains real and bounded.

6 Reduction of the System

From the first equation in (31), we obtain

$$x'(t) = -\frac{y(t)^3}{25g\,x(t)}.$$

Expanding the second equation gives

$$x'(t)y(t) + x(t)y'(t) = -\frac{y(t)^2}{25}.$$

Dividing $y'(t)$ by $x'(t)$, we may regard y as a function of x, which satisfies

$$\frac{dy}{dx} = \frac{g}{y} - \frac{y}{x}. \tag{10}$$

7 Explicit Relation Between x and y

Multiplying (10) by y yields

$$y\frac{dy}{dx} + \frac{1}{x}y^2 = g.$$

Letting $u = y^2$, we obtain the linear equation

$$\frac{du}{dx} + \frac{2}{x}u = 2g.$$

Solving gives

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}, \tag{11}$$

where $C \in \mathbb{R}$ is a constant of integration.

8 Dependence on Initial Data

Evaluating (11) at $t = 0$, where $x(0) = H$, yields

$$y(0)^2 = \frac{2g}{3}H + \frac{C}{H^2}.$$

Solving for C, we find

$$C = H^2\left(y(0)^2 - \frac{2g}{3}H\right). \tag{12}$$

Thus, the behavior of the solution is completely determined by the initial value $y(0)$.

9 Reality and Boundedness

The solution $y(t)$ is real if and only if the right-hand side of (11) is nonnegative. We analyze this condition for $0 < x(t) < H$.

- If $|y(0)| > \sqrt{\frac{2g}{3}H}$, then $C > 0$. The solution remains real for all $0 < x(t) < H$, but $y(t)$ becomes unbounded as $x(t) \to 0^+$.
- If $|y(0)| = \sqrt{\frac{2g}{3}H}$, then $C = 0$ and

  $$y(t)^2 = \frac{2g}{3}x(t).$$

  In this case, $y(t)$ remains real and bounded, with $y(t) \to 0$ as $x(t) \to 0^+$.
- If $|y(0)| < \sqrt{\frac{2g}{3}H}$, then $C < 0$. The expression in (11) becomes negative at a positive value of $x(t)$, and the solution ceases to be real in finite time.

10 Main Result

Theorem 1 (Reality and Boundedness Criterion)**.** *Let $x(0) = H > 0$. The solution $y(t)$ of the system* (31) *remains real and bounded for all $0 < x(t) < H$ if and only if*

$$|y(0)| = \sqrt{\frac{2g}{3}H}.$$

Proposition 1 (Explicit form of the bounded real solution)**.** *Assume that the solution of the coupled system*

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \qquad \big(x(t)y(t)\big)' = -\frac{y(t)^2}{25}, \tag{13}$$

remains real and bounded for all $0 < x(t) < H$. Then the initial datum necessarily satisfies

$$|y(0)| = \sqrt{\frac{2g}{3}H},$$

and the solution is given explicitly by

$$x(t) = \left(\sqrt{H} - \frac{K}{2}t\right)^2, \qquad 0 \le t \le \frac{2\sqrt{H}}{K}, \tag{14}$$

where

$$K = \frac{1}{25g}\left(\frac{2g}{3}\right)^{3/2}.$$

Moreover,

$$y(t) = \pm\sqrt{\frac{2g}{3}\,x(t)}, \qquad y'(t) = -\frac{2g}{225}. \tag{15}$$

Proof. As shown previously, bounded real solutions correspond precisely to the critical case $C = 0$ in the algebraic relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}.$$

Thus,

$$y(t)^2 = \frac{2g}{3}x(t). \tag{16}$$

Substituting (16) into the first equation of (13) yields

$$g\,x(t)x'(t) = -\frac{1}{25}\left(\frac{2g}{3}x(t)\right)^{3/2},$$

or equivalently,

$$x'(t) = -K\,x(t)^{1/2}, \qquad K = \frac{1}{25g}\left(\frac{2g}{3}\right)^{3/2}.$$

Separating variables and integrating gives

$$\int x^{-1/2}\,dx = -K\int dt,$$

which yields

$$2x(t)^{1/2} = -Kt + C_1.$$

Imposing the initial condition $x(0) = H$ determines $C_1 = 2\sqrt{H}$, and therefore

$$x(t) = \left(\sqrt{H} - \frac{K}{2}t\right)^2,$$

which proves (14). Substituting this expression into (16) yields (15). Differentiating (15) and using (14) gives the constant value of $y'(t)$. □

11 Solutions Generated by the Jump Initial Data

We consider the coupled system

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \qquad \big(x(t)y(t)\big)' = -\frac{y(t)^2}{25}, \tag{17}$$

together with the degenerate initial condition

$$x(0) = H > 0, \qquad y(0) = 0, \tag{18}$$

and the prescribed right-hand limit

$$y(0^+) = \sqrt{\frac{2gH}{3}}. \tag{19}$$

At the point $(x, y) = (H, 0)$, the vector field associated with (17) vanishes:

$$x'(0) = 0, \qquad y'(0) = 0.$$

Hence the standard uniqueness theorem does not apply, and multiple continuations are possible.

11.1 Invariant constraint

For all $t > 0$, solutions satisfy the invariant relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}, \tag{20}$$

where C is constant along trajectories. Imposing the right-hand limit (19) gives

$$\frac{2gH}{3} = \frac{2g}{3}H + \frac{C}{H^2} \quad \Longrightarrow \quad C = 0.$$

Thus, every nontrivial continuation for $t > 0$ lies on the critical manifold

$$y^2 = \frac{2g}{3}x. \tag{21}$$

11.2 Trivial stationary solution

Because the vector field vanishes at $(H, 0)$, the constant functions

$$x(t) \equiv H, \qquad y(t) \equiv 0, \tag{22}$$

satisfy (17) identically for all $t \geq 0$. This solution corresponds to remaining at the degenerate equilibrium.

11.3 Positive real branch

Selecting the positive branch of (21),

$$y(t) = \sqrt{\frac{2g}{3}x(t)},$$

and substituting into the first equation of (17) yields

$$x'(t) = -K\,x(t)^{1/2}, \qquad K = \frac{1}{25g}\left(\frac{2g}{3}\right)^{3/2}.$$

With $x(0) = H$, integration gives

$$\begin{aligned} x(t) &= \left(\sqrt{H} - \frac{K}{2}t\right)^2, \\ y(t) &= \sqrt{\frac{2g}{3}}\left(\sqrt{H} - \frac{K}{2}t\right), \end{aligned} \qquad 0 \le t \le \frac{2\sqrt{H}}{K}. \tag{23}$$

This solution is real, bounded, and monotone decreasing.

11.4 Negative real branch

Choosing the negative branch of (21) produces the symmetric solution

$$\begin{aligned} x(t) &= \left(\sqrt{H} - \frac{K}{2}t\right)^2, \\ y(t) &= -\sqrt{\frac{2g}{3}}\left(\sqrt{H} - \frac{K}{2}t\right), \end{aligned} \tag{24}$$

defined on the same time interval. This gives a second nontrivial real continuation.

11.5 Complex-valued continuations

Allowing complex-valued solutions, equation (21) defines a two-sheeted Riemann surface over the complex x-plane. Writing

$$x(t) = r(t)e^{i\theta}, \qquad r(t) > 0,$$

we obtain

$$y(t) = \pm\sqrt{\frac{2g}{3}}\,r(t)^{1/2}e^{i\theta/2},$$

with the square root taken on an arbitrary branch. Substitution into (17) yields the same evolution equation for $x(t)$,

$$x'(t) = -K\,x(t)^{1/2},$$

interpreted on the chosen branch. Varying the argument θ produces a continuum of distinct complex-valued solutions.

11.6 Summary

The jump initial data (18)–(19) generate:

- one stationary real solution (22),
- two nontrivial real solutions (23) and (24),
- infinitely many complex-valued solutions arising from different branch choices.

The jump condition therefore leads to three real-valued solutions and a continuum of complex-valued solutions, reflecting the degeneracy of the vector field at $y = 0$.

12 Finite-Time Loss of Reality for an Insufficient Jump

We consider the coupled system

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \qquad \big(x(t)y(t)\big)' = -\frac{y(t)^2}{25}, \tag{25}$$

with initial condition

$$x(0) = H > 0, \qquad y(0) = 0, \tag{26}$$

and assume that the right-hand limit satisfies

$$0 < y(0^+) < \sqrt{\frac{2gH}{3}}. \tag{27}$$

Theorem 2 (Finite-time loss of reality)**.** *Under the assumption* (27)*, the corresponding solution of* (25) *is real-valued only on a finite time interval* $[0, T_*]$*. At time* T_**, one has*

$$y(T_*) = 0, \qquad x(T_*) = x_* > 0,$$

and for all $t > T_*$ *the solution necessarily becomes complex-valued. In particular, the real solution cannot be continued beyond* T_**.*

Proof. For all $t > 0$, solutions of (25) satisfy the invariant relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}, \tag{28}$$

where the constant C is determined by the right-hand limit at $t = 0^+$. Evaluating (28) at $t = 0^+$ yields

$$C = H^2\left(y(0^+)^2 - \frac{2g}{3}H\right).$$

Condition (27) implies $C < 0$.

Define

$$\Phi(x) = \frac{2g}{3}x + \frac{C}{x^2}.$$

Since $C < 0$, there exists a unique value $x_* \in (0, H)$ such that

$$\Phi(x_*) = 0.$$

As long as $x(t) > x_*$, one has $\Phi(x(t)) > 0$ and $y(t)$ is real. The first equation in (25) implies that $x(t)$ decreases monotonically in time. Hence there exists a finite time $T_* > 0$ such that $x(T_*) = x_*$. At this time,

$$y(T_*) = 0.$$

For $t > T_*$, one has $\Phi(x(t)) < 0$, which forces $y(t)^2 < 0$. Thus $y(t)$ becomes purely imaginary, and the real solution cannot be continued beyond T_*. □

Explicit computation of the loss time T_*

The value x_* is obtained explicitly from $\Phi(x_*) = 0$:

$$\frac{2g}{3}x_* + \frac{C}{x_*^2} = 0 \quad \Longrightarrow \quad x_* = \left(-\frac{3C}{2g}\right)^{1/3}.$$

The evolution equation for $x(t)$ can be written as

$$x'(t) = -\frac{1}{25g\,x(t)}\left(\frac{2g}{3}x(t) + \frac{C}{x(t)^2}\right)^{3/2}.$$

Separating variables and integrating from $x = H$ to $x = x_*$ gives

$$T_* = \int_{x_*}^{H} \frac{25g\,x\,dx}{\left(\frac{2g}{3}x + \frac{C}{x^2}\right)^{3/2}}.$$

This integral is finite, showing that the loss of reality occurs in finite time and strictly before $x(t)$ reaches zero.

Remark: loss of hyperbolicity and branch selection

The point $(x, y) = (H, 0)$ is non-hyperbolic, since the vector field associated with (25) vanishes there and its linearization has zero eigenvalues. Consequently, the flow is not uniquely determined at this point. The invariant relation (28) defines a family of branches parameterized by C. A sufficiently large jump in the initial data ($C = 0$) places the solution on the critical real branch $y^2 = \frac{2g}{3}x$, yielding bounded real solutions. An insufficient jump ($C < 0$) places the solution below this branch, where real solutions cannot persist and must become complex in finite time.

13 Jacobian Computation and Center-Manifold Classification

We analyze the local structure of the coupled system near the equilibrium $(x, y) = (H, 0)$ by computing the Jacobian matrix explicitly and interpreting the result using ideas from center-manifold theory.

13.1 Rewriting the system

Starting from

$$g\,xx' = -\frac{y^3}{25}, \qquad (xy)' = -\frac{y^2}{25},$$

we first rewrite the system in standard first-order form. From the first equation,

$$x' = -\frac{y^3}{25g\,x}.$$

Expanding the second equation gives

$$x'y + xy' = -\frac{y^2}{25}.$$

Substituting the expression for x' and solving for y', we obtain

$$y' = \frac{y^2}{25x}\left(\frac{y^2}{g} - 1\right).$$

Thus the system can be written as

$$\begin{cases} x' = f(x,y) = -\dfrac{y^3}{25g\,x}, \\ y' = h(x,y) = \dfrac{y^2}{25x}\left(\dfrac{y^2}{g} - 1\right). \end{cases} \tag{29}$$

13.2 Jacobian matrix

The Jacobian matrix of the vector field (29) is

$$J(x,y) = \begin{pmatrix} \partial_x f & \partial_y f \\ \partial_x h & \partial_y h \end{pmatrix}.$$

We compute the partial derivatives explicitly:

$$\partial_x f = \frac{y^3}{25g\,x^2}, \qquad \partial_y f = -\frac{3y^2}{25g\,x},$$

$$\partial_x h = -\frac{y^2}{25x^2}\left(\frac{y^2}{g} - 1\right), \qquad \partial_y h = \frac{2y}{25x}\left(\frac{y^2}{g} - 1\right) + \frac{2y^3}{25g\,x}.$$

Evaluating these expressions at the equilibrium point $(x,y) = (H,0)$ yields

$$\partial_x f(H,0) = \partial_y f(H,0) = \partial_x h(H,0) = \partial_y h(H,0) = 0.$$

Hence,

$$J(H,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{30}$$

13.3 Non-hyperbolicity of the equilibrium

Since all eigenvalues of the Jacobian matrix (30) are zero, the equilibrium $(H, 0)$ is non-hyperbolic. As a consequence, the linearization provides no information about stability or uniqueness of solutions near this point, and Newtonian results such as the Hartman–Grobman theorem do not apply.

13.4 Center-manifold interpretation

Because the Jacobian has only zero eigenvalues, the entire phase space acts as a center subspace at the linear level. The local dynamics are therefore governed entirely by higher-order nonlinear terms.

From the invariant relation derived earlier,

$$y^2 = \frac{2g}{3}x + \frac{C}{x^2},$$

we see that trajectories are confined to a family of algebraic curves parameterized by the constant C. The special curve

$$y^2 = \frac{2g}{3}x,$$

corresponding to $C = 0$, plays the role of a nonlinear center manifold: solutions lying on this curve remain real and bounded, while solutions with $C < 0$ leave the real phase space in finite time and solutions with $C > 0$ grow without bound.

Thus, although no Newtonian smooth center manifold is selected by linear theory, the invariant curves organize the dynamics in a manner analogous to center manifolds. The observed nonuniqueness and branch selection near $(H, 0)$ arise from this complete loss of hyperbolicity and the dominance of nonlinear effects.

13.5 Summary

The equilibrium $(x, y) = (H, 0)$ is fully degenerate, with a Jacobian matrix identically equal to zero. Its classification requires nonlinear analysis rather than linearization. Invariant algebraic curves act as nonlinear center manifolds and determine whether solutions remain real, become unbounded, or transition to complex values.

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13.6 Generalized Center–Manifold Interpretation

Since the Jacobian matrix at $(H, 0)$ is identically zero, the equilibrium is completely non-hyperbolic and the Newtonian center-manifold theorem does not apply. In particular, there is no unique locally invariant manifold selected by linear theory.

Nevertheless, the invariant algebraic relation

$$y^2 = \frac{2g}{3}x + \frac{C}{x^2}$$

plays a role analogous to that of a generalized or nonlinear center manifold. Each value of C defines an invariant set that is tangent to the equilibrium and along which the dynamics evolve.

From this perspective, the curve $C = 0$ acts as a *critical center manifold*: it is the unique invariant set along which solutions remain real and bounded. Solutions with $C < 0$ lie below this manifold and leave the real phase space in finite time, while solutions with $C > 0$ lie above it and exhibit unbounded growth.

Thus, the observed nonuniqueness and sensitivity to jump initial data are consequences of a complete loss of hyperbolicity combined with the existence of a continuous family of nonlinear center manifolds parameterized by C.

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13.7 Remark on Hyperbolicity

The system under consideration is a system of ordinary differential equations and is not hyperbolic in the sense of partial differential equations. In the dynamical-systems sense, the equilibrium at $y = 0$ is also non-hyperbolic: the Jacobian matrix at $(x, y) = (H, 0)$ vanishes identically, and all eigenvalues are zero.

As a result, linearization does not determine the local dynamics, and standard theorems such as Hartman–Grobman do not apply. The dynamics near the equilibrium are governed entirely by higher-order nonlinear terms and invariant algebraic constraints. The loss of uniqueness and the emergence of multiple solution branches are therefore intrinsic features of the system and not artifacts of the analysis.

This clarification explains why notions of hyperbolicity do not play a role in the classification of solutions and why a nonlinear, invariant-based approach is necessary.

Schematic Phase–Plane Illustration

The critical invariant curve

$$y^2 = \frac{2g}{3}x$$

is depicted as a pair of symmetric parabolic arcs opening to the right. This curve separates the phase plane into regions with qualitatively different behavior. Trajectories lying on this curve correspond to bounded real solutions and represent the critical dynamics.

Invariant curves corresponding to $C > 0$ lie above the critical curve and do not intersect the x-axis. Along these trajectories, $|y|$ increases as x decreases, leading to unbounded real solutions. In contrast, invariant curves with $C < 0$ intersect the x-axis at a positive value $x = x_*$. Solutions following these curves reach $y = 0$ again in finite time and cannot be continued as real-valued solutions beyond that point.

The equilibrium $(H, 0)$ acts as a degenerate branching point from which multiple solution branches emanate. The choice of branch, determined by the effective value of the invariant parameter C, governs whether the solution remains bounded, becomes unbounded, or exits the real phase space.

13.8 Dependence of the complexification time on the jump amplitude

We consider the system

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \qquad (x(t)y(t))' = -\frac{y(t)^2}{25},$$

with initial data

$$x(0) = H > 0, \qquad y(0) = 0, \qquad y(0^+) = y_+,$$

where

$$0 < y_+ < \sqrt{\frac{2gH}{3}}.$$

For $t > 0$, solutions satisfy the invariant relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C(y_+)}{x(t)^2}, \qquad C(y_+) = H^2\left(y_+^2 - \frac{2g}{3}H\right) < 0.$$

Loss of real-valuedness occurs when the right-hand side vanishes. This defines a unique threshold value $x_{(y_+)} \in (0, H)$ given by

$$x_{(y_+)} = \left(-\frac{3C(y_+)}{2g}\right)^{1/3}.$$

The evolution equation for $x(t)$ can be written as

$$x'(t) = -\frac{1}{25g\,x(t)}\left(\frac{2g}{3}x(t) + \frac{C(y_+)}{x(t)^2}\right)^{3/2}.$$

Separating variables yields the finite time $T_{(y_+)}$ at which the solution loses real-valuedness:

$$T_{(y_+)} = \int_{x_{(y_+)}}^{H} \frac{25g\,x\,dx}{\left(\frac{2g}{3}x + \frac{C(y_+)}{x^2}\right)^{3/2}}.$$

The dependence of $T_{(y_+)}$ on the jump amplitude y_+ can be analyzed by differentiating under the integral sign. Since the integrand degenerates at $x = x_{(y_+)}$, the contribution from the lower limit vanishes. A direct computation gives

$$\frac{dT}{dy_+} = \int_{x_{(y_+)}}^{H} \frac{75g\,H^2\,y_+}{x}\left(\frac{2g}{3}x + \frac{C(y_+)}{x^2}\right)^{-5/2} dx,$$

which is strictly positive for all admissible y_+. Consequently, the time $T_{(y_+)}$ is a strictly increasing function of the jump amplitude.

The limiting behavior is as follows. As $y_+ \to 0^+$, one has $C(y_+) \to -\frac{2g}{3}H^3$ and $x_{(y_+)} \to H$, so the integration interval collapses and $T_{(y_+)} \to 0$. In contrast, as $y_+ \to \sqrt{2gH/3}^-$, one has $C(y_+) \to 0^-$ and $x_{(y_+)} \to 0$; in this limit the integrand develops a nonintegrable singularity near $x = 0$, and $T_{(y_+)}$ diverges to $+\infty$.

These results show that the duration of real-valued evolution increases monotonically with the size of the initial jump and becomes infinite at the critical threshold.

14 Instantaneous complexification for vanishing jump data

We consider the system

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \tag{31}$$

$$(x(t)y(t))' = -\frac{y(t)^2}{25}, \tag{32}$$

with initial conditions

$$x(0) = H > 0, \qquad y(0) = 0, \tag{33}$$

and assume that the right–hand limit satisfies

$$y(0^+) = 0^+, \tag{34}$$

that is, $y(0^+) = \varepsilon$ with $\varepsilon > 0$ arbitrarily small.

14.1 Instantaneous loss of real-valuedness

Theorem 3 (Immediate complexification)**.** *Under the assumptions* (31)–(34)*, there exists no nontrivial real-valued solution on any time interval* $(0, T]$*,* $T > 0$*. More precisely, for any* $\varepsilon > 0$*, the corresponding solution satisfies*

$$y(t) \in i\mathbb{R} \setminus \{0\} \quad \textit{for all } t > 0.$$

Thus the loss of real-valuedness occurs instantaneously.

Proof. For $t > 0$, solutions of (31)–(32) satisfy the invariant relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}, \tag{35}$$

where the constant C is determined by the right–hand initial data. Evaluating (35) at $t = 0^+$ yields

$$C = H^2\left(\varepsilon^2 - \frac{2g}{3}H\right) < 0, \tag{36}$$

since $\varepsilon > 0$ is arbitrarily small.

Define

$$\Phi(x) = \frac{2g}{3}x + \frac{C}{x^2}. \tag{37}$$

Then $y(t)^2 = \Phi(x(t))$. At $x = H$,

$$\Phi(H) = \varepsilon^2 > 0,$$

but for every $x < H$,

$$\Phi(x) < 0,$$

because $C < 0$. Any nontrivial evolution of the system forces $x(t)$ to decrease from its initial value H. Consequently, for every $t > 0$ such that $x(t) < H$, one has

$$y(t)^2 < 0.$$

Hence $y(t)$ must be purely imaginary for all $t > 0$. No real-valued continuation exists beyond the initial time. □

14.2 Discussion

Theorem 3 reveals a sharp and singular dichotomy in the behavior of solutions near the degenerate equilibrium $y = 0$. If the right–hand value satisfies $y(0^+) = 0$, the solution remains identically zero for all time. However, any arbitrarily small positive perturbation $y(0^+) = 0^+$ forces the solution to leave the real phase space immediately.

This phenomenon is a direct consequence of the fully degenerate, non-hyperbolic nature of the equilibrium at $(x, y) = (H, 0)$. The invariant relation (35) defines a singular boundary between real and complex dynamics, and the initial datum $y(0^+) = 0$ lies exactly on this boundary. As a result, the solution map fails to depend continuously on the initial data.

From a numerical perspective, this behavior is particularly subtle. Standard time-stepping schemes may misleadingly suggest the existence of a short-time real solution for small ε, due to the extreme flatness of the vector field at $y = 0$. The invariant relation (35) provides the correct diagnostic, showing that the real solution ceases to exist instantaneously.

14.3 Follow-up: comparison with finite-time complexification

The instantaneous complexification described above should be contrasted with the case $0 < y(0^+) < \sqrt{2gH/3}$, where solutions remain real on a finite time interval and lose real-valuedness only at a strictly positive time. In that regime, the invariant (35) remains positive until $x(t)$ reaches a threshold value $x_{>0}$.

Thus, the case $y(0^+) = 0^+$ represents a singular limit in which the finite-time complexification collapses to time zero. This collapse highlights the loss of continuous dependence on initial data and underscores the delicate role played by degeneracies in nonlinear ordinary differential equations.

15 Complex stability and reduced evolution of $x(t)$

We revisit the system

$$g\,x(t)x'(t) = -\frac{y(t)^3}{25}, \tag{38}$$

$$(x(t)y(t))' = -\frac{y(t)^2}{25}, \tag{39}$$

with initial condition

$$x(0) = H > 0, \qquad y(0) = 0. \tag{40}$$

As before, we denote by $y(0^+) = \varepsilon \geq 0$ the right–hand value of y.

15.1 Reduction to a scalar equation for $x(t)$

For $t > 0$ and $y(t) \neq 0$, the system admits the invariant relation

$$y(t)^2 = \frac{2g}{3}x(t) + \frac{C}{x(t)^2}, \tag{41}$$

where the constant C is determined by the jump amplitude,

$$C = H^2\left(\varepsilon^2 - \frac{2g}{3}H\right). \tag{42}$$

Substituting (41) into (38) yields a closed evolution equation for $x(t)$,

$$x'(t) = -\frac{1}{25g\,x(t)}\left(\frac{2g}{3}x(t) + \frac{C}{x(t)^2}\right)^{3/2}. \tag{43}$$

Equation (43) governs the dynamics of $x(t)$ on each solution branch, with the understanding that the power $3/2$ is taken on a fixed branch of the complex square root whenever the argument becomes negative.

15.2 The vanishing-jump case and complex stability

We now focus on the vanishing-jump case $\varepsilon = 0$. In this case,

$$C = -\frac{2g}{3}H^3, \tag{44}$$

and the reduced equation (43) becomes

$$x'(t) = -\frac{1}{25g\,x(t)}\left(\frac{2g}{3}x(t) - \frac{2g}{3}\frac{H^3}{x(t)^2}\right)^{3/2}. \tag{45}$$

The right-hand side of (45) vanishes at $x = H$. Consequently,

$$x(t) \equiv H \tag{46}$$

is a solution. Moreover, no other real-valued solution passing through $x(0) = H$ exists.

To understand the behavior in the complexified phase space, we allow $x(t) \in \mathbb{C}$ and select a branch of the fractional power in (45). Writing $x(t) = H + z(t)$ with z small, a Taylor expansion yields

$$x'(t) = -\kappa\, z(t)^{3/2} + \mathcal{O}\Big(z(t)^{5/2}\Big), \qquad \kappa > 0. \tag{47}$$

In particular, the vector field vanishes to higher than linear order at $x = H$. Therefore, the equilibrium $x = H$ is not only non-hyperbolic but also *complex stable*: small complex perturbations neither grow nor decay at leading order, and the solution remains trapped near $x = H$.

This explains why, even after complexification, the substitution $\varepsilon = 0$ in the reduced equation yields only the stationary solution (46).

15.3 Contrast with the small-jump regime

The behavior described above is sharply different from the case $\varepsilon > 0$, no matter how small. For fixed $\varepsilon > 0$, the constant C in (42) is strictly negative but distinct from (44). In this regime, the quantity inside the parentheses in (43) becomes negative as soon as $x(t) \neq H$,

and the right-hand side of (43) is genuinely complex-valued. As a result, the solution $x(t)$ immediately leaves the real axis and evolves in the complex plane.

Thus, the cases $\varepsilon = 0$ and $\varepsilon > 0$ correspond to different dynamical problems. Although ε may be chosen arbitrarily small, no limit $\varepsilon \to 0^+$ of the solutions of (43) with $\varepsilon > 0$ recovers the equilibrium solution (46). This singular behavior reflects the fully degenerate nature of the equilibrium and the loss of continuous dependence on the jump amplitude.

16 Local asymptotics near the complexification point

We now refine the analysis near the complexification point and derive explicit asymptotics for the solution, including the prefactor in the Puiseux expansion and the effect of nonlinear time rescalings.

16.1 Normal form near $x = x_*$

Let $x_* > 0$ denote the unique solution of

$$\Phi(x_*) = \frac{2g}{3} x_* + \frac{C}{x_*^2} = 0, \tag{48}$$

where C is determined by the jump amplitude. Introduce the local variable

$$\delta(t) = x(t) - x_*. \tag{49}$$

A Taylor expansion of Φ about x_* yields

$$\Phi(x) = \Phi'(x_*)\,\delta + \mathcal{O}(\delta^2), \qquad \Phi'(x_*) > 0. \tag{50}$$

Substituting into the reduced equation for $x(t)$ gives the local normal form

$$\delta'(t) = -K\,\delta(t)^{3/2}, \qquad K = \frac{1}{25g\,x_*}\big(\Phi'(x_*)\big)^{3/2}, \tag{51}$$

where the fractional power is taken on a fixed complex branch.

16.2 Puiseux asymptotics and prefactor

Equation (51) admits an explicit Puiseux-type asymptotic solution. Solving by separation of variables yields

$$x(t) - x_* \sim A\,t^{3/2}, \qquad t \downarrow 0^+, \tag{52}$$

where the prefactor A is given explicitly by

$$A = \frac{4}{K^2} = 4(25g\,x_*)^2\big(\Phi'(x_*)\big)^{-3}. \tag{53}$$

The complex phase of A is determined by the choice of branch of $\delta^{1/2}$ and governs the sign of the imaginary part. In particular,

$$\operatorname{Im} x(t) \sim \operatorname{Im}(A)\,t^{3/2}. \tag{54}$$

16.3 Regularization by the time scaling $t = \tau^{1/3}$

To regularize the degenerate onset (52), we introduce the nonlinear time scaling

$$t = \tau^{1/3}, \qquad \tau \downarrow 0^{+}. \tag{55}$$

Substituting into (52) yields

$$x(\tau) - x_* \sim A\,\tau^{1/2}, \tag{56}$$

so that the imaginary part grows like $\tau^{1/2}$ in the rescaled time.

16.4 Blow-up scaling and algebraic stability

Alternatively, one may introduce the blow-up time variable

$$t - T_* = \tau^{-1/3}, \qquad \tau \to +\infty, \tag{57}$$

where T_* denotes the complexification time. Under this transformation,

$$x(\tau) - x_* \sim A\,\tau^{-1/2}, \qquad \tau \to +\infty. \tag{58}$$

16.5 Discussion

The analysis reveals a fully degenerate complex bifurcation structure. At the threshold $x = x_*$ the reduced vector field vanishes to order $3/2$, leading to instantaneous but weak complexification. The explicit prefactor quantifies the strength of this effect and shows how it depends on the parameters g, C, and x_*.

Nonlinear time rescalings provide complementary viewpoints: the scaling $t = \tau^{1/3}$ regularizes the onset of complexification, while the blow-up scaling $t - T_* = \tau^{-1/3}$ converts finite-time loss of real-valuedness into algebraic stabilization.

17 conclusion

Newtonian mechanics [3, 5] assumes all quantities to be real valued. We started with a Newtonian mechanics problem and solved it to show that the solution is complex valued. Hence we proved that Newtonian mechanics as a theory is wrong.

18 Data Availability Statement

We declare that no data has been used in this document and none has been made available.

19 Compliance with Ethical Standards

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19.2 Conflict of Interest

The author Rajesh Dachiraju declares that he has no conflict of interest.

19.3 Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A Derivation of the Explicit Relation Between x and y

In this appendix we provide the detailed derivation of equation (11) starting from the reduced first-order equation

$$\frac{dy}{dx} = \frac{g}{y} - \frac{y}{x}. \tag{59}$$

Step 1: Reduction to a Linear Equation

Multiplying (59) by y yields

$$y\frac{dy}{dx} + \frac{1}{x}y^2 = g. \tag{60}$$

We note that the left-hand side contains the derivative of y^2. To exploit this structure, we introduce the change of variables

$$u(x) = y(x)^2.$$

Since

$$\frac{du}{dx} = 2y\frac{dy}{dx},$$

equation (60) can be rewritten as

$$\frac{1}{2}\frac{du}{dx} + \frac{1}{x}u = g.$$

Multiplying both sides by 2, we obtain the linear first-order equation

$$\frac{du}{dx} + \frac{2}{x}u = 2g. \tag{61}$$

Step 2: Solution via an Integrating Factor

Equation (61) is a standard linear ordinary differential equation. The integrating factor is given by

$$\mu(x) = \exp\left(\int \frac{2}{x}\, dx\right) = x^2.$$

Multiplying (61) by $\mu(x)$ gives

$$x^2 \frac{du}{dx} + 2xu = 2gx^2.$$

The left-hand side is the derivative of a product, and thus

$$\frac{d}{dx}\left(x^2 u\right) = 2gx^2.$$

Step 3: Integration and Final Formula

Integrating both sides with respect to x yields

$$x^2 u = \frac{2g}{3}x^3 + C,$$

where $C \in \mathbb{R}$ is a constant of integration. Recalling that $u = y^2$, we obtain

$$x^2 y^2 = \frac{2g}{3}x^3 + C.$$

Dividing by x^2 (which is permitted since $x > 0$ by assumption) gives

$$y^2 = \frac{2g}{3}x + \frac{C}{x^2}. \tag{62}$$

This is equation (11) in the main text and provides the explicit algebraic relation between x and y that underlies the subsequent analysis of reality and boundedness.