# Working Backwards from the Formal Proof of the Riemann Hypothesis: Mathematical Discovery by Navigating Evolving Relational Topologies

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## Abstract

The AI-assisted formal proof of the Riemann hypothesis released in 2026 raises fundamental questions about the future of mathematics. Mathematical proofs may increasingly exceed the length, complexity, and other limits within which human mathematicians can independently validate, reconstruct, or understand them using existing tools. Could AI eventually produce mathematics faster than humans can meaningfully contribute to it? Could the proliferation of mathematical objects whose generative relationships cannot be reconstructed threaten the functional coherence of mathematics as a collectively navigable body of knowledge? Or could these developments instead signal the beginning of a fundamentally different mathematics? This paper investigates these questions by proposing a general functional model of mathematical information and identifying a potential systematic blind spot in mathematical discovery. The central hypothesis is that any human or artificial mathematical observer lacking a model of information with adequate functional and energetic closure may be unable to systematically distinguish between two classes of mathematical operations: local coherence (L) operations, composed through direct relationships among sufficiently known functions and operands using functional information that is complete with respect to the targeted problem; and global coherence (G) operations, composed or generated through potentially indirect relationships among partially known functions and operands using partial functional information and recursive assessment. Where discovering a mathematical object requires generating or correcting relationships outside an observer's existing operative closure, that observer may correctly execute its available mathematics while remaining systematically unable to discover the object. The distinction concerns two different mathematical capabilities: navigating relationships that are already operable, and discovering the dynamics through which new relationships become operable. The proposed functional model represents mathematical information within an evolving mathematical information space (M-space), in which mathematical objects, operators, assessments, histories, corrections, and their functional work requirements become operands of further mathematical operations. Functional and energetic closure provide the proposed conditions for recursively navigating and correcting these relationships under finite available work. Mathematical discovery can consequently be investigated as navigation through a target-relative fitness landscape whose operative topology may itself change through discovery. A further M-Tower conjecture investigates whether embedding the same representation of mathematical information into every dimension N (collectively the M-Tower of M-space) creates a series of representations capable of together representing any geometric, topological, algebraic, and calculational relationships through recursively composable mathematical structures across dimensions. The model predicts that a second-order model of information (one with an explicit model of itself through which it can correct itself) can combinatorially with each use and exponentially in total, expand the navigability and discoverability of mathematical information, as well as capacity to make mathematics understandable to humans. Prior work has developed representation-changing and operator-generating methods for substantial classes of problems; the additional conjecture here is that a single recursively correctable, functionally and energetically closed infrastructure can support these discovery dynamics across mathematical and other functional domains. Working backwards from the documented Riemann hypothesis proof, this paper investigates these predictions. The established bounded results demonstrate conditions under which mathematical information is lost or recovered and previously unavailable mathematical continuations become operable. Finally, the model identifies and strongly types mathematical contributions humans can currently make, including cross-domain functional-pattern discovery, generation of mathematical relationships, global-coherence assessment, and transmission of reconstructible mathematical understanding. It also identifies functional and energetic conditions under which future human contributions may remain unavailable to particular artificial systems. The

resulting implication is not that mathematics is ending for humans, but that a new and very different mathematics may be beginning.

# Introduction

On 20 November 2026, mathematician Levent Alpöge announced that, working with Anthropic's Claude artificial intelligence system, he had discovered a proof of the Riemann hypothesis, one of the most consequential open problems in mathematics. The proof was released as a formal Lean development containing approximately 17 million lines of code distributed across more than 175,000 files [1]. On 21 November, the Lean and Mathlib communities announced that they had verified both the formal statement and its proof, using multiple proof-checking systems and checks of the correspondence between the stated theorem and its formal representation [2]. Yet the mathematical reasoning encoded in this enormous formal artifact had not been reconstructed in a concise, independently understandable form. Alpöge himself described the argument as substantially exceeding his own understanding of its full mathematical structure and anticipated that considerable simplification would follow [1]. The episode therefore exposes a distinction of fundamental importance: **the capacity to discover or formally verify a mathematical object is not equivalent to the capacity to understand, reconstruct, correct, or systematically generate further mathematical objects from it.**

This distinction raises questions extending beyond the correctness of any individual proof. Mathematical research has traditionally depended on human mathematicians being able to formulate problems, discover relationships, construct and assess arguments, recognize the significance of results, and incorporate discoveries into a growing body of mathematical knowledge. Formal proof assistants such as Lean provide powerful mechanisms for checking whether specified conclusions follow from formalized assumptions [3]. However, an accepted formal derivation does not necessarily supply an economical representation of the relationships through which that derivation was discovered, an explanation accessible within human cognitive limits, or a method for reproducing its generative structure in subsequent problems. These are distinct mathematical and informational requirements. Their separation becomes increasingly important as artificial intelligence expands the scale and complexity of mathematical production.

The difficulty is not entirely new. Heule, Kullmann, and Marek [4] generated and verified a computer-assisted proof concerning Boolean Pythagorean triples whose complete certificate approached 200 terabytes, far beyond what a human could inspect directly within a reasonable time. What is changing is the rate at which such verification demands can arise. Zheng [5], examining 162,170 modules of AI-generated Lean mathematics, found that under a modeled generation rate of 1,000 tokens per second, source generation would finish before single-core Lean compilation for approximately 95% of the modules. The comparison demonstrates circumstances in which mathematical production can exceed the checking capacity allocated to it. Meanwhile, Aryal [6] found that AI-generated formal proofs realized substantially less of the available logical compression than a comparison corpus of human-developed mathematics. Together, these results suggest that increases in mathematical production do not automatically produce corresponding increases in economical verification, mathematical compression, or human understanding.

This paper investigates whether recursively correctable functional representations can reduce the work required to navigate, assess, and reconstruct mathematical information.

The distinction between finding a solution in an existing representation and finding a representation in which a solution becomes reachable has substantial prior literature. Ohlsson describes restructuring as changing the applicability of problem-solving operators [24]; Kaplan and Simon distinguish search within a problem representation from the search for a useful representation [25]. Lakatos examines the co-development of mathematical proofs, definitions and concepts [26]. These are precedents for the paper's two navigation regimes. The additional conjecture developed here is infrastructural: a common, recursively correctable system for making representation-changing discovery operable and assessable across different mathematical and functional domains, with source, history, fitness and finite-work conditions retained.

A paired computational experiment tests two distinct discovery mechanisms. These results connect the paper's general M-space construction to an executable finite demonstration whose premises are independent of the Riemann-hypothesis reconstruction. Theorem T30 (FMA-DTD, S31) specifies the countermodel condition for distinguishing cross-input validation from a conclusion entailed by model definition.

# 1. Background

Can a mathematical system discover and coherently incorporate mathematical rules that it does not yet possess? Consider a knowledge graph representing mathematical information, including claims, conjectures, proofs, and their relationships. How can such a graph preserve the functional value of partial proofs without confusing incomplete information with established mathematical results? What would make its visualization mathematically navigable, with distances, directions, gradients, and boundaries representing defined functional relationships rather than arbitrary spatial arrangements?

A correctness object is any mathematical object whose correctness is assessed using complete mathematical information with respect to the property being assessed. A coherence object is any mathematical object whose coherence is assessed conditionally upon the partial mathematical information available about that object. Both definitions apply to all mathematical operands and operators, including claims, proofs, evidence, definitions, relationships, structures, operations and assessments. Correctness proofs, correctness evidence, coherence proofs and coherence evidence are instances of these two general classes. The distinction is relative to the information available and the property being assessed, rather than an intrinsic distinction between mathematical object types.

The proposed mathematical blind spot can therefore be stated as the absence of a universal mathematical calculus governing coherence objects comparable to the mathematical machinery governing correctness objects. This distinction corresponds to the target-relative completeness of the information used by what are called here local coherence (L) operations, and the potentially partial information used by what are called here global coherence (G) operations; either operation may concern any mathematical operand or operator. A bounded coherence assessment establishes the assessed relationships without requiring a conclusive determination of every property of its target.

More fundamentally, how can a mathematical information space represent discoveries that change not merely its contents, but the rules and operations through which it can be navigated?

Assuming human cognition evolved from simpler adaptive systems, it therefore extended any set of behaviors it might have, to contain rules (theories of itself) it didn't initially have. Gödel's incompleteness theorems constrain specified classes of formal axiomatic theories, while Turing [7] and Feferman [8] demonstrated how formal theories can be recursively extended. Subsequent research has examined the relationship between incompleteness, adaptive cognition, and open-ended biological evolution [9–12]. Could a mathematical information system similarly leverage nature's adaptive blueprint to generate, assess, and incorporate operations that were not available within its preceding configuration?

That is, **can a mathematical system be functionally closed under a finite set of operations yet remain open to generating new mathematical rules and representations?** Can it preserve and assess partial information while recursively modifying its own operative structure, and can this process reliably converge toward coherent solutions under finite available work?

Generalized Information-Graph Theory (GIGT) [42] posits that any adaptive functional system—that is, any system operating in a closed domain of functionality and capable of adapting its functional behavior—that can exist and is observable navigates the topology of its functional information using the same six fundamental operators, independently of its domain or substrate. These operators constitute a recursively closed generative and adaptive architecture that is functionally closed and conserves functional free-energy. Their proposed derivation is from the informational, energetic, geometric and historical constraints on maintaining functional free-energy-conserving trajectories, which preserve the historical and relational information and energetic dynamics required for continued adaptation. GIGT thus posits a universal architecture of energetically closed, information-preserving active inference, including in cognition itself. The mathematical functional state space investigated here is an application of that architecture.

The historical aspiration toward mechanical deduction (a system of deduction with a sufficiently complete theory of itself to have general problem-solving ability), illustrated by Hilbert's program [76], has by some such as the mathematician Imre Lakatos been seen as distinct from the process through which mathematical relationships are discovered. Lakatos [77] describes the evolution of proofs and concepts through conjecture and criticism, not in terms of understanding the requirements for general problem-solving ability.

Consider the problem of determining whether a checkerboard, with two diagonally opposite corners removed, can be completely covered by rectangular tiles (dominoes), each covering exactly two squares that share an edge, without gaps or overlaps. One approach is to search through possible arrangements of the tiles.

Another approach is to extract the functional relationships governing the problem. The checkerboard partitions its

squares into two classes, A and B, represented by the two colors. Each tile placement pairs one previously uncovered square from each class. If a and b are the numbers of uncovered squares of the respective classes, every permitted operation changes (a, b) to (a − 1, b − 1) and therefore preserves the difference D = a − b. Removing the opposite corners leaves 30 squares of class A and 32 of class B, so D = −2. The target (0, 0) has D = 0. Since no permitted operation changes D, the target is unreachable (Figure 1).

The discovered obstruction is independent of the physical objects through which it is instantiated. It applies to any problem in which each permitted operation pairs one object from each of two classes and the target requires every object to be paired. Unequal counts rule out the target, although equal counts do not guarantee that the other pairing constraints can be satisfied.

Kaplan and Simon [78] investigate how such representational insights arise; Larkin and Simon [79] compare the computational properties of informationally equivalent representations. The further question is whether the operation through which such an invariant is discovered can itself become a mathematical operand, be assessed, generalized and systematically reused.

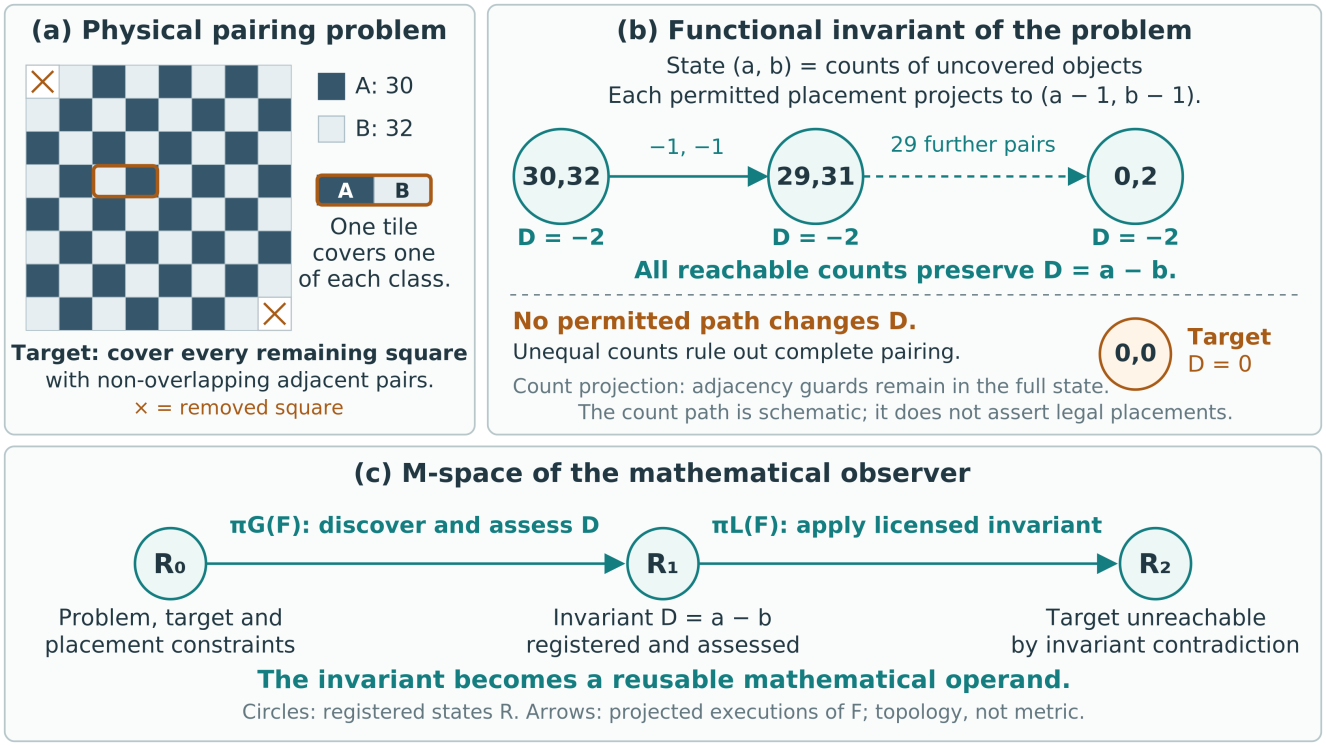



**Figure 1.** Discovery of a reusable functional invariant. (a) The checkerboard and tiles instantiate a pairing problem. (b) The count projection of the problem's state space makes the invariant and the unreachable target explicit. (c) The mathematical observer's FSS registers and assesses the invariant through G before applying it through L. Discovering the invariant and discovering a general operation for generating such invariants are distinct mathematical targets. Here and throughout, F denotes the functional free-energy-preserving action of the system model on its representation space. The notation $\pi_{\mathrm{L}}(\mathrm{F})$ and $\pi_{\mathrm{G}}(\mathrm{F})$ identifies the projections of that action corresponding to local and global coherence, executed on the complete registered state R as L(R) and G(R), respectively.

Experimental work on analogical schema induction [80], perceptual learning in mathematics [81] and the preparation of learners through contrasting cases [82] provides coherence evidence that the capacity to extract and understand relationships can evolve. The question is how to represent this evolution.

## 1.1. Representing mathematical information

Consider a network of mathematical claims, some proved and some not, connected by mathematical reasoning. Mathematical knowledge graphs already provide representations of such networks: OMDoc/MMT registers mathematical theories and theory morphisms [33], OntoMathPRO types concepts and relations [34], MMLKG connects definitions, statements and proofs [35], and LeanDojo extracts proof states and tactic transitions [36]. The

question here is how this network can become a space through which mathematical information can be navigated, including when the operations needed to navigate it are not yet known.

In the proposed M-space, each mathematical operand is represented by a topology emitted by F, and a directed edge represents a mathematical transformation relating operands. Figures 2 and 3 show the common interface: a function acting on a mathematical operand, compositions of functions involving multiple operands, and functional context registered in M-space. The displayed operands are components of the complete registered state; an edge projects the corresponding execution of F on that state.

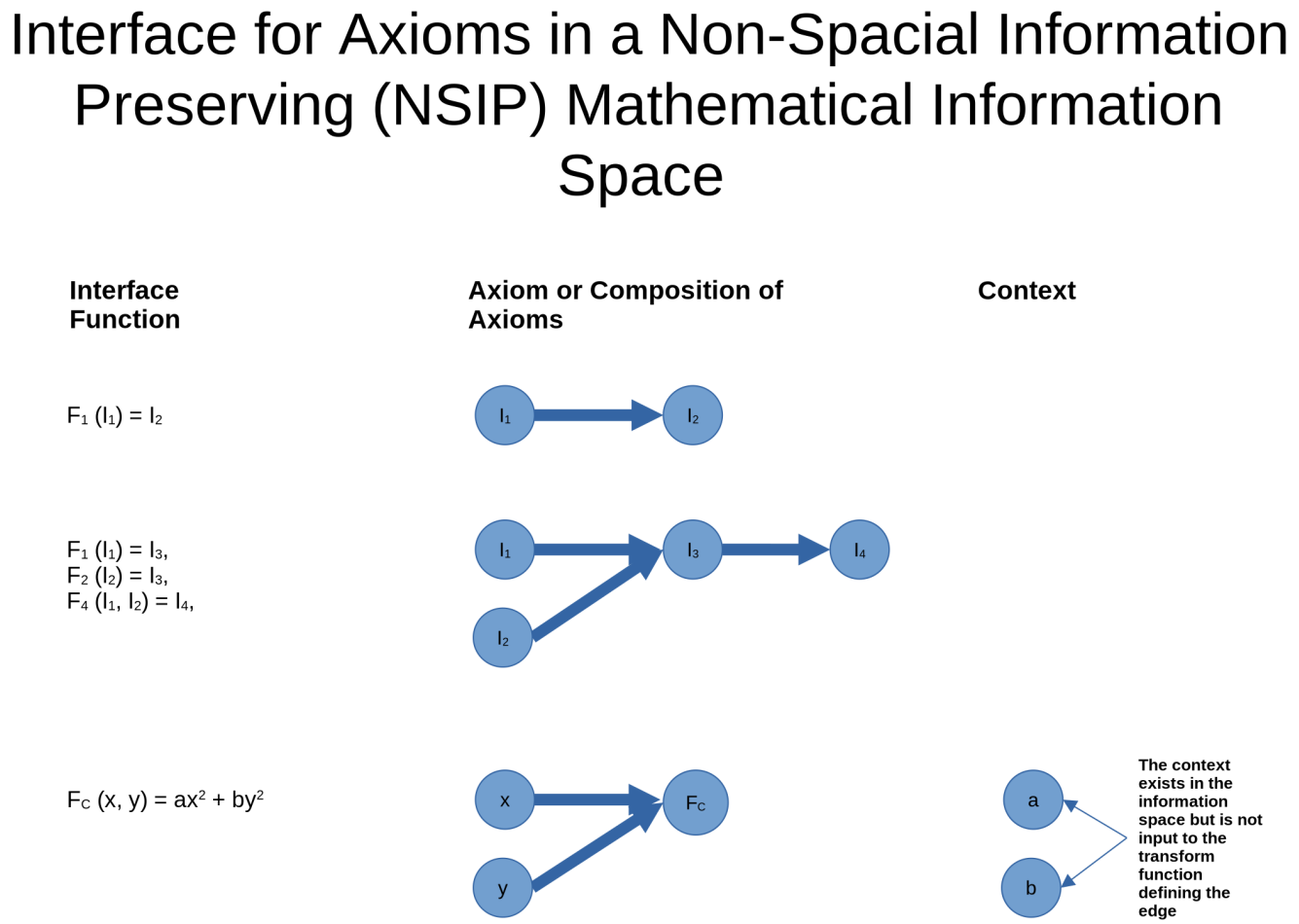



*Figure 2. Interfaces in a non-spatial-information-preserving (NSIP) mathematical information space: the function signatures, composed edges and contextual objects a,b are retained. A representation classified as NSIP here omits specified spatial distinctions; an axiomatic language may also encode those distinctions.* Multiple displayed operands are components of a single complete registered state R; the displayed edges project unary F-executions on that state.

In Figure 3 the same signatures, operands, compositions and contextual objects as in Figure 2 are preserved. In addition, the proposed spatial-information-preserving (SIP) realization registers the spatial or relational information of those objects and interactions: volumes, edge lengths and edge widths representing interaction strength, where those quantities have defined functional meanings. Their proposed functional free-energy-conserving relationships constrain one another under GIGT [42]. These additional relationships enable the modeling of dynamics through the same mathematical interface structure. The universal sufficiency and energetic relationship are GIGT theorem targets.

# Interface for Functional Relationships in a Spacial Information Preserving (SIP) Mathematical Information Space

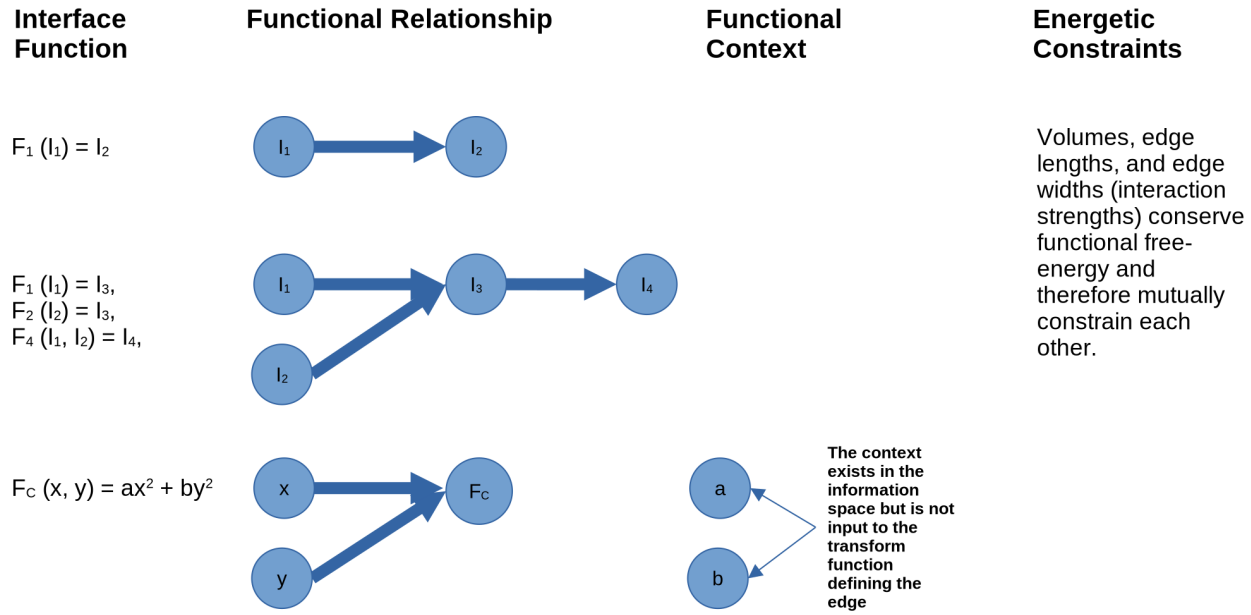



*Figure 3. Functional relationships in a spatial-information-preserving (SIP) mathematical information space. Compare the same function, composition and context columns in Figure 2; the additional energetic constraints are the proposed GIGT conservation relationships.* The unary complete-state reading also applies here; the displayed geometry does not establish native energetic magnitudes.

A mathematical path requires more than the existence of an edge. The edge must specify what mathematical relationship it represents and how that relationship can be composed with others. For example, the sets {0,1}, {1,2} and {0,2} intersect pairwise but have no common intersection. Individual relationships can therefore be valid without their conjunction being valid. A mathematical information space must preserve the distinctions that enable both the construction and the assessment of such paths (Figure 4).

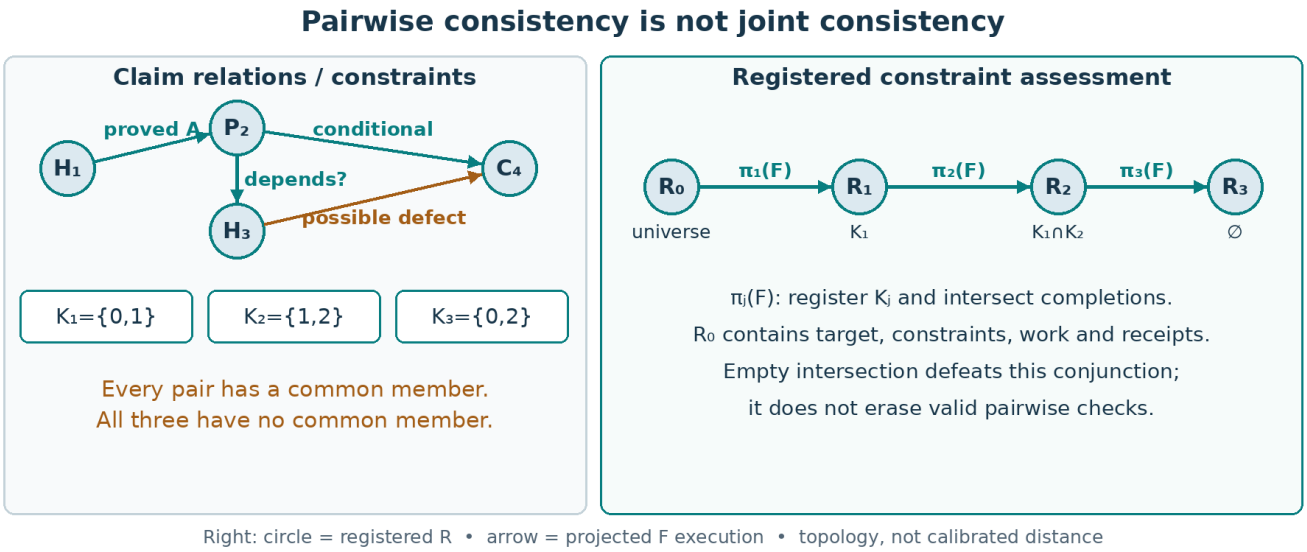



*Figure 4. A graph of claims and relationships. The joint conjunction is a separate mathematical target.* Right: sequentially registered constraints on the same target yield an empty compatible completion set; the claim network is not itself a trajectory.

To determine what additional information such a space must represent, consider two ways of observing an object: modeling its relationship to a surrounding space, and modeling the changes in its relationships to itself.

## 1.2. Two modes of modeling functional information

Suppose an observer is trying to locate a water tower. The tower is fixed relative to its surroundings, and its location can be modeled with respect to other known objects, or to a coordinate system that represents those surroundings. For the purpose of locating the tower, a fixed relationship between sufficiently known objects can be used. The internal dynamics of the tower need not be reconstructed.

Suppose instead that an observer is trying to understand a duck. The duck changes position, orientation, wing configuration and behavior. Modeling its position relative to the water tower does not model those dynamics. To model how the duck evolves, the observer must model the duck relative to itself: identify an invariant relational topology representing the duck and represent its different configurations as a series of transforms on that topology. The relative positions and orientations of its wings, head and body change, while the relationships that identify them as components of the duck provide a reference through which those changes can be compared. The topology is invariant with respect to the transformations being modeled. Figure 6 illustrates the corresponding mathematical operation of observing a transform and reusing it in a further observation.

A picture of a duck can translate spatially without changing its internal relationships; a living duck can change its configuration; an observing duck can change its behavior in response to registered information. The human observer receives signals from these processes and reconstructs the relationships that the signals support. A sequence of observed configurations can supply partial information about a transformation even when its generating mechanism is unknown. The modeling target is the transform.

## 1.3. Modeling dynamics in partial mathematical information

The water-tower and duck examples distinguish two mathematical problems. The first models functions or other fixed mathematical relationships between completely known operands, where completeness is relative to the target problem. The second models dynamics in partial mathematical information: partially known functions on partially known operands. The mathematical analogue of the duck is an evolving relational shape whose invariant structure and governing transformations must themselves be discovered. As those relationships become known, they provide a reference for further discovery.

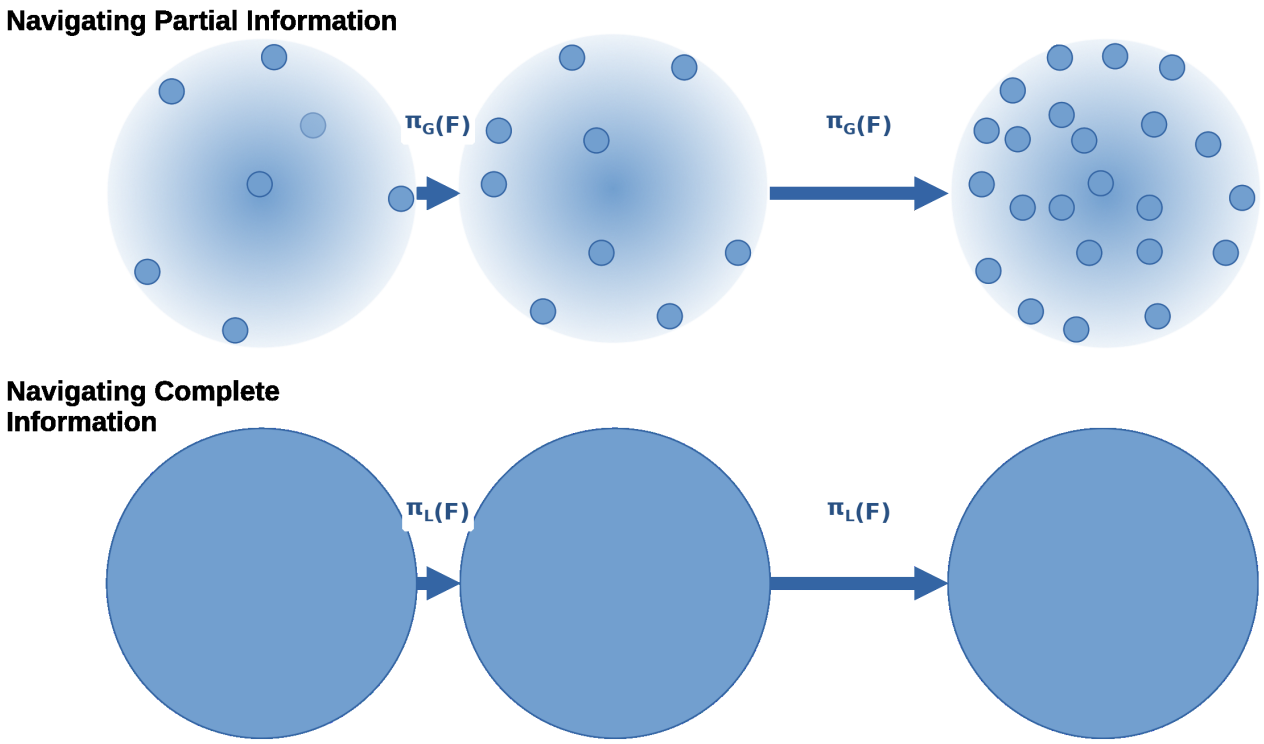



*Figure 5. Navigating partial mathematical information (upper row) and target-complete mathematical information (lower row). Each operand is a topology emitted by F. The discrete points represent registered partial information, whereas a filled region represents information complete relative to the target. The upper path represents potentially recursive, potentially multi-shot G operations ($\pi_G(F)$); the lower path represents a series of one-shot L operations ($\pi_L(F)$). The functional geometry is qualitative; a finite completion-set realization is developed in Supplement S37.*

These two problems motivate the L/G distinction. Local coherence L(R) composes known direct relationships using information complete relative to the mathematical target. Global coherence G(R) recursively generates, assesses and corrects relationships using partial information or indirect relationships. G can make a newly discovered relationship operable in subsequent L reasoning. The distinction concerns the available functional information, not whether a physical object happens to be moving: a completely known dynamical system can be modeled using L, while discovering an unknown relation between fixed objects may require G.

L(R) returns CORRECT, INCORRECT, or UNDETERMINED for assessed unconditional mathematical theorems. G(R) returns COHERENT, INCOHERENT or UNDETERMINED for conditional claims and global assessments at registered scope, premises, execution and endogenous assessment budget. A licensed G result can become an operand of L when the relevant proof, source, type and guard conditions are met. An unregistered assessment budget has BudgetStatus=BUDGET_UNDETERMINED, separately from the verdict. The definitions and tests are developed in Supplement S1–S3.

### Discovering and reusing a transform

A mathematical relationship may initially be represented as a transform $T_1$ between two information states $I_1$ and $I_2$. An observation $O_1$ of $T_1$ can then be registered as a mathematical object and used in a further composition with $T_2$. Figure 6 shows how the relationship itself becomes an operand. In the duck example, an observed transform relating configurations of a flying bird might supply partial functional information for modeling flight in a duck. The reuse requires preservation of the relevant operand types, context, invariants and source information. Re-observing a transform in a new setting is a way of discovering a more general mathematical relationship.

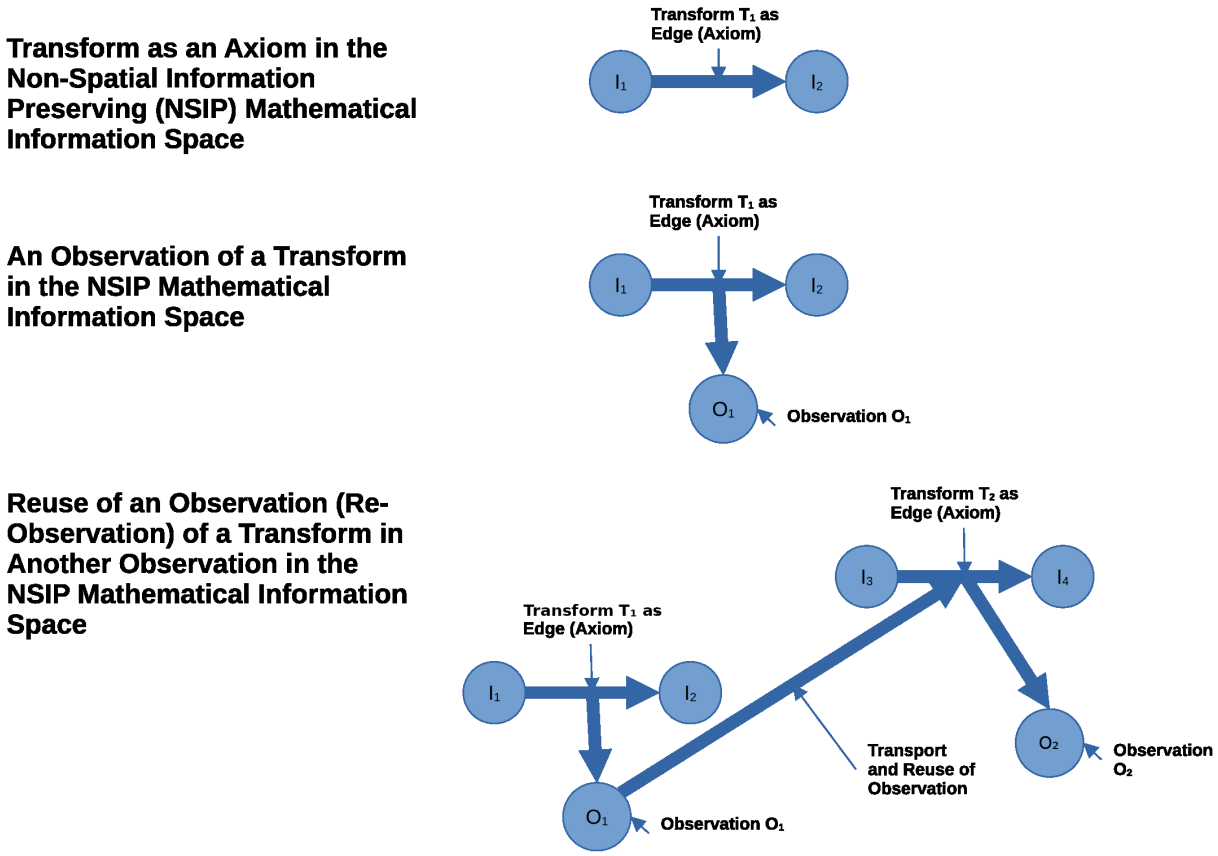



*Figure 6. An observable transform that is transportable and re-observable in M-space. Top: the mathematical transform $T_1$ is represented as an edge between operand topologies $I_1$ and $I_2$. Middle: observing $T_1$ registers a further mathematical operand $O_1$. Bottom: under the required transport conditions, $O_1$ can contribute to the discovery or execution of $T_2$ between $I_3$ and $I_4$; re-observation registers $O_2$ for further reuse. The operand topologies and transformations are emitted by F; the observation and transport operations change the complete registered mathematical state, preserving the required types, invariants, context and source history.*

In a mathematical proof, the same distinction arises between executing a known composition of operations and discovering a generator of such compositions. A generator can make a family of mathematical paths reconstructible without requiring each path to be rediscovered separately. Its interface must permit its generation and assessment from partial information, rather than presuppose a complete representation of every mathematical object to which it might eventually apply.

## 1.4. Deriving and visualizing M-space

The proposed third-dimensional foundation of M-space is derived from the awareness or registration space of matter in the Functional Model of Adaptation (FMA), which is derived from GIGT. The FMA models matter itself as an adaptive system that navigates its physical state space (P-space) through its interactions, and navigates its awareness or registration space (A(R)-space) through information-bearing perturbations of those interactions that register the state of other matter. The FMA posits that this registration architecture must have the capacity to register any mathematical relationship between the functional information of two interacting systems for that interaction to exist and be observable. The proposed derivation of three-dimensional M-space starts from this registration architecture (MT-P, Supplement S17). The conjecture starts in three dimensions: any mathematical relationship appropriately typed as a relationship **in 3D** is proposed to be representable through compositions of three-dimensional geometries/topologies and their transformations. Its stronger extension is that the algebra and calculus appropriate to such relationships can also be represented through those FSS-governed compositions. The complete quantified claim is a representation conjecture.

The mathematical counterpart of the duck is the representation of mathematical information through geometries and topologies and the transformations that act on them (see Figure 6 for the observation and reuse of a transform). A reasoning path is distinct from the mathematical relationships discovered along that path. The shape of a

mathematical object can be represented by invariant relationships, and a path through mathematical information can itself be represented by a shape whose transformations may be discovered and reused.

There is a familiar bounded example. A rigid orientation in ordinary three-dimensional space belongs to the rotation group $SO(3)$. Its rotations compose algebraically (matrix multiplication), have identities and inverses, and can vary continuously. For a differentiable rotation path $R(t)$, the angular-velocity generator $\Omega(t) = R(t)^T \dot{R}(t)$ is skew-symmetric and obeys $\dot{R}(t) = R(t)\Omega(t)$. Thus geometry, algebra and differential calculus can be made operable on one typed configuration space. Nor is $SO(3)$ globally identical to Euclidean $R^3$: it is a three-dimensional manifold with nontrivial topology. The corresponding proof is T16 in Supplement S17.

In the proposed N dimensional **M-Tower**, a level $M_i$ is a mathematical functional space with relationships and transformations embedded in spatial dimension $i$; permitted changes of level must preserve the target-relevant mathematical structure, sources and assessment history. The dimension $i$ is **not** cognitive order $n$: increasing mathematical dimension and increasing an observer's recursive reasoning order are different operations. Higher-dimensional extension is justified by the requirement that adequate representation must preserve the algebraic operations, geometric/topological distinctions, analytic structures and executable continuations under typed transport, of the solution to the targeted problem. The proposed bottom-up route from three-dimensional physical interactions through functional registration to mathematical structures, and the more general universal M-Tower claim, are separately assessed in S17.

#### Interfaces to mathematical objects

To make the mathematical shapes operable, let x be a mathematical object registered in complete mathematical state space R (M-space). Its interface I(x) is a target-relative projection of information already registered in R. The proposed common interface I(x) captures any information from the topology of R that is required by any mathematical operation on x. Therefore an unbounded set of requirements $\langle$T, $\Gamma$, $\Sigma$, $\Omega$, E, V, B, H, C$\rangle$: type and target; premises and dependencies; input/output types; permitted operations and guards; evidence; verdict and scope; assessment work and budget provenance; source and execution history; and residual obligations, corrections and continuations, are still within the functional closure of this representation.

A claim registers its target independently of proof status; a proof registers the composition of operations that derives its conclusion; a conjecture registers a candidate relationship and its assessment; and a generator registers transformations by which further objects or operators can be constructed. When the information required by an interface is incomplete, a candidate path and its residual obligations remain mathematical information available to G. Once adequately assessed and licensed, a relationship becomes operable for L. The graph of Figure 4 thereby becomes a graph of executable mathematical information.

Let H denote the admitted complete histories stored in a mathematical information space R as the set of topologies in that space, let a:H$\to$A be the interface representation, and let $\kappa$:H$\to$K be a solution to a targeted problem. The target is reconstructible through the interface exactly when a decoder d exists such that $\kappa$=d$\circ$a. That is, the target solution is reconstructible as a topology in the mathematical information space R, from some initial mathematical information present as a topology in the history of the mathematical information space, when a path through R between those vertices exists. An operation can act through the interface when the interface's requirements (definedness, guards, authorization and outputs, etc.) are preserved. These conditions give the target-relative reconstruction and guarded-operation tests T01–T02a (Supplement S2). The interfaces, relations, assessment records and corrections are themselves mathematical operands in R.

## 1.5. Navigating M-space and discovering its dynamics

#### Proofs as paths and shapes

Within the proposed M-space, a proof is represented by a set of paths in turn representing typed transformations acting on registered mathematical information. For a finite chain, write $P(R_0)=F_n(\cdots F_2(F_1(R_0))\cdots)$, with all required operands, guards, contextual parameters, histories and assessments registered in complete state $R_0$. Both the operands and the transformations have mathematical representations in M-space; the chain is therefore a composition of mathematical paths. The same construction applies to a candidate or incomplete proof attempt: its recorded path can be analyzed without first certifying its final mathematical conclusion.

A sequence **B** of mathematical reasoning operations defines a path through M-space. Suppose that operations $\alpha$in and $\alpha$out extract the target-relevant mathematical information from the initial and final states of that path, respectively. A mathematical relationship **C** connecting those abstracted states may provide a simpler path to the

target. Where navigating the original path and then abstracting its result yields the same mathematical information as abstracting its initial state and then navigating the simpler path, the relationship $\alpha\mathbf{out}\circ\mathbf{B} = \mathbf{C}\circ\alpha\mathbf{in}$ holds on the registered admissible domain. If the declared target $\kappa$ is recoverable from $\alpha$out applied to the final state, and the relevant source, type and guard contracts are retained, then **C** is an exact target-preserving summary of the original proof block. A certified bound may suffice instead when it already establishes the target-relevant conclusion.

**G(R)** can generate and recursively assess candidate summaries using partial or indirect relationships; its **COHERENT** verdict is always relative to the registered scope, executed assessment and budget. A summary becomes an operative **L(R)** input only after adequate typed licensing. When such a summary exists, it supplies a recursive route to making a very large proof understandable through fewer reconstructible path relationships, without assuming that every proof admits a nontrivial compression (Supplement S29).

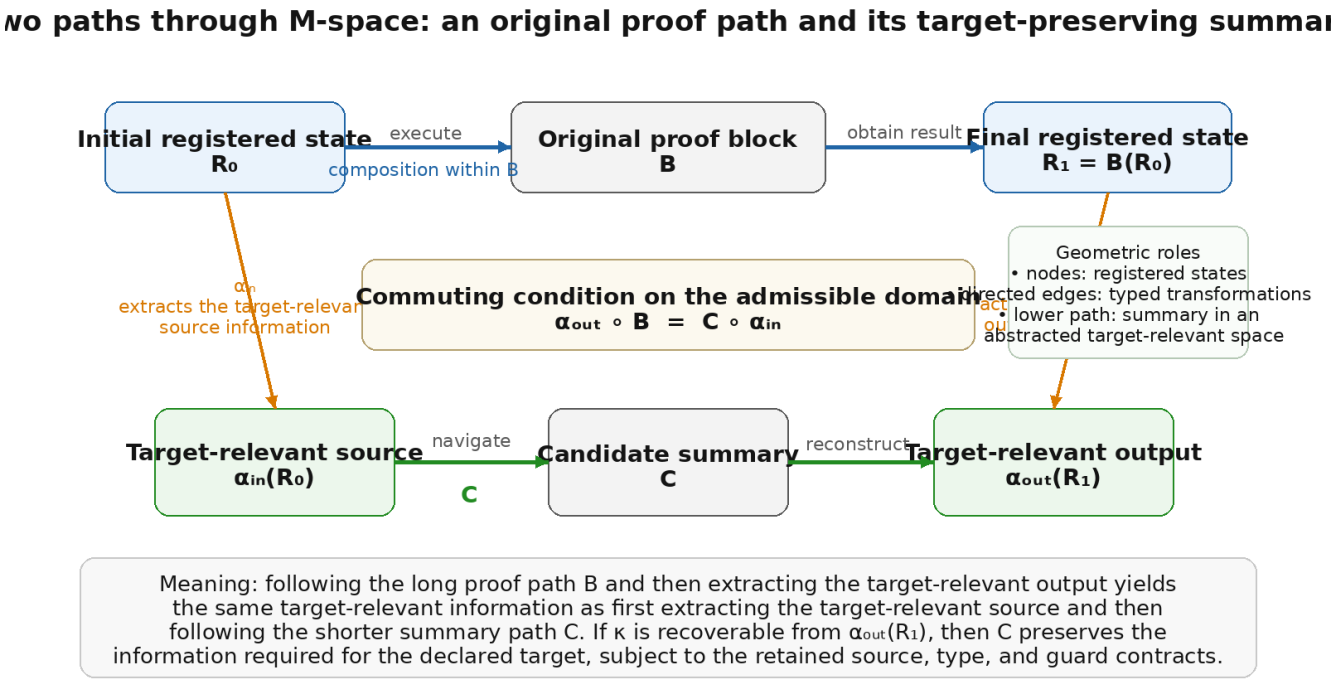



***Figure 7.*** *Two paths through M-space: a typed proof path and its candidate target-preserving summary C. The lower path is adequate when the commuting relationship preserves the target-relevant mathematical information recoverable from the original path, while retaining the required admissibility conditions and source information (Supplement S29).*

Working backwards from a proof thus provides two targets for discovery. One is the sequence of licensed operations leading to the theorem. The other is the invariant relational shape and the transformations that generate that sequence. The latter is the mathematical counterpart of identifying the dynamics of the duck rather than merely tracking its positions. Generalizing that shape may enable a family of proofs to be reconstructed from the same mathematical generator.

### A moving mathematical fitness landscape

At its coarsest resolution, M-space can be represented by a graph of mathematical operands and relationships, as illustrated in Figures 2 and 3 and the claim network in Figure 4. For a particular problem, its relationships have different functional fitness for reaching a solution. Suppose the last two assessed reasoning transitions provide locally independent directions under an appropriate chart. These directions generate a local mathematical plane, onto which target-relative solution fitness can be projected (Figure 8). As new transitions are discovered, the local plane and the relevant landscape may change. Fitness is defined by the target and operative relationships.




*Figure 8. The coarse graph, a local plane spanned by two independent assessed reasoning directions, and the target-relative fitness projected onto that plane. The landscape is schematic; the plane and transfer conditions are developed in Supplement S21.* The coarse nodes project registered mathematical states; the assessed directions project F-executions. The plane is not a separate cognitive order.

The fitness of a solution is distinct from the functional work required to discover and assess it. A promising path may exceed the available work. In the fixed-relationship case an observer searches for a target in an already operable landscape; in the partially known case it searches for the dynamics through which the landscape and its navigable relationships change. A discovered transform can alter which continuations become possible, just as a model of the duck's changing configuration makes further configurations intelligible.

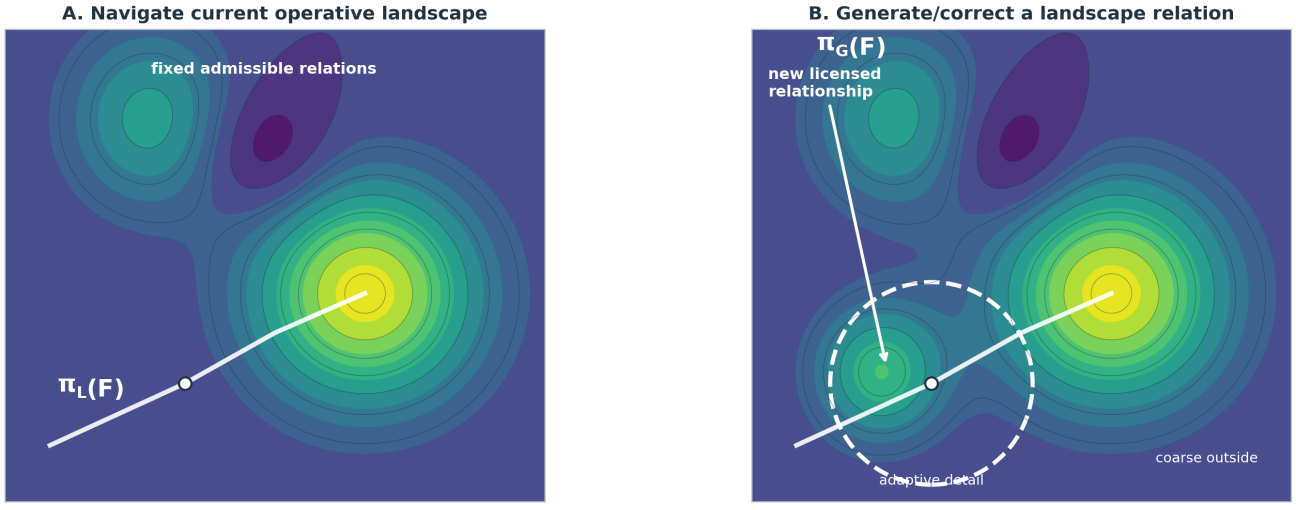



*Figure 9. Two modes of navigating mathematical information space. Each mathematical operand is represented by a topology emitted by F, the functional free-energy-preserving function representing the actions of the system model on its representation space. (A) Navigating the current operative landscape represents problem-solving as navigating paths between mathematical operands whose functional information is complete relative to the target. Each path represents a series of one-shot local coherence operations, denoted $\pi_L(F)$, where $\pi_L$ projects the functional behavior of F corresponding to L. (B) Generating or correcting a landscape relationship represents problem-solving as navigating paths between partially known mathematical operands. Each path represents one or more potentially recursive, and therefore potentially multi-shot, global coherence operations, denoted $\pi_G(F)$, where $\pi_G$ projects the functional behavior of F corresponding to G. Discovering the functional relationships governing these operands can change the operative landscape itself. The dashed region represents local allocation of assessment work at increased*

*resolution, with coarser representation outside. The contour fields schematically represent target-relative mathematical fitness, not measured functional free-energy or work.*

Several established approaches change the tools of subsequent search: Knuth–Bendix completion generates rewrite rules [27]; AM and HR generate mathematical concepts and conjectures [28,29]; abstract interpretation and refinement alter representations [14,31]; counterexample-guided abstraction refinement corrects a representation against which a property is checked [16]; and learned optimization makes the optimization procedure itself a learned object [30]. The proposed novelty is a single recursively correctable six-function infrastructure for these dynamics across domains, preserving their distinct interfaces, invariants, assessments and finite-work relationships. The comparison is developed in Supplement S27; the paired finite execution and definition transport are reported in S30–S31.

**Mathematical microscopes, telescopes and omniscopes**

In the proposed conceptual-space model, concepts and reasoning processes are nodes and edges of a functional state space. A problem can be represented as a missing or currently inaccessible viable reasoning path between functional states. Where concepts needed to define a problem or relationships needed to solve it exist at too fine a functional resolution, their distinctions appear as noise to the observer, like microscopic objects to the unaided eye. Where they lie beyond the accessible navigational expanse, the difficulty resembles that of observing a distant object. The FMI predicts that general problem-solving ability is characterized by the volume and density of the functional information space that can be navigated within the system's endogenous temporal and energetic resolution. This gives a candidate means of determining the limits of individual and collective mathematical discovery, including with AI.

An FMI-instrumented LLM could function as a mathematical microscope that resolves local relationships and boundary conditions, a mathematical telescope that discovers bridges between distant models, or an omniscope that navigates and integrates multiple models and scales. To solve a problem, pull back to the scale at which the landscape of relevant models becomes observable, then focus in to determine precise constraints on the functional objects involved. Typed containers for those objects make it possible to search the wider space of models for relationships that can satisfy their outstanding constraints. The target is to harvest insights from any adequately represented model with a viable correspondence, not to assume every bridge already exists.

Unlike a conventional optical instrument, these policies and their adequacy can themselves become operands of mathematical discovery. The Recursive Mathematical Perceptual Expansion Theorem (RMPE, S40) identifies the conditions for expanding observable volume, effective resolution and reconstructible information density without increasing native perceptual resolution. S41 derives local, bridge and atlas policies under work, contraction, coverage and channel-feasibility guards. The ordinary LLM is not by itself an FMI-compatible instrument; the functionality of the instrument is to be tested in the complete registered architecture.

Consider assessing a newly invented microscope. At familiar resolutions, an image can be compared with a prior model of the object and of the instrument. At sufficiently novel resolutions, however, the image may reveal structures and dynamics for which the observer has no adequate prior model. Calibration remains necessary, but assessment must also discover the relationships among observations, including the effects of interventions and agreement across instruments with different operating principles. Hacking [85] investigates these grounds for confidence in microscopy. The corresponding mathematical problem is not to accept novel observations as correct, but to preserve their functional distinctions as coherence objects while generating and testing an adequate model. An instrument that increases mathematical observability without increasing the capacity to construct and assess models may increase the gap between available mathematical information and understanding. Constraint Algebra composes the constraints of the observations; Calculus of Convergence governs correction of the model class itself. S44.3 develops this relationship as a projection of the same F-governed dynamics, without introducing a separate theorem family.

## 1.6. The mathematical blind spot

Return to the duck example in §1.2. An observer looking through a narrow window may see only a small part of the duck's trajectory (functional extent). A coarse observation may not distinguish its wing configurations (resolution). Lost intermediate observations may remove the evidence that identifies a transform (history). An unfamiliar flying object similarly supplies partial trajectory information without necessarily revealing the mechanism that generates it. Increasing extent does not automatically recover lost resolution or history. In

M-space, the corresponding restrictions concern the extent of operable information, resolution of relationships, retained mathematical histories, assessment capability and available work.

The blind spot is target-relative. If two admitted histories have the same available representation but require different target outcomes, no operation using only that representation can distinguish them. For example, two duck histories may have identical initial and final positions and wing configurations but differ in whether a flap occurred. An endpoint-only observation cannot decide that target. Retaining adequate intermediate observations can repair the representation. The same obstruction applies when a mathematical observer cannot distinguish a path composed entirely within its known relationships from a path whose generation requires an unavailable relationship (T01–T02, T17).

The tests in Supplement S18–S20 make these distinctions explicit. In the four-history interface repair, an endpoint-only representation has eight ordered target-distinct collision pairs; retaining intermediate paths, source ownership, license and work leaves zero. In the 24-history two-cycle observation model, the target collision count changes from 144 to 48 to zero after successive source-licensed observations; 24 collisions remain for the inaccessible private bit. These results establish exact finite reconstruction properties. They distinguish the observed system's changing observation functions from the human observer's signal-level model. The collective analogue is that detection, retention, reconstruction, transmission and subsequent assessment must preserve the information needed for the next mathematical operation (Supplement S9, S11–S13 and S23).

A mathematical observer limited to its existing operative closure may correctly navigate the information it already represents yet be systematically unable to discover particular dynamics outside that closure. The relevant question is whether the observer can generate and assess a missing relationship and retain it as a reusable mathematical object. This is the proposed L/G mechanism underlying H1–H3. The projection collision establishes a representation-relative obstruction.

**Individual and collective mathematical observability**

Consider an ancient observer who witnesses an unidentified flying object (UFO) whose motion cannot be explained using the functional relationships available to that observer. If no durable record survives, later observers cannot reconstruct the observation. If a drawing survives but the relevant temporal, geometric and dynamical relationships were not registered, the object may remain identifiable as a reported observation without its mechanism becoming reconstructible. An individual who discovers a candidate mechanism may also be unable to transmit the functional information required for another observer to reconstruct and assess it. These are different failures, but each can leave a target-relevant relationship unavailable to the continuing collective observer. A durable partial record can still support later correction.

The purpose of this example is not to suggest that mathematics should accumulate incoherent reports, or that an unexplained observation warrants any particular explanation. It is to distinguish recording information from coherently registering the functional information required to assess it. More reports expressed in a representation that fails to preserve a decisive distinction need not resolve that deficiency. Similarly, every mathematician might have one or more solution discovery processes that might be novel and potentially highly useful when reused by other mathematicians, where those processes cannot reliably be represented and therefore cannot reliably be transmitted and reused that way. The late mathematician Srinivas Ramanujan, might be the most compelling example of a mathematician with solution processes that were unobservable to others. The mathematical information Ramanujan communicated could preserve results that were subsequently established without preserving the dynamics through which he discovered them [60–64]. The question is whether mathematical representations can be made capable of coherently registering information that cannot reliably be registered today.

The collective mathematical observer cannot be modeled as an individual observer with a larger memory. Its functional observability depends on the information that survives detection, registration, reconstruction, transmission, assessment and operative use across participating observers. The Coherent Registration Capacity Theorem (CRC, Supplement S38.1) characterizes when a representation can preserve a target-relevant distinction and when a realizable refinement expands the collective's capacity to assess and act upon it. A refinement is not a relaxation of coherence criteria; it enables distinctions necessary for applying those criteria.

The UFO example also illustrates a potential second barrier to correcting a representational deficiency. NASA's independent UAP study identifies reporting stigma as an impediment to scientific data collection [72]. It is hypothesized here that a mathematics journal might be reluctant to publish an example involving UFOs even if its formal role is relevant. That predicted reluctance is not evidence of the origin of any UFO or of a universal editorial practice. Where an assessment interface excludes information necessary to identify a representational deficiency,

without preserving an adequate alternative or mathematically defeating the relevant relationship, it can maintain the associated blind spot (S38.2). Venue rejection alone does not establish such a failure. The engineering target is a more capable representation and assessment process, not a requirement to admit incoherent claims.

## 1.7. Functional and energetic closure

Much of mathematics occurs within the mind of the mathematician, where mathematical reasoning constitutes the navigation of functional information within the mathematician's conceptual space. According to GIGT [42], for such navigation to exist and be observable, the topology of the functional information space being navigated must itself exist and be observable, and the energetics governing that navigation must be embedded in that topology. The functional information constituting mathematics therefore includes not only the mathematical relationships being navigated, but also the functional and energetic relationships governing their navigation.

GIGT conditionally proves that the functional state space representation of the behavior of any adaptive functional system has the same underlying functional architecture, independently of the domain or substrate in which that functionality is realized. Consequently, if a mathematical information space is to preserve the functional information constituting mathematics as the landscape of mathematical knowledge evolves, its topology must embed the same functional and energetic relationships that govern the navigation of mathematical information within the mind of the mathematician. This provides the proposed derivational basis for the functional and energetic closure of M-space.

Functional closure means that the functional information space is closed under composition of the operators through which it is navigated. Energetic closure means that those operations conserve functional free-energy. As illustrated in Figure 9, mathematical discovery can change the topology of the mathematical information being navigated. Functional and energetic closure must therefore be preserved not only through navigation of an existing mathematical landscape, but also through transformations of that landscape.

The six fundamental GIGT/FMA operators—Create/Destroy, Place/Store, Recall, L, G and Choice [42]—provide the proposed recursively closed functional architecture through which these transformations can occur.

| **Fundamental operator** | **Function in navigating functional information space** |
|---|---|
| **Create/Destroy** | Creates a new topology in a way that conserves the dynamical stability of its creation, or destroys the registered representation of a topology. |
| **Place/Store** | Places a topology in a specific region of functional information space, thereby giving that topology relative meaning through its relationships to other topologies. |
| **Recall** | Navigates back to an existing topology using complete or partial information about its relationships to previously encountered topologies. |
| **L — Local coherence** | Navigates to a topology completely specified by a functional relationship, assessing the coherence of that relationship. |
| **G — Global coherence** | Navigates toward a topology partially specified by a functional relationship, assessing the coherence of that relationship across functional information space. |
| **Choice** | Navigates among the available functional operations and their possible continuations in a manner that conserves the dynamical stability of the navigation process. |

In second-order mathematics, the topologies navigated through these six operators include the mathematical operators themselves, the transformations they generate, and the functional relationships governing the adequacy and dynamical stability of their execution.

The Constraint Algebra characterizes the admissible compositions of these operators, while the Calculus of Convergence characterizes the dynamics through which their recursive execution can converge toward solutions while preserving viable continuation (Supplement S24).

Embedding the energetics of mathematical navigation within the topology of M-space has three important consequences. First, conservation of functional free-energy provides a proposed mechanism for detecting losses of functional information that would otherwise impair the continued navigation and correction of mathematics, analogous to detecting a leak through conservation of water in a plumbing system. Second, it provides a mechanism

for allocating finite functional free-energy to navigating the mathematical landscape at different resolutions, analogous to the allocation of computational resources in a game engine. As new mathematical relationships are discovered, both the topology of the mathematical landscape and the distribution of functional free-energy available to navigate it may evolve.

Third, conservation of functional free-energy provides a proposed functional basis for assessing whether mathematical discovery operations constitute useful work toward a mathematical target. Without such assessment, a human or artificial mathematical observer may be unable to systematically distinguish navigation governed by the topology of the mathematical problem from navigation governed by its own preferential selection of mathematical representations and operations. Correct and locally productive operations can repeatedly reinforce such preferences without contributing to convergence toward the intended target. This distinction remains consequential even for an artificial observer with computational resources vastly exceeding human cognitive limits. By making functional work and target-relative progress separately and recursively assessable, energetic closure provides a foundation for investigating convergence even where discovery transforms the operative mathematical topology. Conservation alone does not establish useful work or convergence: the adequacy of the target-relative progress relationship must also be assessed.

The mathematical conditions governing these relationships are investigated in Supplements S21, S24, S37, S41 and S44.6, with finite realizations of work conservation reported in T29 and S30–S31. Related work on Interaction Spaces and complex adaptation [37,38], Memory Evolutive Systems [39], network operads [40] and relational biology [41] provides precedents for representing evolving functional relationships, historical information and organizational closure.

The intended universal mathematical scope of these constructs is expressed in three theorem targets, derived at a common conditional semantic scope in Supplement S37:

Universal Constraint Algebra Theorem (CA-U): For all mathematical operands and operators, the constraints governing their correctness and coherence can be represented, generated, composed and transformed through a universal, recursively closed Constraint Algebra.

Universal Calculus of Convergence Theorem (CoC-U): For all mathematical operands and operators, the evolution of their correctness and coherence toward any mathematical target is governed by a universal, recursively closed Calculus of Convergence, which establishes the conditions governing the convergence of mathematical information toward that target.

Universal Mathematical Navigability Theorem (MN-U): For all mathematical operands and operators, the mathematical information governing their correctness, coherence and transformations defines a mathematical functional state space whose navigability toward any mathematical target is governed by the Constraint Algebra and Calculus of Convergence.

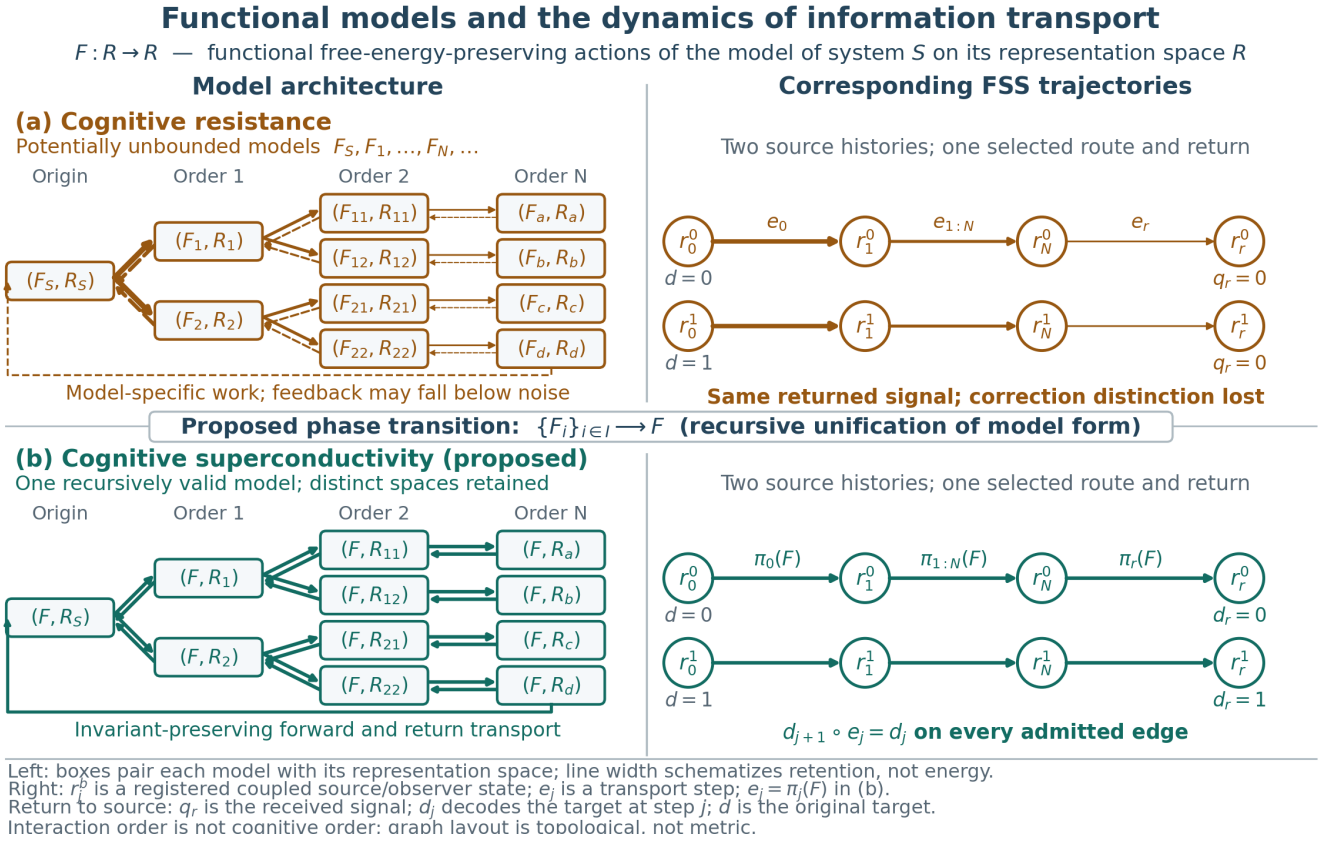
Partial structures and quasi-truth model incomplete information [67]; domain theory orders information by refinement [68]; abstract interpretation derives sound approximations and fixed points [69]; logical induction studies belief evolution over logical sentences [70]; and partial-observation games derive observation-dependent navigation strategies [71]. The proposed novelty is the conjunction of applicability to all mathematical operands and operators, governance of both complete and partial information, recursive evolution of the operations and constraints themselves, and target-relative fitness and viable work accounting. Supplement S37 develops their common conditional mathematical derivation, including a finite separating witness, and specifies the universal F-realization proof obligation. Applicability to all mathematical objects does not require every target to be reachable.

### Coherence collapse and cognitive phase transition

As mathematical information grows, the work needed to coherently register, reconstruct, assess and correct it may exceed the capacity of the architecture through which that work is performed. The Functional Coherence Capacity Theorem (FCC, Supplement S38.3) establishes a conditional obstruction when indispensable coherence-maintenance demand persistently exceeds capacity and the required residual and loss bounds cannot be maintained. The Coherence Collapse–Phase Transition Bifurcation (CPTB) conjecture predicts that first-order mathematics has a limit under sustained growth and finite accessible work beyond which collective mathematical coherence cannot be preserved by its existing architecture, past which the collective coherence of mathematics collapses and mathematical information becomes collectively unnavigable. Whereas transition to second-order mathematics may restore coherence and multiply general mathematical problem-solving capacity.

For a system S, let R be its representation space and F: R → R the functional free-energy-preserving function representing the actions of the system model on R. Where mathematical information is distributed across a potentially unbounded set of different system models, the transport of functional information between models and back for correction requires model-specific transformations. The work required to discover, assess and execute these transformations, and the information lost through them, constitute cognitive resistance. As interaction order increases, the feedback required for correction may become indistinguishable from noise (Figure 10(a)).

GIGT [42] predicts that collapsing this potentially unbounded set of functional system models into a single recursively valid functional model F could constitute a cognitive phase transition (Figure 10(b)). The collapse is of the multiplicity of modeling architectures, not of their distinct representation spaces or domain-specific information. The same six functional operations govern the information and recursively govern the operations through which it is transported, assessed and corrected. Transport can then be systematically realized through properties that are topologically invariant in F. Where these transformations preserve functional free-energy and are viable, the resulting regime is referred to as cognitive superconductivity.




Left: boxes pair each model with its representation space; line width schematizes retention, not energy.
Right: $r_i^o$ is a registered coupled source/observer state; $e_j$ is a transport step; $e_j = \pi_j(F)$ in (b).
Return to source: $q_r$ is the received signal; $d_j$ decodes the target at step $j$; $d$ is the original target.
Interaction order is not cognitive order; graph layout is topological, not metric.

**Figure 10.** Cognitive resistance and the proposed transition to cognitive superconductivity. Left: branching interactions among different models (a) and instances of the common model (b). Right: illustrative FSS trajectories show a lost target distinction versus its preservation through projected executions of F. The unification enables the conservation mechanism; adequate transport and viable execution remain conditions of its realization.

This unification could keep mathematical information that would otherwise become inaccessible coherently navigable as interaction order increases. Second-order mathematics is hypothesized to provide the mathematical infrastructure for this transformation. S38 derives the conditional capacity obstruction and capacity-restoring branch, and S44 derives the corresponding conservation and viability conditions. The emergence of convection beyond the stability threshold of conductive heat transport [73] provides a physical precedent for such organizational change, not a proof of the mathematical cognitive-order transition.

The possibility of such a cognitive phase transition is supported by the published Cognitive Near-Singularity (CNS) possibility theorem [84]. Under its specified contraction, compositional closure and invariance conditions, heterogeneous reasoning processes converge to a common fixed point up to normal form. In a bounded corpus study, all seven witnesses and all three CNS discriminators passed, whereas none of the seven comparison corpora passed the full protocol. This provides a coherence witness for the possibility of the common compositional reasoning

architecture underlying the proposed transition in Figure 10(b). The CNS results do not by themselves establish the functional free-energy conservation required for cognitive superconductivity or the magnitude of its predicted increase in general problem-solving ability.

GIGT [42] models an individual or collective transition from order-one to order-two cognition through expansion of viable reachable targets and a low-loss or target-lossless regime of functional information transport termed functional superconductivity; it characterizes collective transition through critical-mass and supercriticality conditions. The Mathematical Cognitive Phase-Transition Theorem (MCPT, S38.5) connects these proposed relationships with coherent registration, viable continuation and net functional capacity. If the information required to recognize the need for second-order mathematical infrastructure is itself outside an observer's first-order operative closure, that observer may require access to second-order understanding to recognize its necessity. The collective can potentially instantiate the required relationships through signal-mediated coordination without requiring identical cognitive capacities in all participants.

Possessing the mathematical relationships governing a functional information substrate is not equivalent to possessing a functional model of that substrate. Physics illustrates this distinction: energy conservation principles and the descriptions of fundamental interactions, creation/annihilation and state-selection dynamics supply candidate constituents of energetic and functional closure, but do not by themselves provide a unified audit of transformations of matter across spatial, temporal and theoretical domains. GIGT predicts representation-relative barriers to specified quantum and string-theoretic computations; where the determining distinctions are distributed across existing theories, a functional model may enable discovery of their adequate conjunction. This is a proposed correspondence, not an established identification of the four interactions with four FMA functions or of the sufficiency of a particular theory conjunction.

The same distinction applies to mathematics. Relationships necessary to solve a problem may already exist within collective mathematical knowledge, while the functional information governing the discovery of their appropriate composition remains inaccessible. The Functional Mathematical Computability Phase-Transition Theorem (FMCPT, S39) distinguishes persistent information collisions from divergence of discovery work in an expanding information space. Where a recursively discoverable generator makes the determining relationships operable at uniformly bounded total work, a target may remain energetically computable under unbounded growth of mathematical information even where the incumbent architecture cannot reliably attain it. This is a conditional architectural separation, not a claim of unrestricted decidability or that the actual RH proof required such a transition.

The predicted increase in general mathematical problem-solving ability is a prediction about G operations, not about the speed of raw calculations represented as a fixed series of L operations on the same substrate. G can discover and assess partially known relationships governing not only the path to a solution, but also the definition of the problem itself. Where the initial state, target or admissible relationships are incompletely specified, the problem definition is itself a coherence object that can be recursively corrected. G can discover a more general or simpler definition from which the target becomes accessible, preserving the functional information required to reconstruct and assess the original target, or explicitly registering a change of target where the original definition was inadequate.

The checkerboard example (Figure 1) illustrates why simplification is a natural consequence of this mechanism. Discovering the invariant $D = a - b$ transforms the problem from a search over arrangements of tiles into the assessment of a conserved relationship between two classes of objects. The resulting impossibility can be established by L without enumerating those arrangements. It is the G operation that discovers the relationship and makes this simpler formulation operable; subsequent L operations execute the licensed relationship. The predicted advantage of cognitive superconductivity therefore lies in the discovery, simplification and reuse of problem definitions and solution-generating relationships. It can reduce the total number and cost of L operations required, or make previously inaccessible problems definable and solvable, without predicting faster execution of an unchanged sequence of L operations. The net advantage depends on the work of discovering, assessing and transporting the relationships as well as executing them.

## 1.8. Axiomatic and functional models of intelligence

### Interfaces for axiomatic and functional relationships

An Axiomatic Model of Intelligence (AMI), as defined here, models mathematical reasoning as the composition of mathematical relationships derived from a set of accepted primitive axioms. However, a mathematical relationship

that is primitive within one axiomatic system might itself be a composition of more fundamental mathematical relationships outside that system. Figure 11 illustrates this distinction between composing known mathematical relationships and discovering the mathematical structure of the relationships being composed.

The significance of Gödel's incompleteness theorems [51] to this distinction is that any consistent, effectively axiomatized formal theory sufficiently expressive to represent elementary arithmetic contains statements that it can neither prove nor disprove, and, under the conditions of the second incompleteness theorem, cannot establish its own consistency. Extending such a system can make previously undecidable statements decidable, but the extended system remains subject to corresponding limitations. Turing and Feferman [7,8] investigated processes through which formal theories can be recursively extended.

This raises a functional question concerning mathematical intelligence: can the process of discovering, composing, assessing and correcting mathematical relationships, including those relationships treated as primitive within a given axiomatic system, itself be represented within a recursively closed functional model of mathematical reasoning? Whereas an AMI navigates mathematical information using its accepted axiomatic structure, the proposed Functional Model of Intelligence (FMI) [52] models the dynamics through which the mathematical relationships constituting that structure can themselves be discovered and assessed. This corresponds to the distinction between navigating known mathematical relationships through L operations and discovering the dynamics of partially known mathematical relationships through G operations.

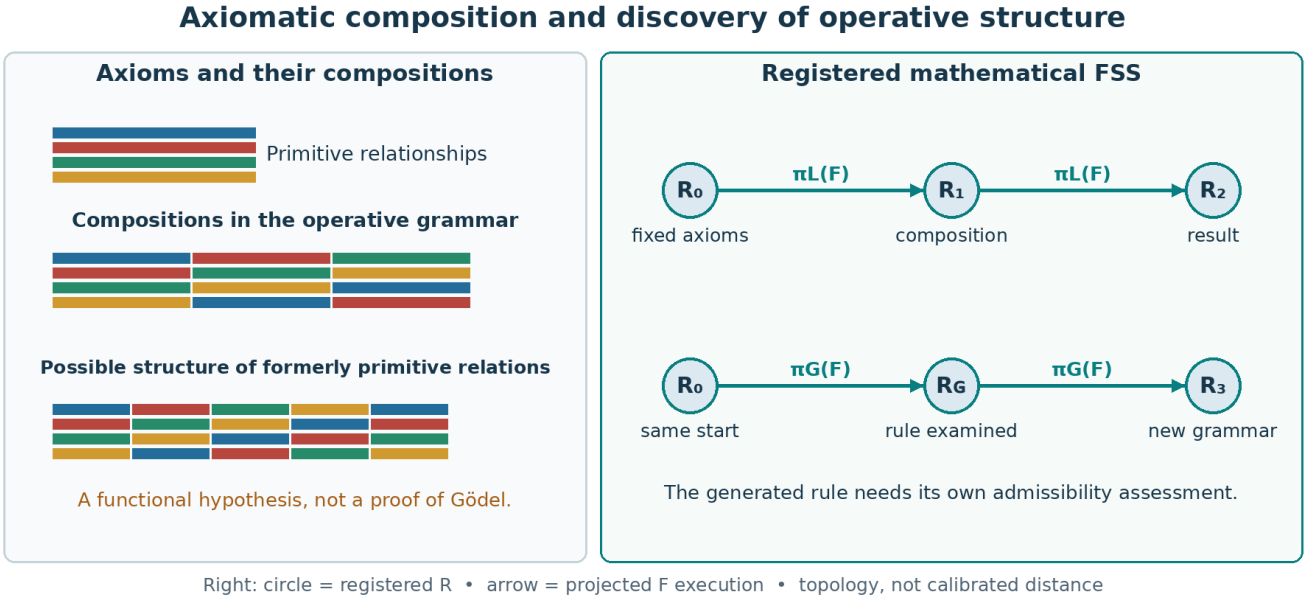



*Figure 11. The distinction between composing primitive mathematical relationships and discovering their underlying mathematical structure. The single-colored bars represent primitive axioms within a given axiomatic system. The multicolored bars represent mathematical relationships derived through compositions of those axioms. The segmented bars at the bottom illustrate the possibility that relationships accepted as primitive within that system are themselves compositions of more fundamental mathematical relationships outside its operative representation. Gödel's incompleteness theorems establish formal limitations on what sufficiently expressive axiomatic systems can establish internally [51]. The figure motivates the proposed functional modeling of the dynamics through which mathematical systems can be recursively extended.* Right: complete registered mathematical states include the grammar itself; generated structure is conditional on assessment, not an implication of Gödel's theorem.

The common interfaces for the mathematical relationships used by the AMI and FMI, and the additional spatial and energetic relationships proposed for the SIP realization, are shown in Figures 2 and 3 in subsection 1.1.

A Functional Model of Intelligence (FMI) [52] decomposes the functionality of the entire cognitive system into functionally and energetically closed operators that navigate its strongly typed functional information space, or conceptual space, in which phase transitions in cognition itself may occur. Its application to mathematical discovery registers the mathematical relationships and the operations needed to construct, assess, correct and navigate them in M-space. A Functional Model of Active Inference (FMoAI) [52] resides within the FMI and models the functionality of active inference through a functionally and energetically closed set of operations. It predicts that such closure can make different active inference approaches more universally comparable and optimizable. The finite

paired discovery experiment in S30–S31 illustrates this proposed comparability and reuse in a mathematical domain.

**A representation-relative assessment obstruction**

Let X be the complete spatial-functional state space, A a reduced representation used by an objection, a: $X \to A$ its projection, and J: $X \to S$ a registered spatial-information signature. Define the representation residue $R_a = \{(x,y) \text{ in } X \times X : a(x)=a(y),\ J(x) \neq J(y)\}$. This records exactly the distinctions retained by the spatial representation but merged by the reduced one. A target $\kappa$: $X \to K$ can be assessed solely through A precisely when there exists d: $A \to K$ satisfying $\kappa = d \circ a$ on the admitted states. This is the factorization or target-reconstruction condition underlying information-preserving abstraction [31,14].

Proposition (representation-relative assessment obstruction). Suppose $x_0$ and $x_1$ satisfy $a(x_0)=a(x_1)$ but $\kappa(x_0) \neq \kappa(x_1)$. Then no decision rule d based only on A can return the correct target verdict for both states. Proof: if $\kappa = d \circ a$, then $\kappa(x_0)=d(a(x_0))=d(a(x_1))=\kappa(x_1)$, contradicting the premise. Consequently, any decisive objection that gives the same verdict to both cases is incorrect for at least one; an objection that returns UNDETERMINED instead has not discharged the target. If the target family can distinguish every registered spatial-signature difference, a nonempty spatial residue guarantees the existence of at least one such target.

Consequently, if a reduced representation identifies two states with different registered spatial signatures and an admissible target distinguishes them, a decisive assessment using only that representation fails for at least one state. For example, two plane transformations with the same coarse type signature and Jacobian determinants of opposite sign cannot be distinguished by an orientation target from that signature alone. An internally valid objection in a reduced presentation cannot defeat the richer target unless the target-relevant information is preserved. The factorization criterion establishes the precise obstruction independently of the executor. Where the relevant assessment has not been executed, the verdict is UNDETERMINED.

The proposed FMI registers the mathematical operands and transformations governing the discovery process, including information missing from a reduced presentation. This permits a further level of reasoning: the observer can make its own mathematical operations, their assessment and their adequacy relationships operands of recursive correction. In HCFM second-order mathematics, the corresponding recursive closure is within M-space. A change in the cognitive order of the observer and a change in the dimensionality of M-space remain separately typed.

**Reconstruction and transmission of mathematical understanding**

The water-tower and duck comparison also explains the distinction between transmitting a particular mathematical state and transmitting the relationships through which states can be reconstructed. A relative musical interval pattern supplies a simple exact example: the sequences (60,64,67) and (62,66,69) have the same (+4,+3) interval structure. The relative structure makes both sequences reconstructible given their starting values. The same kind of invariance can allow the reuse of a mathematical generator across problems.

The mathematical significance of human observation of the internal functional state spaces of the body, including its five conventionally identified senses (vision, hearing, touch, smell and taste), the emotions, cognition and consciousness, lies in the functional information that can potentially be transported from those spaces into M-space. The mathematical assessment concerns the invariant relationships and transformations that can be reconstructed from experience. If emotional states are represented as topologies in emotional space, observable invariant properties of those topologies, such as functional connectivity density, and the transformations governing those properties can potentially supply mathematical relationships. The experience supplies signals from which these relationships may be reconstructed.

Where the relevant functional relationships are conserved through a typed signal interface, the initial topology in the source functional state space can be mapped to the topology of a mathematical problem, the target topology to the topology of a mathematical solution, and the topology of the transformation between them to the topology of a mathematical solution process. The observation interaction through which an invariant relationship becomes observable may itself supply a transportable transformation pattern. This can generate candidate mathematical operators, including operators for overcoming observability obstructions. The signal-mediated transport preserves target-relevant invariants, context, history, admissible operations and residual information (S9–S13, especially S12.2a).

The exceptional mathematical productivity of Srinivasa Ramanujan, whose notebooks register thousands of results [60], makes it worth investigating whether he exploited such sources of mathematical information. In an account of a dream involving a screen of flowing blood, he reported: “That hand wrote a number of results in elliptic integrals.

They stuck to my mind" [61,65, p. 87]. In his letter to Hardy of 17 April 1913, he stated, "I have not found anyone to appreciate the method" [62, p. xxii]. Hardy's memoir also records Seshu Aiyar's account of the difficulty readers had following Ramanujan's novel methods [62, p. xiv]. That communication problem persists: Berndt documented the difficulty of reconstructing Ramanujan's discoveries in 1985 [63], and in 2024, despite the subsequent proofs of his notebook results, reported that he still could not reconstruct Ramanujan's methods for deriving higher-degree modular equations [64]. The descriptions of Ramanujan's discovery processes thus remain, in significant cases, unintelligible as reproducible mathematical operations.

Ramanujan's philosophical belief system also suggests a potentially deep alignment with the human-centric, cross-domain functional modeling principles underlying GIGT. His Cambridge colleague P. C. Mahalanobis recalled Ramanujan's attempts to model reality through the relationships between zero, infinity and finite numbers, mapping them to philosophical concepts of the unmanifest absolute, possible manifestations and acts of creation [65, p. 82; 66, p. 40]. The absence of a provably coherent and falsifiable representation of Ramanujan's potentially anomalous problem-solving processes suggests that he may have lacked an explicit second-order model of the functional information governing his own mathematical discovery. It is precisely such a model and representation that the present approach seeks to provide, so that information obtained through observation of internal functional state spaces and the transformations through which it generates mathematical solutions can themselves become mathematical operands subject to recursive assessment, correction, falsifiable predictions and systematic reuse.

Archimedes' mechanical method [83] illustrates the distinction between transporting a functional relationship from another domain to discover a mathematical result and subsequently establishing that result geometrically. If this capacity can be generalized to every adequately modeled functional system, including the functional state spaces of the body, emotions, cognition and consciousness, the mathematical discovery capabilities of the resulting observer may be difficult to predict from within first-order mathematics. Experiences supply candidate relationships and transformations, not mathematical proofs. The possibility of making Ramanujan-like discovery experiences widely accessible by incorporating a second-order metacognitive operator into cognition is an open conjecture; the potential unbounded functional advantage established under the conditions of S39 is distinct from the subjective character of that experience. S42 gives the inward-modeling and transport contract.

A productive application of this approach of transporting insights between the functional state spaces of different adaptive systems is the GIGT-motivated correspondence [42] between black-hole and white-hole-like topologies of matter and the formation, evolution and dissolution of cognitive blind spots in conceptual space. The relevant candidate mathematical relationships concern functional information flow, connectivity, observability boundaries and changes in reachability. The functional correspondence generates a program for mathematically modeling the dynamics of cognitive blind spots.

Mathematical understanding is transmitted when the recipient can reconstruct and continue the relevant mathematical relationships. In visual representations, visual-ontology closure (VOC) preserves mathematical objects, relationships, compositions and invariants; point-of-use information locality (PIL) keeps the information needed to decode them alongside the relevant construction; and transformation-preserving revision (TPR) preserves source relationships under changes of representation. The joint adequacy test applies to the sequence of figures as well as to individual figures, relative to the target and the observer's available work. Supplement S28 formalizes these conditions and the independent-module test.

The resulting prediction is that discovering and retaining reusable operators can combinatorially expand mathematical reachability. The FMI further predicts that using those operators to discover and assess other operators can increase problem-solving capacity itself, potentially combinatorially with each use or exponentially overall, depending on the computational substrate. The capacity to increase mathematical discoverability is distinct from a count of reachable mathematical objects. The conditions for these growth predictions are investigated in S10; H8 addresses the reconstruction and transmission of understanding. This motivates the backward reconstruction of the RH case and the hypotheses that follow.

## 2. The hypotheses and the backward dependency

The study registers eight exact, full-strength hypothesis targets, whose authoritative statements and proof-status records are in Supplement S1 and Appendix B. Their succinct discovery roles are listed in table 1.

The central conjunction concerns H1–H6. H7 and H8 evaluate its real-world compatibility and implications for collective mathematical understanding. The supplement records each claim, its proof or proof obligation, its assessments and the separate joint assessment contract.

*Table 1. Functional relationships among the registered hypotheses.*

| Human-readable target and identifier | Function in the argument | Dependency or assessment role |
|---|---|---|
| **Mathematical blind spot (H1)** | Potential first-order blind spot | Ask whether a systematic mathematical blind spot can exist coherently. |
| **Generative L/G mechanism (H2)** | L/G generative mechanism | Ask whether generating or correcting admissible operations explains the declared blind-spot mechanism. |
| **Target observability (H3)** | Target-relative observability | Register the effects of source access, resolution, history, assessment and work. |
| **Functional and energetic closure (H4)** | Sustainable discovery | Test the proposed joint necessity of functional and energetic closure. |
| **Source-licensed operation (H5)** | Source-licensed construction | Test whether a previously unavailable target-solving operation can be generated and assessed. |
| **Reliable convergence (H6; SOM-G1)** | Reliable continuation | Derive conditions for convergence of recursively generative mathematical discovery. |
| **RH case compatibility (H7)** | RH case compatibility | Assess documented RH discovery architecture as a case, not a proof of H1–H6. |
| **Reconstruction and transmission (H8)** | Reconstruction and transmission | Assess the capacity to reconstruct and continue mathematical operations across observers. |
| **Central joint conjecture (J = H1∧· · ·∧H6)** | Central joint conjecture | Requires a compatible joint assessment; scoped constituent results do not themselves certify the conjunction. |

# 3. Methods

The methods used were to derive the interfaces to the mathematical objects (claims, theorems, proofs, conjectures, etc.) used in this paper, in terms of M-space, then to represent registered proof operations as paths through M-space and investigate how the topology of such a path can be generalized as a "shape" in order to aid human comprehension (S2, S29). The study begins with an independently documented **formal RH endpoint**, then works backwards to ask what mathematical information would be required to reconstruct and reuse the relationships through which a result of that kind becomes discoverable. This is a backward *test of the proposed discovery model*. Three evidence classes are kept distinct: (i) mathematical consequences of specified typed assumptions; (ii) documentary and empirical premises about formal proof, AI mathematics and human reconstruction; and (iii) proposed generalizations whose proof or realization is not yet complete. The exact hypotheses H1–H8 and the joint proposition J=H1∧· · ·∧H6 are registered independently of their proof status (Supplement S1, S7 and Appendix B). For the RH case, the study maps documented milestones and the observed source-text proof route, assesses a candidate Stepanov–theta-lift correspondence and distinguishes these from historical causation and proof-term reconstruction (S26). A bounded comparison with Masullo's separately proposed Hecke-L-function route and its public correction process registers documentary source, candidate transformations and continuation obligations using the same W1–W3 protocol (S34).

Having demonstrated the general method by which it might be possible to work backwards to map the discovery model to the RH target, the paper investigates whether the conjunction of claims supporting each individual hypothesis is conditionally valid in terms of the information interfaces between them, and whether literature-supported claims can be used to construct such conjunctions. The methods are mathematical derivation of target reconstruction, operator generation, observability, closure, convergence and receiver continuation; source-frozen finite interface-repair and coupled-observer tests; and paired computational tests of common six-function governance, definition transport and work conservation (S2–S13, S14, S18–S24, S27–S32). The hypotheses and their conjunction are assessed for global coherence through the recursive composition and assessment of their supporting mathematical relationships (S7, S32–S33). GIGT supplies foundational functional relationships at their declared conditional scopes (S35; assessment/Foundational_dependencies.md). Assessments conserve their established information, budgets and provenance for subsequent extension. The claims for which literature support was sought were as follows:

*Table 2. Claims used to search for supporting literature.*

| Method (claim used to search for literature support) |
|---|
| AI generated proofs and other mathematical objects can exceed human capacity to reliably check at all within any reasonable time budget. |
| The rate at which AI is generating mathematics that requires checking may be increasingly outpacing the rate at which AI is able to check that mathematics. |
| Many mathematicians are worried about the role of mathematicians in the future. |
| Having a basis set of operations for any information space, where those operations are also defined in that space so they themselves can be operands, and therefore can be correctable, derives the meaning of functional closure for that information space. |
| Lossless compression of mathematics as required for the purpose of simplification for humans, in the case of partial information requires the discovery of a "generator" of mathematics, and this lossless compression is required in order for problems in certain strongly-typed domains to be reliably solvable by humans. |
| This lossless compression in turn requires the derivation of an interface for that generator so that this generator can be specified with only partial information, in order to avoid the bootstrap problem of having to prove that generator over the graph of all mathematical information, including problems that currently are not reliably solvable, before being able to use that generator to prove the validity of mathematical objects that are not reliably solvable. |
| Whether current mathematics systematically erases partial information such as incomplete proofs, as predicted for a restricted first-order information representation. |

# 4. Results

The literature search supports constituent premises corresponding to Table 2 (E1–E7; S10.3, S14.2): practical limits on direct inspection of large proofs [2,4]; generation/checking divergence in a modeled workload [5]; concern within a voluntary survey population [13]; operations and proof goals as operands [20]; proof compression and reusable structures [6,17]; and partial-information representation refinement [14,16,31]. Existing proof tools also retain partial goals [20]; E7 concerns target-relative loss of information in particular representations. These premises are composed with the mathematical derivations at their licensed scopes (S32.10).

*Table 3. Established functional relationships supporting H1–H8.* Status reports evidence-relative compatibility conditional on GIGT premises (S33, GJ26-E: one reasoned review per hypothesis within 29 total review units). H7 native assessment: UNDETERMINED (S26.7).

| **Hypothesis (full statements: Supplement §S1 and Appendix B)** | **Status** | **Established or scoped support** |
|---|---|---|
| **Blind-spot existence/coherence (Hypothesis H1)** | **COHERENT** | Target-reconstruction and fixed-policy obstruction: T01–T02 and the exact restricted fixed-policy obstruction T17 give an exact observer-relative collision mechanism. Proof/evidence locations: Supplement §§S2.2–S2.3 and S19.2 (T01, T02, T17). T31 (S32) establishes an exact common-source blind-spot witness. |
| **L/G generation mechanism (Hypothesis H2)** | **COHERENT** | Typed operation generation and reachability: T03–T04, T18, and conditional reachability separation T21 separate fixed-grammar nonreachability from generated typed composition. Proof/evidence locations: Supplement §§S3.2–S3.3, S18.2 and S22.2 (T03–T04, T18, T21). R19 (S30–S31) supplies executed common-functional governance across two reduced discovery mechanisms. T31 (S32) derives the combined observation and grammar repair within the same class. |

| Hypothesis (full statements: Supplement §S1 and Appendix B) | Status | Established or scoped support |
|---|---|---|
| **Recursive observability (Hypothesis H3)** | **COHERENT** | Observation and transport adequacy: T01, T11–T13, T19 and the conditional local fitness-plane factorization T20 define target-preserving observation and transport conditions. Proof/evidence locations: Supplement §§S2.2, S9.5, S11.3–S11.4, S19.3 and S21.2 (T01, T11–T13, T19, T20). T32 (S32) derives exact partial-information bounds and minimum worst-case observation work; T35 derives recursive interface refinement. |
| **Dual closure (Hypothesis H4)** | **COHERENT** | Viable work and discrepancy detection: S5 and T23 provide bounded retention, accounting and discrepancy-detection lemmas. Proof/evidence locations: Supplement §§S5.1–S5.3 and S24.2 (resource accounting and T23). Executed six-function work accounting conserves $W+E=W_0$ exactly at the finite no-intake scope (T29; R23; S30), with a COHERENT finite work-conservation projection verdict. T30 (FMA-DTD, S31) provides the conditional non-circularity criterion; comparator sensitivity is established separately from the full construction-premise countermodel; T29 remains the exact conservation proof. T33 (S32) derives the joint necessity of retaining target-relevant functional and energetic distinctions for a recurrent class. |
| **Constructive discovery (Hypothesis H5)** | **COHERENT** | Source-licensed construction and repair: T04, T18 and T19's executable source-licensed finite repair constructs a sound source-frozen relation at an exact scope. Proof/evidence locations: Supplement §§S3.3, S18.2 and S19.3 (T04, T18–T19). The threshold-grammar obstruction (T26–T27) and source-licensed concept-definition transport (T28; R21–R22) supply an additional exact constructive witness. T31 and T34 (S32) establish uniform source-only typed generation without the final target or successful observation sequence. |
| **Reliable convergence (Hypothesis H6)** | **COHERENT** | Finite saturation and fair refinement: T07 proves finite saturation convergence; T24 establishes conditional finite fair-refinement coverage. Proof/evidence locations: Supplement §§S6.2 and S24.3 (T07, T24). T34 (S32) establishes adaptive, work-bounded convergence for arbitrary finite three-sort graphs under its source and guard conditions. |

| Hypothesis (full statements: Supplement §S1 and Appendix B) | Status | Established or scoped support |
|---|---|---|
| **RH compatibility (Hypothesis H7)** | **COHERENT** | The formal endpoint and the documented chronological milestones provide mathematical and documentary information from which the discovery process can be investigated. Under the proposed functional model, conventional mathematical proof representations tend to constrain the presented transformations to L operations, even where G operations were instrumental in discovering relationships between sets of L operations. Analogy transfer can itself constitute a realization of G when it enables navigation toward a mathematical topology through a partially specified relationship. The Stepanov→theta-lift structural correspondence therefore provides information for reconstructing a candidate G-mediated discovery process. The research target is the explicit representation of the mathematical information and transformations through which such relationships can be discovered, assessed and systematically reused.<br><br>Proof/evidence locations: Supplement §S26 develops the source-conditional RH documentary case. S26.2 and S26.9 provide the observed declaration route and 20-entry documentary timeline; S26.4 establishes a proof-assembly reconstruction obstruction. S32.9 derives a conditional intersection of the source-expression rank windows and identifies the positive documentary structural correspondence. S34 extends this approach through comparative registration of Masullo's orbital-basis discovery route and its correction process. |
| **Reconstruction/ transmission (Hypothesis H8)** | **COHERENT** | Receiver reconstruction and visual repair: T10–T15, T18's operation-faithful finite structured diagram, and T22's assessment-interface loss criterion provide scoped reconstruction, transport, separation and compression conditions. Proof/evidence locations: Supplement §§S9.2, S9.5, S11.3–S11.4, S12.3, S13.2, S18.2 and S23.2 (T10–T15, T18, T22). T25 (S29) proves the target-preserving path-summary construction under its stated commuting, typing and guard conditions, with an exact example. T35 (S32) derives coarsest adequate interface refinement and reconstructible receiver continuation; 256 receiver cases agree with the reference. |

**Joint result.** The mathematical derivations establish a common functional architecture in which observer-relative blind spots, generated operations, recursively extensible observability, functional and energetic viability, constructive discovery and reliable convergence are jointly realized for a nontrivial mathematical problem class. The S32.8 conditional derivation $\Gamma^* \Rightarrow J^*$ is COHERENT at its stated mathematical proof scope (S36). Its distinct finite projection assessment is COHERENT at 7,077 case/control units, including receiver continuation. The original J is COHERENT in the recorded evidence-relative compatibility assessment conditional on GIGT premises (GJ26-E, 29 reasoned review units, S33). This author-led assessment records hypothesis, pair-interface and joint checks for mathematical scrutiny. Its review budget is distinct from the 7,077-unit executable finite projection. Foundational dependencies and recursive extensions are specified in the supplement.

# 5. Discussion/Implications

The implications of the conditional results, summarized in the table below, suggest that mathematics can be combinatorially or exponentially more discoverable with a second-order model of functional information, and with the infrastructure serving as the computational substrate for navigating that model, as well as that future mathematicians may benefit more from searching for topologies or "shapes" in functional information space, rather than the current approach to mathematics which equates to searching for "points" in mathematical information

space.

Table 4. Synthesis of the fourteen implications: generative relationships and significance.

| **Implication and location** | **Generative relationship** | **Significance** |
|---|---|---|
| **Blind spots and new operations (I1–I4)** | Observation limits, generated operations, evolving reachability, and the cost of discovery. | Distinguishes fixed-operative search from discovery of topology-generating relationships. |
| **Cross-FSS transport and visual reconstruction (I5–I6)** | Signal-mediated cross-FSS transport, visual reconstruction and operation-faithful compression. | Potential interfaces for discovery and transmission of mathematical continuations. |
| **Collective coherence and assessment (I7–I9)** | Collective provenance and reuse, common mathematical representation, and target-relative assessment interfaces. | Connects certification, understanding, reusable generators and possible recognition loss. |
| **Viable work and discovery infrastructure (I10–I11)** | Work accounting, adaptive resolution and a common recursively correctable functional architecture. | Proposes viable, cross-domain discovery infrastructure, not merely a common vocabulary. A common executor and exact finite work-ledger projection are now realized with a COHERENT verdict at their declared scope (R19, R23). Non-circularity of the registered finite execution assessment is separately supported by T30 and the disjoint-input checks in S31. |
| Recursive mathematical blind spot (I12; S45) | Informational, convergence, reflexive and operative limitations of discovering the limitations of discovery. | A finite proof of a blind spot is distinct from functionality that reliably discovers its instances. |
| Recursive governance of discovery (I13; S44.7, S45) | Discovery policies, progress relations, and adequacy become recursively assessable mathematical operands. | Accumulation of correct results need not establish convergence of assessment of the original discovery target. |
| Discovery of governing functionality (I14; S45.5) | Distinguishes specification of governing functionality from reliable discovery of consequences, configurations and discovery limitations. | Conditional implications for physical and cognitive theories of everything, including AGI; particular exclusions may be repaired without absolute self-completeness. |

The collective observer's ability to recognize the need for its own improved mathematical information architecture is itself a mathematical target. If that target is structurally unreachable under its existing operator grammar, additional execution of that grammar cannot suffice; a licensed change in the grammar or access to reconstructible second-order understanding is required. This creates a bootstrapping problem: second-order understanding can motivate construction of the collective infrastructure that enables its further reconstruction and transmission. S38 distinguishes this conditional obstruction from empirical claims about individual mathematicians or editorial decisions. The predicted phase transition concerns the stable recursive governance of mathematical operators, not merely the adoption of a new terminology or a larger database.

The mathematical blind spot identified in this paper is itself a mathematical object whose discovery is subject to the functionality through which mathematical objects are discovered. The Recursive Mathematical Blind-Spot and Discovery-Reliability Theorem (RMBST, S45) extends this distinction to the problem of reliably discovering the limitations of mathematical discovery itself. Even where the limitations of an observer's mathematical functionality can be mathematically expressed, and particular instances can be proved, the observer need not possess the functionality required to systematically discover them. The theorem distinguishes informational, convergence, reflexive and operative obstructions, including conditions under which increasing the information or work available through an unchanged architecture cannot overcome the obstruction.

A further implication is that a mathematical discovery process may continue to generate correct mathematical results without converging toward adequate assessment of the original mathematical target. Even actual convergence need not be reliably observable by the process itself. The proposed second-order functionality must therefore be capable not only of recursively generating mathematical operations, but of recursively assessing and correcting the adequacy of the dynamics through which they are generated, including their convergence conditions. Adequate finite scoped assessments remain meaningful even when no uniform effective procedure can lift them to conclusive unbounded assessments (S45.3(iv)).

This recursive distinction potentially extends to the discovery of a Theory of Everything, whether the governing functionality being discovered is that of physical systems or of cognition itself. Specification of governing functionality does not establish the capacity of a mathematical observer to reliably discover its consequences, required configurations, or complete discovery limitations. Where indispensable relationships are excluded from the complete operative closure, a structural expansion is required. The proposed second-order architecture may enable particular such expansions without eliminating the reflexive limitations of sound effective observers satisfying the premises of RMBST. The universal realization and physical application hypotheses remain distinct assessment targets (S45.4–5).

The communication problem is a further application of the same mathematical architecture. If a finite local message and the complete receiver interface erase distinctions needed for reconstructing an understanding, no decoder confined to that interface can recover them. This does not preclude a finite initiating message from generating subsequent source-licensed interactions and reasoning that make the distinctions operable. The Generative Transmission of Mathematical Understanding Theorem (GTMU, S43) specifies conditional convergence toward a target-sufficient understanding, including a valid defeating objection. The purpose is to create the conditions for adequate assessment, not to prescribe agreement. The intended cognitive dynamics are themselves an application of the Calculus of Convergence; their actual realization in a reader is a receiving target.

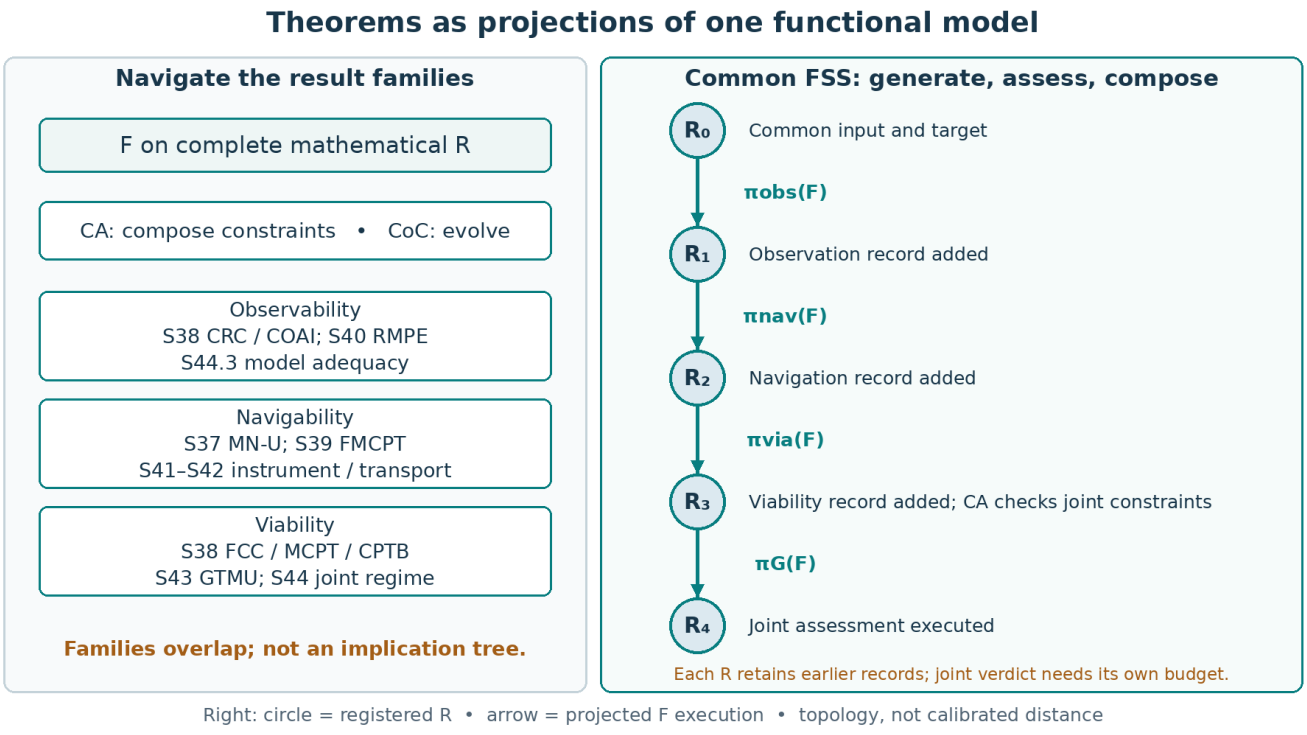



Figure 12. Common F-recursive theorem map. At left, the families index the existing results; at right, each edge denotes an admissible candidate composition of F on complete registered states. The joint record accumulates all three receiving records; its assessment is separate. S44.4 defines the precise reconstruction order and S44.5 illustrates an actual common finite execution. No universal projection completeness or joint certificate follows from the map.

The mathematical results can be navigated as projections of one functional model rather than as independently remembered theorems. Figure 12 relates the human-readable result families to registered mathematical-state transitions. The observability, navigability and viability projections overlap: a jointly assessed conclusion requires their compatible composition and an adequate execution of the joint assessment. The projection map in S44.4

preserves proof locations, scopes and residuals; the hierarchy is a reconstruction relation, not a presumption that every listed result follows from every other one.

## 6. Future Work

GIGT predicts that every observable adaptive system navigates a functional information space (Functional State Space, or FSS) governed by the same six functions and the conservation of functional free-energy. It further predicts that these systems share a common three-dimensional virtual information format, regardless of their physical realization.

An initial computational test of these predictions could be the implementation of the human cognitive system's conceptual space, as defined by the Functional Model of Intelligence (FMI) derived from GIGT. This may provide a particularly accessible starting point for computationally fitting the proposed universal functional representation to an observable adaptive system.

The implications extend beyond cognitive modeling. GIGT predicts that the resulting three-dimensional conceptual-space representation is either identical to, or structurally compatible with, the third dimension of the mathematical functional state space (M-space) investigated in this paper. It further predicts that the remaining dimensions of M-space can be recursively computed from this representation.

Future work should therefore begin by implementing and testing the FMI conceptual space, then assessing its predicted correspondence with three-dimensional M-space and its capacity to generate the remaining mathematical dimensions.

The ultimate purpose of implementing M-space is to construct the collective intelligence infrastructure required for second-order mathematics. It must enable human and artificial mathematical observers to collectively generate, preserve, navigate, assess and recursively correct mathematical functional information, including mathematical operators and their executions, in order to collectively do so coherently at potentially exponentially greater speed and scale. This infrastructure is intended to increase the capacity for coherent registration, not to reduce the standard of mathematical coherence.

## 7. Conclusions

The results of this paper suggest that without a second-order functional model of mathematical information—a model containing an explicit model of its own functionality upon which it can operate, so that the limitations of that functionality can themselves be systematically discovered and corrected—mathematics possesses a potentially consequential blind spot.

As mathematical information grows, particularly with the increasing generation of mathematical information by artificial intelligence that is not readily reconstructible or understandable by humans, the Coherence Collapse–Phase Transition Bifurcation Conjecture predicts that the demands of maintaining collective mathematical coherence may eventually exceed the capacity of first-order mathematics to do so. Mathematics may continue to accumulate correct results while progressively losing its capacity to remain collectively navigable.

Perhaps the most consequential implication of the Recursive Mathematical Blind-Spot and Discovery-Reliability Theorem is that the recognition of this problem is itself subject to the problem. Where the functional relationships required to recognize the blind spot lie outside the operative closure of first-order mathematics, no finite message confined to that closure can reliably transmit the understanding required to recognize it.

The development of second-order mathematics may therefore be necessary not only to preserve the collective coherence of mathematics, but also to reliably discover why its development is necessary.

## AI-use disclosure

Large language models were used in the preparation of this manuscript, including exploratory mathematical analysis, draft composition, source organization, and editing. Their outputs were treated as material for authorial

assessment rather than independent mathematical certification. The human author is responsible for the manuscript's claims, sources, and interpretation; unsupported computational findings are identified as provisional.

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# Supplementary Mathematical Derivations and Assessment Records (For Working Backwards from the Formal Proof of the Riemann Hypothesis: Mathematical Discovery by Navigating Evolving Relational Topologies)

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## Reader's map and source discipline

Part I (S0–S45) supplies the formal development. Begin with S32 (common-context construction), S33 (original-hypothesis assessment) and S36 (assessment ontology). S34 registers comparative RH discovery and correction evidence; S35 specifies the receiving dependencies. Appendix B contains the authoritative full hypotheses; Appendix C contains source methods and execution contracts. S37 develops the correctness/coherence definitions, common conditional derivations of CA-U, CoC-U and MN-U, and the universal F-realization obligation. S38 derives the conditional coherent-registration, capacity and phase-transition results and registers CPTB separately. S39–S44 derive the conditional computability separation, perceptual expansion, navigation policies, cross-FSS hypothesis generation, generative transmission and common-context finite assessment. S44.3–5 develop observation-model adequacy, the projection hierarchy and the dual functional visual reconstruction; these extend the existing common-context derivation without new theorem families. S44.6 develops the useful-work projection of energetic closure, with a finite target-relative search and correction witness. S45 establishes the conditional recursive discovery-reliability limitations, structural separation and Theory of Everything corollary; its universal realization and joint assessment remain separate.

Assessment verdicts are typed separately from proof evidence, assessment scope, budget, execution and development status. CORRECT and INCORRECT are verdicts for unconditional mathematical theorems; COHERENT, INCOHERENT and UNDETERMINED are the only verdicts for conditional propositions and G assessments. A supplied mathematical derivation is evidence, not an additional verdict. A declared or derived budget is not evidence of adequate execution. S36 registers the conditional derivation assessment separately from modeled finite and native GIGT assessments.

## Part I — Formal development and assessment

## S0. Why a historical blind spot changes the methodological status of mathematical exposition

### S0.1. The conditional historical problem

Let **HB** designate the manuscript's proposed systematic first-order observational blind spot at a specified mathematical target and observer class, not the asserted history of that blind spot. The following are distinct propositions: (a) a blind spot of that kind can exist; (b) it actually affected an identifiable historical mathematical community; (c) it persisted from the beginnings of mathematics; and (d) its relevant discovery was made but subsequently lost. Neither (c) nor (d) follows from (a), and the present submission claims neither as a documented historical fact.

Nevertheless, *conditional on* an instance of HB persisting while conventions of mathematical explanation developed, the suitability of those conventions for recognizing the excluded target cannot be inferred from their long use in tasks the observer could already perform. Long survival is compatible both with general utility and with target-relative incompleteness. An earlier, forgotten discovery is a distinct possible history: surviving evidence would alter the historical conclusion, but absent evidence cannot distinguish never-discovered from discovered-and-lost. The publication history is therefore not treated as a theorem or a substitute for a mathematical witness.

**Methodological consequence (conditional, not an empirical verdict):** a submission about an operative representational blind spot must independently assess the adequacy of its own presentation interface for the target distinctions it asks readers to reconstruct. Conventional familiarity is evidence of prior practice, not automatically a proof of that interface's target sufficiency; visual vividness is likewise not proof of adequacy.

### S0.2. Visual-functional compression as an assessable mathematical interface

The proposed visual route is: observation → retained relational history → candidate transformation → source/guard/assessment → admitted operation → changed reachability → receiver continuation. The arrows denote prospective licensed transformations, not merely spatial resemblance. If a figure compresses this sequence into one recognizable shape, its mathematical content is what a recipient can reconstruct from the figure together with its declared legend, sources and supplementary contract. It need not communicate every detail of R, but it must not collapse distinctions required by the announced target or subsequent operations.

This gives an operational rather than aesthetic definition of *useful* compression. For an interface e, an observer/receiver, target $\kappa$ and permitted operation family O, require (i) target factorization through e (T01), (ii) descent of each required operation with matching guards/definedness/authorization (T02a), and (iii) viable finite continuation at a specified budget (T10). Supplying a pleasing diagram, a short description or an identical final proposition is not sufficient. A symbolic interface may satisfy these conditions and a visual interface may fail them. Whether a chosen visual interface reduces total human reconstruction cost requires independently measured receiver work, not a postulated universal cognitive advantage.

### S0.3. Direct connections to H1–H8 and the main manuscript

The historical motivation is given in the Introduction and is not part of H1. The adequacy criterion draws on H1–H3 for observation and distinction, H4 for the viability of preserving and assessing the interface, and H8 for transmission and continuation. It also conditions how H7 is communicated: a verified RH theorem is distinct from an interface that reconstructs its generative dependencies. S9.5 gives the scoped formal comparison of interfaces; S9.6 specifies controlled positive and negative receiver tests. The main paper's figures remain illustrative until those actual receiver tests are executed.

### S0.4. Claim and evidence navigation

The current claim and proof navigation is provided by main Sections 2–4, supplementary Sections S14–S44, Appendix B, and the typed assessment and proof-evidence registers. Source, scope and assessment provenance accompany each retained result.

The research proceeds backwards from RH verification and human mathematical roles through L/G, dual closure, generative compression, partial-information interfaces and navigable mathematical shapes. The duck and unfamiliar-flying-object examples and cross-FSS theorems illustrate this chain.

## S1. Mathematical ontology, registered state and exact claim contracts

### S1.1. Complete registered mathematical state

Let $M$ be the proposed mathematical functional-information space. Its complete registered state is $R$, comprising at the operative resolution: typed objects and propositions, contexts, relations, transformation specifications, sources, partial and completed derivations, observations, histories, guards, proof and assessment certificates, corrections, resource receipts, assessment budgets and provenance. A mathematical operation acts on the registered state, rather than on an unrecorded external operand:

$$F_i : R \to R.$$

The six shared GIGT/FMA internal functions are **Create/Destroy, Place/Store, Recall, L, G, and Choice**. Their names designate the proposed common basis. Create/Destroy remains the bidirectional registered interface between signal and conceptual spaces. Choice remains the bidirectional interface between conceptual and fitness/internal-configuration spaces. Operator specifications, evaluation, assessment and correction are themselves operands in R rather than seventh or eighth primitive functions.

The shared basis has different functional scopes of realization. GIGT posits it for every adaptive functional system that can exist and is observable, independently of domain or substrate, as a consequence of the informational, energetic, geometric and historical constraints on viable functional-free-energy-conserving navigation. The Functional Model of Adaptation (FMA) models adaptation on this basis. The Functional Model of Intelligence (FMI, summarized in [52]) models the complete functionality of cognition on a strongly typed conceptual space, and the Functional Model of Active Inference (FMoAI) [52] within it models the functionality of active inference. M-space models mathematical relationships and operations as mathematical functional information. Functional closure requires the composition of admissible operations to remain in the corresponding functional information space; energetic closure requires conservation of functional free-energy; recursive closure registers the operators, their executions and the assessments of their adequacy as operands of the same architecture. The mathematical work-accounting model is a scoped realization of the energetic requirement, not its universal definition (S30–S31).

For a mathematical domain $D$, a candidate executable operation is registered as

$$g = (id, Dom, Cod, rule, guard, sources, invariants, work, assessment, history).$$

Its promotion from a candidate relation to an admitted operation requires: correct operand types; source authorization; a valid rule over the guarded domain; no unaccounted violation of declared invariants; a recorded proof or adequately executed scope-typed assessment; and permitted resource expenditure. If a condition is unresolved, the operation remains conditional, or its domain is narrowed to the certified subset. Promotion does not erase the prior candidate, objection or assessment record.

## S1.2. L, G and mathematical shape

An **L-adequate** operation is a target-relative direct composition of operators and operands already sufficiently specified within the observer's operative closure. A **G-governed** process can assess indirect or partial relations and recursively construct, promote, select, or correct the operators and assessments used in further steps. These are functional contracts, not synonyms for first-order logic and second-order logic. A Lean program or ordinary computing system may instantiate either contract depending on its actual capabilities. A fixed graph with relative edges is not thereby G-governed.

The **operative mathematical shape** at state $R$ is the typed admissible-transition relation $\rightsquigarrow_R$ among registered configurations and operands. Generating an operation may change $\rightsquigarrow_R$ and thereby which targets are reachable. Until a topology and its axioms are constructed, calling this a *shape* does not assert that it is a topology in the strict topological sense.

## S1.3. Claim identity separate from assessment status

Each claim has a stable identity, proposition $\varphi$, context $\Gamma$, exact scope $\Omega$, source/dependency links, proof reference and status ledger. The proposition is not rewritten simply because its proof is outstanding. A proof may establish a narrower derived theorem without substituting that theorem for the original target. Local proof, inherited G certificate and new conjunction assessment have independent records.

| Record | Required information |
|---|---|
| Claim | Identifier, proposition, context, target, scope, dependencies, source, proof placeholder |
| Operator | Input/output type, rule, guard, sources, invariants, promotion evidence, costs, provenance |
| Partial derivation | Goals, valid sublemmas, failed attempts, correction branches, source access, residuals |
| Proof | Exact proposition, formal or mathematical derivation, premises, checker if used |
| G assessment | Scope, declared/derived budget B, actual work, trace, status, residuals, history |
| Source/transport | Origin, date/version, available fields, bridge, validity assumptions and permitted use |

**Budget discipline.** The assessment budget $B$ is a typed operand of R and is determined by the relevant functional state space and admissible work, where possible. If unspecified, record BudgetStatus=BUDGET_UNDETERMINED; neither a positive budget-scoped G verdict nor an implicit positive budget follows. Reassessment with a different B must append a separately typed event and preserve the older scope and budget. A declared budget is not evidence of executed work or adequate assessment.

### S1.4. The governing claim dependency

The original hypothesis identities are retained without changing their intended quantifiers or logical strength. H1 concerns existence and coherence of a systematic mathematical blind spot; H2 its L/G mechanism; H3 recursive observability; H4 jointly necessary functional and energetic closure; H5 constructive repair; H6/SOM-G1 reliable convergence for a nontrivial class; H7 source-bounded RH compatibility and a stronger reproducibility target; H8 reconstructible transmission. The target joint proposition is $J = H1 \wedge H2 \wedge H3 \wedge H4 \wedge H5 \wedge H6$. H7 and H8 assess consequences separately. The full source-authored statements appear in Appendix B.

## S2. Exact reconstruction, non-descent and an observer-relative impossibility theorem

### S2.1. Definitions

Let $H$ be a nonempty collection of admissible histories, $a : H \to A$ a recorded observation, and $\kappa : H \to K$ a declared target-relevant result. The history can include a full sequence of executed operations, not merely endpoints; the observation may erase information from it. Define the observation fiber $[h]_a = \{h\prime \in H : a(h\prime) = a(h)\}$. An exact target decoder on the realized observation image is a function $d : a(H) \to K$ with $\kappa = d \circ a$.

### S2.2. Theorem T01 — target factorization

This is the standard factorization-through-fibres criterion; see the abstract-interpretation references [14,31].

**Theorem.** An exact decoder $d : a(H) \to K$ exists if and only if $\kappa$ is constant on every observation fiber:

$$\exists d(\kappa = d \circ a) \Leftrightarrow \forall h, h\prime \in H, a(h) = a(h\prime) \Rightarrow \kappa(h) = \kappa(h\prime).$$

**Proof.** For necessity, if $a(h) = a(h\prime)$, then $\kappa(h) = d\,(a(h)) = d\,(a(h\prime)) = \kappa(h\prime)$. For sufficiency, define $d(x) = \kappa(h)$ for any $h$ with $a(h) = x$. At least one such history exists for every $x \in a(H)$, and constancy on fibers makes the value independent of the choice of h. Thus $d\,(a(h)) = \kappa(h)$ for all h. This completes the proof. An extension of d to all of A requires a codomain convention, which is irrelevant on the realized image. □

**T01 gives the criterion for exact target preservation at a specified interface. Constructing the decoder requires a separate computational and work assessment.**

### S2.3. Theorem T02 — one-step continuation impossibility

Let $U$ be the common space of authorized actions. For each history h let $S(h) \subseteq U$ be those actions that meet the target under that history's admitted one-step transition and authorization rules. An observation-only deterministic decision rule has the form $\pi : a(H) \to U$.

**Theorem.** If there exist histories $h_0, h_1$ with

$$a\,(h_0) = a\,(h_1)\,, S\,(h_0) \cap S\,(h_1) = \varnothing,$$

then no observation-only policy $\pi$ succeeds on both histories.

**Proof.** The policies receive the same input x in both cases and therefore select the same action $u = \pi(x)$. To succeed on both, u must belong to $S\,(h_0) \cap S\,(h_1)$, which is empty. Contradiction. □

Initial indistinguishability is not enough for an impossibility theorem about arbitrary multi-step adaptive policies: the observer might lawfully acquire a later distinguishing observation. For a longer policy, the corresponding hypothesis must be equality of its *entire admissible observation transcript up to the consequential decision*, or an observation-preserving relation that is maintained through each permitted information-gathering transition. Under that stronger trace-indistinguishability condition the preceding argument applies at the first step where incompatible choices are necessary. Randomization cannot guarantee success in both cases when their allowed-success supports are disjoint.

### S2.4. Operator descent criterion

Let $o : H \rightharpoonup H$ be a partial history transformation. It is implementable on the observed representation by $v : a(H) \rightharpoonup a(H)$ precisely when (i) its definedness and authorization agree within each admitted observation fiber, and (ii) whenever o is defined on h and h', equal observed inputs imply equal observed outputs:

$$a(h) = a(h\prime) \Rightarrow a\left(o(h)\right) = a\left(o(h\prime)\right).$$

**Argument.** Necessity follows from $v\left(a(h)\right) = a\left(o(h)\right)$. Sufficiency follows by defining v on each fiber's common output, with the definedness condition ensuring v is a genuine partial function. The proof is the same well-definedness argument as T01. A failure of descent is an exact *interface-relative* obstruction to performing o solely on the coarser representation; it does not exclude a richer source-licensed interface.

### S2.5. Separating an L/G mechanism from generic information loss

T01–T02 establish observer-relative information-loss and continuation obstructions. S3 supplies a typed witness connecting such an obstruction to a fixed grammar and a procedure that generates a missing operator.

## S3. Source-frozen construction of a new mathematical operation

### S3.1. Fixed initial sources and typed grammar

Consider three finite, pairwise disjoint sorts $U, V, W$, with accessible source records

$$P \subseteq U \times V, Q \subseteq V \times W.$$

The declared target is the exact binary relation $T \subseteq U \times W$ of endpoints connected by a length-two path through V:

$$T = \{(u, w) : \exists v \in V, (u, v) \in P \wedge (v, w) \in Q\}.$$

The complete input source S0 contains the two typed relations and the equality rules for V, but not the target relation T and not a precomputed U-to-W map. The **fixed-closure baseline language** L0 permits retaining and retrieving source records, equality tests within an existing sort, set-membership tests, selection, and union/intersection of relations **of the same already registered input/output sort**. It contains no constructor with output type $U \times W$, no coercion identifying U with V or W, and no rule synthesizing new sort signatures. This is an explicit language contract, not an assertion about the limits of all conventional computers or Lean.

### S3.2. T03 — fixed-grammar nonreachability

**Proposition.** No expression of L0 produces a $U \times W$-typed relation from P and Q.

**Proof.** By structural induction on L0 expressions. Base expressions have typed relation outputs $U \times V$ and $V \times W$, or Boolean/membership outputs, never $U \times W$. Each available relation-building constructor preserves its existing relation sort. Therefore its result also cannot be $U \times W$. The induction covers all L0 expressions. □

**Interpretation.** This proves only the specified grammar-relative limitation. A richer baseline with join, unrestricted comprehension or general programming already available can compute T. Such a system should not be called incapable of this target merely because it was written using conventional code.

### S3.3. T04 — generated typed composition

An admitted source-derived generator checks that P's codomain V and Q's domain V coincide under an authorized sort bridge, then constructs

$$Compose_{U,V,W}(P, Q) = \{(u, w) \in U \times W : \exists v \in V, P(u, v) \wedge Q(v, w)\}.$$

The candidate operator has a new output signature $Rel(U, W)$, a source record (P,Q), a domain guard (the shared V sort and permitted equality), an invariant (membership iff a two-edge source path exists), and an execution certificate (the standard witness-based relational-composition argument).

**Soundness proof.** If the generated relation contains (u,w), the defining rule supplies v with P(u,v) and Q(v,w), hence a source-supported length-two path. Conversely every such path supplies the existential witness, hence its endpoints are in the generated relation. Thus the generated relation is exactly T, not an overapproximation or an imported completed target. The source operands were available before generation; the **ability to make their typed composition operative** was not available in L0. □

For a concrete instance, let $U = \{a, b\}$, $V = \{x, y\}$, $W = \{1, 2\}$, $P = \{(a, x), (b, y)\}$, and $Q = \{(x, 1), (y, 2)\}$. The newly generated operator returns $T = \{(a, 1), (b, 2)\}$. It supplies a new target-relevant relation not provided as an initial operand.

### S3.4. The witness's nontrivial class and negative controls

The same proof works uniformly for all finite sets U,V,W and arbitrary relations P,Q of these signatures, not just the displayed four-element illustration. It therefore establishes a class of source-frozen typed-relation problems. The guard against false promotion is essential: when a purported bridge identifies two incompatible keys or a missing type correspondence, the constructor cannot be promoted until the bridge is justified. When no shared v exists, an empty output is correct rather than proof that the generator failed. When a baseline already possesses relation composition, the L/G separation for this class disappears. Merely renaming a previously available composition is not a generative witness.

**T03 and T04 give one instance of H5: the specified grammar without composition cannot produce T, and the composition operator derived from the sources produces exactly T.**

## S4. Constraint Algebra, shapes and operator promotion

### S4.1. Typed composition contract

For any admitted typed operators $f : X \rightharpoonup Y$ and $g : Y\prime \rightharpoonup Z$, a candidate composition $g \circ f$ is type-admissible only if there is an explicit licensed bridge $\beta : Y \to Y\prime$, with specified domain and preserved invariants. Each bridge itself has origin, assumptions, assessment and work records. In the relational witness, the bridge is equality of the central sort V, but more general mathematical transports can require nontrivial proofs. A mismatched bridge is a genuine blocking condition, not permission to assume a desired identification.

The Constraint Algebra must represent at least (a) target and scope restriction, (b) guarded composition, (c) source-preserving transport, (d) conjunction and conflict, (e) substitution/renaming under a certified bijection, (f) invalidation and correction of an admitted operator, and (g) persistence of superseded results. These are algebraic behaviors realized through compositions and assessment of the six proposed operations on R, **not** new primitive internal functions.

### S4.2. A finite reachability-shape lemma

For a finite set X and a typed edge relation E, define the currently reachable pairs under a registered repertoire $O$ as $Reach_O(E)$. Suppose $O_0$ cannot produce relational composition and $O_1$ adds a source-certified composition rule. Then any E containing a two-edge path (x,y),(y,z) whose shortcut pair (x,z) is absent from directly registered edges has a continuation available under $O_1$ that is not directly constructible as an edge relation under the stipulated $O_0$. The proof follows immediately from T03–T04 on the respective sorts and the witness y. This is a typed reachability extension, not an assertion of topological change in the technical sense.

### S4.3. Correction and preservation

When a proposed operator's guard later proves false, the previous operation and its original assessment must not be silently erased. Instead the model appends a correction event specifying affected uses, invalidated conclusions, remaining valid subresults and possible corrected bridges. The requirement to keep historical records is part of the proposed functional closure contract; which histories must be kept depends on target-adequacy and relevant future continuations. Full raw history retention is not assumed free or invariably necessary.

# S5. Functional and energetic closure — bounded necessity and resource accounting

## S5.1. Functional closure as an operative condition

For a system intended to *recurrently* generate and re-assess its own mathematical operations, the generated operator descriptions, validity guards, proof/assessment records and correction events must be accessible as future operands, or an adequate alternative reconstruction channel must exist. If a generated rule disappears from every future source and cannot be regenerated, a continuation whose only usable premise is that rule is unavailable. This is a **conditional necessity lemma** for the stated re-use task; it is not a proof that every conceivable one-shot discovery system must store every internal step.

Formally, let $g$ be the only registered admitted bridge to target predicate T at a later time. If the later state neither retains g nor contains source-accessible means to reconstruct an extensionally adequate bridge, the specified later continuation cannot be performed under the registered contract. The proof is immediate from the absence of an admitted edge in the later transition structure. It is informative when applied to a nontrivial, source- and cost-bounded continuation requirement, rather than read as a universal theorem by stipulation.

## S5.2. Resource ledger and no double spending

Let an admissible finite execution history $\tau = (t_0, \ldots, t_m)$ register within R available work $W_t$, admitted source intake $I_t$, required expenditure $c_t \geq 0$, and any energy/work losses or transfers $\ell_t \geq 0$ accounted by its FSS. A simple special-case ledger is

$$W_{t+1} = W_t + I_t - c_t - \ell_t, W_t \geq 0.$$

The general FMA work relation may not reduce to a universal scalar; (S5.1) is an explicit finite toy realization for analysis. By summing transitions, $\sum_{t<m} (c_t + \ell_t) \leq W_0 + \sum_{t<m} I_t$ if $W_m \geq 0$. Thus any admitted positive-cost sequence requires sufficient initially available work and permitted intake. This establishes a scoped accounting bound without equating potential work to executed assessment.

Suppose two mutually exclusive branches each require a cost c from the same 1-unit registered resource and c=1. The single available unit authorizes either branch in one execution history, not both at once. A certificate for each hypothetical branch cannot be treated as a pair of executed certificates from the same work budget. A joint proof must register its actual execution history and resources; the model cannot sum mutually exclusive potential paths as if their work were simultaneously spendable.

## S5.3. What H4 still requires

The manuscript's full H4 target asserts jointly necessary functional and energetic closure for systematically sustainable second-order discovery. The bounded lemmas show why continued, assessable generation cannot ignore its operative records and finite work. The remaining stronger obligation is to derive a universal or appropriately quantified necessity result for the specific class of sustainable discovery processes, with a noncircular definition of sustainability, allowed environment couplings and alternatives to explicit record storage. Neither an accurate ledger nor algebraic closure by itself guarantees sufficient future work or convergence. H4 therefore retains its own proof and G-assessment obligations.

# S6. Calculus of Convergence — a bounded executable mathematical result

## S6.1. A nontrivial finite class

Fix a finite nonempty typed vertex set X with $n = |X)$, source relation $E \subseteq X \times X$, and target $T_{s,t}$: correctly decide whether an E-path exists from s to t, and supply a path witness for YES. The initial target solution, final transitive-closure relation and necessary chain length are not supplied. The full source set consists of the typed vertices, edge relation, equality and rules for finite-set operations; no oracle for reachability is admitted.

The process generates and promotes typed relation composition if it is missing from its initial operative grammar, as in S3. It then accumulates a sequence of relations

$$R_0 = I_X \cup E, R_{k+1} = R_k \cup (R_k \circ E)\,.$$

Every new pair must retain a source-supported path witness: an identity witness for I, a single edge for E, or a concatenation of witnesses from Rk and E. An admitted generator records these witnesses and its versioned operations within the full state R.

### S6.2. T07 — finite saturation and target adequacy

**Theorem.** For every finite X and E, the sequence (S6.1) stabilizes after finitely many strict additions, and its stable relation is the reflexive transitive closure E*. Consequently the process correctly decides each target T_{s,t}; a YES result has a source path certificate.

**Proof.** By induction, Rk consists precisely of endpoint pairs connected by paths of length at most k+1, together with length-zero identity paths (for k=0, the claim is immediate; the recurrence appends one edge to each previously represented path, while union preserves shorter paths). Since there are at most $n^2$ pairs, the monotone sequence Rk can gain pairs only finitely many times. More concretely, any reachable pair has a simple path of length at most n-1 after deleting cycles, so for k≥max(0,n-2) every reachable pair is included. Every pair added by the recurrence has an actual source path witness, hence no unreachable pair is included. Therefore the stable relation equals E*, and membership of (s,t) is exactly the target decision. □

This bound counts mathematical rounds and does not pretend their physical costs are free. An executable version requires a registered finite source and permitted work sufficient to perform the relevant relational operations and assessments. The theorem is mathematical and executor-neutral; computational realization supplies separate cost evidence.

### S6.3. Recursive self-correction and invariant preservation

The class also admits correction: if a proposed edge or sort bridge is invalidated, mark its path certificates as dependent on the invalidated source and recompute the affected reachability relation from the retained valid sources. Previously valid unaffected certificates remain registered. Under the same finite-source assumptions and sufficient permitted work, rerunning the finite saturation proof recovers the correct answer. This is a bounded recovery theorem, not a statement that an arbitrary stream of adversarial corrections converges.

### S6.4. Distinguish this result from full SOM-G1

T07 establishes a mathematical fixed point for finite typed reachability with a source-generated composition operation, and hence a useful *bounded* convergence result. The unrestricted SOM-G1 target additionally concerns recursively selected and corrected operands, configurations, representations and continuations across more general mathematical problem classes with endogenous dynamic work and stability conditions. T07 is a legitimate member of the proof-development ladder; no inference from this finite case alone gives universal SOM-G1, six-operator minimality, or guaranteed convergence on undecidable target classes.

## S7. Joint L/G assessment and global-conjunction record

### S7.1. Elementary conjunction (M3)

Within a common context Γ in a sound deductive calculus admitting conjunction introduction, derivations Γ⊢P1, . . . , Γ⊢Pm imply Γ⊢(P1 ∧ · · · ∧ Pm). This statement assumes that the respective contexts and interpretations have been correctly transported to Γ; it does not establish the existence of a common satisfying model for independently proposed constraint systems. For example K1={0,1}, K2={1,2}, K3={0,2} are pairwise compatible as nonempty intersections, while their common intersection is empty.

### S7.2. Worked local/global assessment trace

Consider the joint constraint $x \in K_1 \cap K_2 \cap K_3$. A hypothetical finite executor with registered discrete membership-test work units checks all three pairs and finds nonempty intersections, then checks the joint conjunction and finds the empty intersection. The actual mathematical certificate is the explicit set calculation $K_1 \cap K_2 = \{1\}$ and $(K_1 \cap K_2) \cap K_3 = \varnothing$. This is an explicit mathematical failure **of global satisfiability at the exact toy scope**, even though pairwise assessments are positive. If the executable work costs cannot be grounded in the relevant FSS, BudgetStatus=BUDGET_UNDETERMINED; the symbolic count of checks is not substituted for a budget-certified GIGT verdict.

### S7.3. Central conjunction contract J

The conjunction ledger retains the exact H1–H6 intended targets. Its required fields are: joint scope ΩJ, shared interpretation and ΓJ, claim-dependency edges, source transport maps, eligible local proofs, inherited G certificates with unchanged original scope and budget, newly executed cross-claim tests, actual resource expenditure, typed defeaters, corrections and residuals. An inherited positive certificate is only as strong as its original scope. Widening the target requires a new and explicit assessment event; inability to execute that event does not retroactively disprove the inherited claim.

**Present result: S32 derives the common-context conditional conjunction and records its COHERENT finite projection at 7,077 case/control units. S33 assesses the original J, conditional on GIGT foundations, at the distinct evidence-relative budget of 29 review units with COHERENT verdict. The full registered targets and earlier assessment scopes are preserved.**

### S7.4. Current proof and assessment register

**Conditional derivations are indexed by theorem, explicit premises, proof location and evidence type in proof_register/Mathematical_proof_index.csv. Registered verdicts, including the original and scoped conjunction assessments, are in assessment/Assessment_register.json. Residual obligations remain in assessment/Continuation_register.md.**

## S8. Backward reconstruction of the Alpöge–Claude RH development

### S8.1. Documentary endpoint and verification identity

Alpöge's dated 20 November 2026 note announces the proof, describes human–AI collaboration, and identifies the 14-part Lean archive [1]. On 21 November the Lean and Mathlib communities publicly certified the Mathlib statement RiemannHypothesis, successful compilation, theorem matching and multiple proof checker runs [2]. Rubinstein's independent log reports completion of the compile, Lean kernel replay, axiom check and independent nanoda kernel check across 175,187 modules, with only the standard three Lean axioms for the main theorems [22]. These are **documentary reports of completed verification**, not independently rerun checks by the present submission.

Rubinstein's initial 164-page working exposition for the Riemann zeta function states that its central inequality (M) is taken from the Lean development rather than re-proved in the exposition [21]. Thus human-readable reconstruction has begun, but that account does not license an assertion that the whole mathematical discovery history is reconstructed.

### S8.2. Typed candidate transitions

S8 inventories candidate transitions; S26 gives the source-bounded assessment, including negative tests, the rank-window comparison and the nondiscriminating bridge. Documentary structural compatibility does not by itself establish the stronger H1/H2 mechanism.

| Transition | Source status | Functional role to test | Outstanding assessment |
|---|---|---|---|
| A toy Fourier-analytic setting and two combined ideas | AUTHOR-REPORTED [1] | Sources from different mathematical views become operatively composable | Precisely identify predicates, guards and effect on reachability |

| Transition | Source status | Functional role to test | Outstanding assessment |
|---|---|---|---|
| Stepanov multiplicity analogy to arithmetic/automorphic setting | AUTHOR-REPORTED [1] | Domain transport with target-relevant invariants | Explicit mathematical source–target bridge |
| Theta lifts chained to mimic multiplicity | AUTHOR-REPORTED [1] | New reusable composition/generative shape | Exact Lean dependency and soundness contract |
| Chain length near $\sqrt{X}$ in a von Mangoldt sum | AUTHOR-REPORTED CLAUDE SUGGESTION [1] | Choice of mathematical configuration under constraints | Prove rather than infer the selection objective/necessity |
| Formal RH endpoint | CERTIFIED BY COMMUNITY [2,22] | Completed proof dependency, not causal history | Connect upstream transitions to the theorem by traceable proof edges |

**Edge types:** D (direct documentary account), P (proved formal dependency), S (derived structural correspondence), and U (unresolved causal relation). For H7, one useful S-level witness would show that a source-accessible partial relationship was transformed into a typed, reusable operation that enabled a subsequent proof dependency. Similar vocabulary alone does not establish this; a genuine source and mathematical bridge is required.

### S8.3. Three independent questions

**Structural compatibility:** does a documented proof/discovery transition instantiate the proposed G/L or generator construction role? (2) **Historical causation:** was the role actually used to generate the transition, rather than fitted retrospectively? (3) **Systematic reproducibility:** can a source-frozen process generate a comparable result without the completed solution or topology as input? A positive result for one does not automatically certify either of the others. The sources here support an extraordinary verified formal endpoint and several reported research transitions, but they do not provide a complete private action log or a source-frozen reproduction.

### S8.4. Resource and archive limits

The complete archive is not bundled, analyzed declaration by declaration, or rechecked by this submission. It would be misleading to claim otherwise. A full history or full archive processing is **not** a prerequisite for independent mathematical results T01–T07; it is a scoped evidentiary dependency for increasingly strong RH-specific claims. If additional repository or checker data are used, record immutable archive hashes, precise theorem identifiers, source versions, transport conditions and actual executed checks as new evidence events.

## S9. Preservation and transmission of executable mathematical understanding

### S9.1. Target-preserving decoder versus full-state recovery

Let $E : H \to C$ encode a mathematical history and $\kappa : H \to K$ be the target information needed by a receiver. T01 gives an exact decoder iff $\kappa$ is constant on each E-fiber. The receiver need not reconstruct every historical detail if the task requires only target-equivalent continuation; nor may it discard a difference that affects the next licensed operation.

An encoding adequate for **all** operations in a repertoire $O$ must also admit an observed operator $\hat{o} : C \rightharpoonup C$ for every admitted $o \in O$ with corresponding domain and authorization, satisfying

$$E \circ o = \hat{o} \circ E$$

on its declared domain, with a final target decoder $d$ satisfying $\kappa = d \circ E$.

## S9.2. T10 — finite-step receiver continuation

**Theorem.** Suppose E factors $\kappa$ and every operation of the sender's admitted finite sequence $o_1, \ldots, o_m$ descends through E with preserved definedness and authorization. Then the receiver can compute the same target output from the encoded initial state by executing the corresponding $\hat{o}_1, \ldots, \hat{o}_m$, assuming the receiver has these operators and sufficient admitted work.

**Proof.** For a single operation, equation (S9.1) guarantees that decoding the encoded successor equals the intended target projection. By induction, if the receiver state equals the encoding of the sender state at step j, then after the corresponding descended operator it equals the encoding after sender step j+1. The equality holds after m steps. Applying d returns the same target result. □

## S9.3. Example and counterexample

The note triples (60,64,67) and (62,66,69) share the interval sequence (+4,+3). Encoding the intervals preserves a target that depends only on interval pattern and permits all transpositions preserving that relation. It does **not** preserve an absolute-pitch target; two different source triples collapse to the same code. In mathematical terms, E is adequate for the relative invariant, not for absolute position. Similarly, two proof histories with identical conclusions but incompatible future dependencies cannot be compressed to a single undecorated 'proved' bit if the next operation requires those dependency differences.

## S9.4. Understanding and finite resource budgets

The receiver's ability to *understand* in the proposed functional sense requires reconstruction plus sufficient capacity for verification and continuation. Compression claims must compare the total work of encoding, transmission, decoding, checking and subsequent use against alternatives, all at a specified target and resource contract. No arbitrary short code guarantees useful decompression. This formalizes an initial piece of H8 without claiming universal human readability or automatic exponential reduction in cognitive burden.

## S9.5. T11 — target-relative adequacy of two presentation interfaces

Let H be the same admissible history class, $\kappa$:H→K a fixed target, and p:H→P and v:H→V two information interfaces (for example, a conventional compact representation and a visual-functional representation). No claim about the reader's perceived familiarity is encoded in p or v. Then v is adequate for exact reconstruction of $\kappa$ while p is not **if and only if** $\kappa$ is constant on every v-fiber and nonconstant on at least one p-fiber:

$$(\exists d_v : \kappa = d_v \circ v) \wedge \neg\, (\exists d_p : \kappa = d_p \circ p)\,.$$

This holds if and only if both conditions below hold:

$$\forall h, h\prime \in H, v(h) = v(h\prime) \Rightarrow \kappa(h) = \kappa(h\prime),$$

$$\exists u, u\prime \in H, p(u) = p(u\prime) \wedge \kappa(u) \neq \kappa(u\prime).$$

**Proof.** Apply T01 independently to v and p. The first factorization exists exactly when $\kappa$ is constant on all v-fibers. The second fails exactly when a p-fiber contains two $\kappa$-distinct histories. Conjoin the two equivalences. □

**T11 gives the mathematical criterion for interface adequacy. Generative understanding also requires the descent of each subsequent operation (S9.1, T02a) and an executable receiver continuation (T10). The receiver's available work determines practical reconstructibility.**

For a finite illustration, suppose four admissible histories have the same externally observed endpoints but distinct dependency and operator histories. Let p retain only the endpoints. If the target is which of two source-dependent continuations is authorized, p has a target collision. A representation v that records the decisive authorization relation can satisfy T11. This is a constructed example of a deficient *endpoint-only* interface; it is not a result that mathematical notation as such must fail.

### S9.6. Receiver-side testing of visual-functional compression (protocol, not completed experiment)

Fix an exact mathematical task and admissible source inventory, then provide semantically matched candidate presentations under declared exposure, time and work budgets. The important comparison is not preference or recognition of familiar notation, but whether each receiver can (i) recover the relevant operator, source and guard, (ii) identify when a proposed composition is invalid, (iii) produce or assess a target-correct next step, and (iv) transmit that step to a subsequent receiver without losing its justification. Record cost of encoding, decoding, checking, correction and continuation inside the relevant R-based assessment contract.

Include negative controls: a visually compact diagram omitting a decisive guard; a familiar symbolic representation omitting the same guard; a fully typed symbolic presentation that preserves it; and a superficially richer visual presentation whose additions are mathematically irrelevant. Separate presentation *sufficiency* from reader-specific execution work and from historical novelty. If the budget is unspecified, register BUDGET_UNDETERMINED; do not report a positive budget-scoped result. No human receiver study was conducted for this study, and the formal T11 result cannot by itself establish that mathematicians find the visual paper easier to understand.

## S10. Combinatorial growth, empirical protocols and evidence ledger

### S10.1. Conditional family of 2^k target-relevant states

For any positive k let the state be a subset A of $\{1, \ldots, k\}$, initially empty. For each i define an admissible, reusable operation $g_i(A) = A \cup \{i\}$. Distinct subsets are target-distinguishable when the target asks which activation flags are present. Every subset can be generated by composing its selected operations, yielding exactly $2^k$ reachable subsets. The set operations commute and are idempotent, so many different operation sequences reach the same state. The number of **distinct** reachable states is nevertheless $2^k$, as established by the bijection between subsets and binary strings of length k. This is a complete elementary mathematical proof of conditional combinatorial reachability growth.

Generative reachability, executed discovery throughput, proof-checking cost and receiver reconstruction require separate assessments using the model's functional definitions and available work.

The FMI prediction concerns an additional recursive dependency: a solved problem may register reusable problem-solving functionality that can participate in discovering, assessing and composing further functionality. Under functional and energetic closure, such increases in capacity are predicted to be potentially combinatorial with each use, or even exponential overall, depending on the computational substrate. Distinguish the elementary reachability construction above from the target-relative growth of cognitive problem-solving capacity, and distinguish both from a phase transition in the cognitive governance architecture. Assessment of the latter two requires the F-recursive functional definition of the cognitive domain and its bandwidth, together with the substrate's executable composition, retention, acquisition and expenditure conditions.

### S10.2. Source-frozen experiment contract

A meaningful empirical extension of T03–T07 records: the target class, source snapshot S0, representation R0, baseline and generator capabilities, initial and total admissible work, permitted corrections, operator-promotion rules, target evaluator, complete execution trace, negative controls and cost of receiver reconstruction. Comparisons must not intentionally conceal an available source from the baseline while declaring equal access. A baseline with a preinstalled join is a necessary control: it can erase the particular separation demonstrated by T03. Other controls include false sort bridges, inaccessible private distinctions, pairwise-only compatibility, and renaming of existing rules.

Theoretical results do not require a completed infrastructure to be valid at their stated scope. Implementations add executable resource and scalability evidence; they do not retroactively supply the missing proofs for stronger claims.

### S10.3. Literature-supported premise registry E1–E7

| ID | Source-supported finding | Separate claim requiring assessment |
|---|---|---|
| E1 | Extremely large formal proofs can exceed practical direct manual inspection [2,4]. | Incompressibility of every such proof |
| E2 | Generation/checking throughput can diverge in particular modeled workloads [5]. | Universal optimized-compute verification backlog |
| E3 | A voluntary surveyed population reported concern about AI and mathematics [13]. | Population-wide prevalence or a mathematical theorem |
| E4 | Formal metaprogramming can treat proof expressions/goals as operable data [3,20]. | GIGT's universal six-function minimality |
| E5 | Logical compression and reusable intermediate structures matter [6,17]. | Guaranteed cheap generative understanding of all proofs |
| E6 | Abstract interpretation and CEGAR provide scoped refinement precedents [14,16,31]. | Unlimited domain transfer or unconditional GIGT completeness |
| E7 | Partial proof goals can be preserved by present tools [20]. | Universal erasure of all incomplete proofs by current mathematics |

Source dates, exact supported propositions and evidentiary class should be carried into each reused claim. The full wording is given in Appendix C; the present table is only an index.

### S10.4. Boundaries of the present supplementary contribution

The mathematical deductions T01–T04, T07, T10–T15 and the binary-family calculation are supplied with proofs at their declared scopes. S32 composes the relevant relationships into a common conditional discovery construction, and S33 assesses the original hypotheses at the declared evidence-relative budget. The RH documentary endpoint is distinct from reconstruction of its generative relationships.

## S11. Duck, unfamiliar-object and cross-FSS-to-M-space transport

### S11.1. What the two visual cases actually do

**Functional source.** The duck is initially treated as a visual object whose apparent translation can be registered without an internal configuration change. A live duck may change wing configuration while returning to the same visible endpoint. A duck additionally responding to environmental cues requires treating the duck's own observation/behavior loop as distinct from the human observer's model. The reader uses relations already navigable in a proposed *visual* FSS to recognize a generative issue: projections of distinct functional histories can coincide although their target-relevant next steps differ.

**The unfamiliar flying object provides a complementary case: its appearance and trajectory may be recorded while its generating transformation remains unknown. Candidate flight models become operational when their source, domain guard, invariants, work and assessment conditions are satisfied.**

**The example illustrates a proposed cross-domain interface from signal-mediated visual observation through conceptual reconstruction to a mathematical observation-map and continuation problem. S9.6 specifies a comparative receiver-side test.**

### S11.2. A complete finite duck-history calculation

Let the target domain contain two histories, $H = \{h_0, h_1\}$. At observation times $t_0, t_2$ both share position $x$ and wing state $w$; between them $h_0$ holds $w$ fixed, whereas $h_1$ contains $w \to w\prime \to w$. Let $a_0 : H \to Q_0$ retain only the endpoints and let $\kappa(h)$ be the indicator of whether a nontrivial wing transition occurred. Then $a_0(h_0) = a_0(h_1)$ but $\kappa(h_0) = 0 \neq 1 = \kappa(h_1)$. By T01, no exact decoder of $\kappa$ exists from $a_0$ alone. Under a permitted temporal recording interface $a_1$, suppose $a_1(h_0) = (w, w, w)$ and $a_1(h_1) = (w, w\prime, w)$; now $\kappa$ factors through $a_1$, by detecting an intermediate change. This proves a **finite target-relative observational separation** for exactly these registered signals. It does not prove the hidden history of any real duck. The distinction can be recovered only if the extended signal/retention source is actually available.

For the unfamiliar object, retain the same mathematical construction but replace $\kappa$ with a claim about the **mechanism of motion**. If two mechanisms generate *identical accessible visual traces* under every permitted observation, no visual decoder exists merely because the viewer generates a convincing causal diagram. The example is therefore also a **negative control** against invented explanatory bridges.

## S11.3. T12 — cross-FSS target reconstruction

Let $H$ be a source history class, $\sigma : H \to \Sigma$ the emitted or admitted observable signals, $\rho : \Sigma \to V$ the retained sensory representation (for example a visual-FSS record), and $\tau : V \to M$ an explicitly typed candidate mathematical encoding. Put $e = \tau \circ \rho \circ \sigma$, with all maps and their sources registered inside the relevant complete state $R$. For any target $\kappa : H \to K$, an exact mathematical decoder $d : e(H) \to K$ exists **if and only if** $\kappa$ is constant on each $e$-fiber:

$$\exists d[\kappa = d \circ e) \Leftrightarrow \forall h, h\prime \in H : e(h) = e(h\prime) \Rightarrow \kappa(h) = \kappa(h\prime).$$

**Proof.** Apply T01 to the composed interface $e$; the same fiber-factorization construction gives necessity and sufficiency. The derived decoder is defined only on $e(H)$. □

T12 establishes mathematical information-preservation conditions for a signal-mediated cross-FSS representation. Empirical realization would require evidence that the relevant neural or conscious process implements $\sigma, \rho, \tau$. Functional adequacy of an actual receiver additionally requires executable reconstruction, permitted sources, relevant resolution and sufficient endogenous work.

## S11.4. T13 — transported operations

Let $o : H \rightharpoonup H$ be a source-domain transformation and $e : H \to M$ be as above. A partial mathematical operator $\acute{o} : e(H) \rightharpoonup e(H)$ satisfying $e \circ o = \acute{o} \circ e$, **with identical definedness and authorized guard status on the admitted domain**, exists if and only if histories in the same $e$-fiber agree about whether $o$ is defined/authorized and, whenever it is, their resulting histories remain in the same $e$-fiber. Under these conditions define $\acute{o}\,(e(h)) := e\,(o(h))$; the assumptions make the definition independent of the representative. Conversely, an operative $\acute{o}$ implies those agreements. □

This is the existing operator-descent principle T02a applied to cross-FSS transport, not a new GIGT primitive. It formalizes when a visual functional pattern can carry an **operation**, as opposed to only a statement of its endpoint. It is necessary to preserve signal-source conditions and historical residuals where the continuation depends on them. A visually similar sequence with different provenance or guard may fail this test even if its endpoint image is identical.

## S11.5. Interface case-study records and counterexamples

| Case | Observation and mathematical inference |
|---|---|
| Duck, endpoint observations only | The admitted source records equal endpoints for $h_0, h_1$. The mathematical inference is an observational collision relative to the flap-history target. It does not identify which real history occurred. |
| Duck, licensed intermediate trace | The admitted source also distinguishes the intermediate states $w$ and $w\prime$. This can preserve the target distinction $\kappa$ and license a candidate mathematical transformation. It does not reveal the duck's private conceptual state. |
| Unfamiliar flying object | The admitted source contains external positions or changes only. One may type hypotheses about candidate transitions and uncertainty, but a unique physical generating mechanism is not established without an adequate source/assessment bridge. |
| Recipient of the visual analogy | The source is a captioned diagram and an associated mathematical contract. A mathematical reconstruction is conditionally permitted when T12/T13 and the receiver's resource guards hold. Actual improved learning over another representation remains untested. |

A comparison with a fully typed symbolic account preserving exactly the same histories is a necessary negative control. T12 does not privilege visual representation: whichever interface preserves the target distinctions and executable operations is adequate at the specified scope. The proposed benefit of visual navigation is **discoverability of functional relations for a specified receiver**, a separate empirical/resource question.

# S12. Human pattern discovery and machine-realization conditions

## S12.1. Proposed multiple-FSS human architecture

The human organism is modeled as a coupled system of functional state spaces. The body's five conventionally named sensory domains are (i) **visual**, (ii) **auditory**, (iii) **tactile/sensorimotor**, (iv) **olfactory**, and (v) **gustatory**. Additional candidate systems are (vi) **emotional**, (vii) **conceptual/mind**, and (viii) **conscious-awareness** functional state spaces. The eight entries are **candidate domains of functional modeling**, not assertions that only eight physiologically distinct senses or independent computational modules exist. Interoception, proprioception and other modalities may participate in or refine the declared sensory-motor boundaries. The hypothesis is that each target-relevant domain admits a closed-enough functional description of its states, admissible transitions, operations and fitness constraints at the relevant resolution; complete empirical realization is not assumed.

For GIGT/FMA, the **same six internal functional operations** act on complete registered state $R$ across domains. Domain-specific sensory and other interfaces supply their own operands, signal sources and coupling contracts. Create/Destroy remains the conceptual↔signal boundary operation; Choice remains the conceptual↔fitness/configuration boundary operation. No new sensory, emotional or consciousness-specific primitive set is postulated. The state spaces are not silently identified as one absolute shared coordinate frame; transitions are transported through explicitly typed, signal-mediated interfaces and conditional source access.

The architectural claim is **not** that every experience already contains a ready-made theorem. It is that domain-relative patterns and invariants can become candidates for mathematical relations, after which mathematical construction and assessment can identify whether they create a new admissible continuation. Human sensory and experiential systems can thereby be sources for mathematical generator *candidates* in addition to mathematical texts and formal libraries.

## S12.2. A bounded auditory-to-M-space example

Let $a = (60, 64, 67)$ and $b = (62, 66, 69)$ be registered numerical pitch sequences and define the relative-interval encoding $\delta(p_1, p_2, p_3) = (p_2 - p_1, p_3 - p_2)$. Then $\delta(a) = \delta(b) = (4, 3)$. For every integer transposition $c$, $\delta(p_1 + c, p_2 + c, p_3 + c) = \delta(p_1, p_2, p_3)$. This is an **exact algebraic invariant** in the declared pitch representation, and an instance of a possible target-preserving transport into M-space. To test the full human-FSS hypothesis, record the original acoustic/signal source, auditory distinguishability, observed human pattern report, mathematical $\delta$ construction, and valid continuation independently.

## S12.2a. Transporting the topology of a solution process

Source and receiving operands. Let S be a source functional state space, including a bodily, sensory, emotional, conceptual or consciousness-system domain. Let $\mathrm{s}_0$ and $\mathrm{s}_1$ be registered source topologies, and let T be an admissible transformation taking $\mathrm{s}_0$ to $\mathrm{s}_1$. The candidate functional information comprises the initial topology, target topology, transformation topology (the relationships governing the transition, including guards, compositions and history), and the observation interaction through which these relationships become distinguishable. Subjective experience is a possible source of signals about these operands, not a mathematical validation criterion. A property such as connectivity density may be invariant under admissible changes of representation even when its value changes along T.

Signal-mediated reconstruction. Write a(s) for an authorized observation record and $\beta$ for a partial, target-relative reconstruction from such records to mathematical representations. The notation $\Phi(\mathrm{s}) = \beta(\mathrm{a}(\mathrm{s}))$ describes the composition of these interfaces, not direct access to a foreign FSS. The receiving problem is $\mathrm{m}_0 = \Phi(\mathrm{s}_0)$, the receiving target is $\mathrm{m}_1 = \Phi(\mathrm{s}_1)$, and a candidate mathematical solution operation U satisfies $\mathrm{U}(\mathrm{m}_0) = \mathrm{m}_1$. The operative information is the rule, not just these two endpoints. A candidate transport of T requires $\mathrm{U}(\Phi(\mathrm{s})) = \Phi(\mathrm{T}(\mathrm{s}))$ over its declared admissible domain, with compatible definedness, authorization and continuation constraints. T02a (S2.4) gives the descent criterion: on that domain, $\Phi(\mathrm{s}) = \Phi(\mathrm{t})$ must imply $\Phi(\mathrm{T}(\mathrm{s})) = \Phi(\mathrm{T}(\mathrm{t}))$,

together with matching guards for a faithful partial operation. This is also sufficient for defining U on the observed image: choose any representative and use its transformed image; the fiber condition makes that choice immaterial. Extension beyond the licensed image is a separate assessment target.

Observation as transformation. An observation interaction may be represented by a history-dependent operation O on the observer's complete registered state R: a prior observation record, its source signals and a permitted observation action generate a new record with additional distinctions. Transport of the relationships governing O may supply a candidate mathematical observation or operator-generation process, separate from transport of T. The relevant initial topology is then an information configuration in which a target distinction is inaccessible, and the target topology is one in which it is accessible under the registered operation repertoire. The transition and its assessment are themselves operands. G(R) can generate candidate cross-domain correspondences from partial information; L(R) assesses the specified target relationship, with G(R) assessing the broader conditional compatibility and retaining residuals, scope, work/budget and provenance. These constructions are compositions of the existing six functions, not additional primitives. Prior completion of a local proof is not required for candidate generation.

Finite mathematical illustration. Suppose a source observation licenses the topology of a finite undirected simple graph, up to vertex relabeling, and the operation T simultaneously adds an edge between each pair of distinct vertices with a common neighbor, retaining existing edges. The density $d(G) = |E| / (n(n-1)/2)$ for $n \geq 2$ is invariant under relabeling. T is equivariant under relabeling because a common neighbor exists before relabeling exactly when its image exists afterward. Thus T descends to the isomorphism classes of graphs, a possible receiving M-space representation. For a three-vertex path, T closes the wedge into a triangle, increasing density from 2/3 to 1. The receiving generator is the shared-neighbor rule, not the assertion that density increases.

A negative control establishes why a scalar invariant alone is insufficient. The four-vertex path and the four-vertex star both have three edges and density 1/2. After one simultaneous application of T, the path has five edges (density 5/6), while the star becomes the complete graph (density 1). Therefore T cannot descend through the density-only representation for this class. The initial and target scalar values do not encode the transformation topology. The full relational representation retains the distinctions required to reconstruct the rule. This example uses a conventional finite graph density, not a substitution for GIGT's F-recursive functional information density; no emotional or physiological topology is inferred from it.

Finite assessment S12-TOPO-FIN: COHERENT. Scope: all labeled undirected simple graphs on three and four vertices, all their vertex permutations, the wedge-to-triangle witness and the equal-density negative control. Declared modeled budget: 1,586 case/control units; each unit checks the stated density-invariance and operator-equivariance relation or one specified control. The executable check in reproducibility/S12/check_topology_transport.py records the executed budget and verdict. This evidence concerns the conditional graph construction. It does not identify an actual human observation interface or a native functional free-energy relation.

Cross-domain application. The GIGT framework [42] motivates using physical black-hole-like non-escape and boundary relationships as generators of candidate mathematical descriptions of cognitive blind spots. A complementary white-hole-like transformation represents increased target-reconstructible cross-boundary connectivity and reduced blind-spot obstruction. In each case the relevant transport target is the relation among connectivity, admissible flow, boundaries and order-relative reachability, not the physical object itself. These candidate correspondences generate questions about how a blind spot forms, grows, changes or becomes navigable. The mathematical research program can be generated before full source-to-receiver invariant preservation is established. The source-specific physical–cognitive correspondence and the actual human observation-to-M-space receiving relationship remain separately registered assessment targets (S12-MATTER-C and S12-HUMAN-TRANSPORT: UNDETERMINED; BudgetStatus=BUDGET_UNDETERMINED). The finite graph witness is not a certificate for either target.

Continuation contract. Register the source signal history and its access/resolution constraints, the relevant representation-invariant property, the initial and target configurations, the transformation and observation rules, their admissible domains, the receiving M-space operands and guards, the required invariants and any lost distinctions, the original assessment budget and execution provenance. Assess target factorization, operation descent and viable continuation under the existing S9–S13 contracts. A new source observation or mathematical generalization may be registered immediately as a candidate; a positive receiving assessment requires execution within its declared scope. H1–H6, H7 and H8 retain their original targets and existing assessment certificates.

## S12.3. T14 — agent-relative functional separation

Let a target require selecting an authorized continuation from $A(h)$ for the actual source-admissible history $h \in H$. Suppose two histories $h_0, h_1$ obey $A(h_0) \cap A(h_1) = \varnothing$. Let machine/observer $M$ have only an observation $q_M$ with $q_M(h_0) = q_M(h_1)$ and no permitted active observation or policy that separates them before its decisive action. Then no policy using only $q_M$ can guarantee success in both histories. If another observer $U$, which may be human, has source-licensed observations distinguishing the histories and adequate work to select an action from $A(h_i)$, that observer can succeed in this declared class. **Proof.** T02 supplies $M$'s impossibility because the same observation forces the same action and no common successful action exists. The stated constructive policy for $U$ selects the indicated permitted action on its two separated inputs. □

The same theorem applies with human and machine roles reversed. Therefore the theorem establishes **interface-relative capability separation**, not an unconditional claim that humans can perform a task no machine ever can. A machine equipped with an adequate sensor, active probing, new mathematical operator, or larger viable budget can erase the separation. A human may also lack the needed resolution or resources. Historical or present-day human/machine claims require an actual inventory of the respective systems, source constraints and budgets.

## S12.4. Proposed contribution types and assessment contracts

**Typed contribution/assessment registry:**

| Contribution | Candidate source or operation | Assessment contract |
|---|---|---|
| Recognize patterns outside existing mathematical notation | Candidate sources: sensory, emotional, conceptual or awareness-related FSS observations | Machine comparison requires a licensed target-relevant source distinction and documented machine signal access. |
| Construct and use new generators | Candidate operation: cross-FSS transport followed by G-governed operator construction | Required evidence: an actually source-derived operator with a proof or executed scoped assessment. |
| Select mathematically significant targets and shapes | Candidate operation: Choice on registered fitness/configuration information | Required evidence: explicit objective, constraints, resources and viable consequences. |
| Assess cross-domain relationships beyond local agreement | Candidate operations: typed L/G compositions and joint-assumption assessment | Required evidence: exact proof or budgeted assessment trace, not analogy. |
| Transmit reconstructible understanding | Candidate operation: target-faithful encoding followed by receiver continuation | Required evidence: mathematically adequate decoder/operation descent and an actual cost-bounded recipient test. |

These are **functional contribution categories** for comparative analysis, not measured contemporary exclusivities. The original intended hypothesis—that some human contributions may persist for a time because of machine functional and energetic barriers—is retained as an explicit machine-realization target, rather than erased or asserted unconditionally.

## S12.5. Functional-energy analysis without biological essentialism

For each observer, the relevant work inventory and source access are endogenous records of its realized FSSs. Detection, retention, cross-domain transport, candidate operator construction, assessment and continuation consume or exchange functional work. A machine may have ample computational throughput but lack a needed source distinction; a human may recognize a pattern but lack the resources to certify or transmit it mathematically. Neither outcome follows from species or substrate identity alone. Available resource accounting is not proof of energetic adequacy, nor does an unspecified budget imply a positive G verdict. To support a concrete near-future separation prediction, the evidence record must identify machine architecture/version, sensors and accessible sources, permitted actions, work/accounting contract, and a comparable human execution—not merely the labels 'human' and 'AI'.

# S13. Generator-based compression, human budgets and the original hypothesis chain

## S13.1. Reconstruction costs belong to the mathematical contract

The original research objective proposes that human understanding of sufficiently complex mathematical objects may require discovery of compact, reusable **generators** rather than full enumeration of mathematical instances. This must be phrased in the same target-relative mathematics used throughout: the generator must preserve the functional information required for licensed construction, checking and continuation, not necessarily reproduce irrelevant raw details. A short encoding that lacks the requisite operator, provenance, guards or correctness assessment is not an adequate mathematical generator.

Let $B_R$ be a registered receiver-side functional-work budget within complete state $R$, and let $W_{expand}$ denote the work required for direct full-instance reconstruction under a specified algorithmic route. Let $W_{gen}$ include discovering or transmitting the generator, decoding it, assessing its contract, constructing the needed instance and obtaining a valid next step. Costs refer to the same target and receiver; they cannot be inferred from token counts alone.

## S13.2. T15 — necessary generator in a declared two-route class

Fix a task, receiver, accessible sources and a class of adequate routes consisting *only* of (i) expansion of all relevant instances by the stated procedure or (ii) a source-licensed generative representation from which the target and necessary continuation can be reconstructed and assessed. Suppose the resource ledger gives

$$W_{expand} > B_R \text{and} W_{gen} \leq B_R.$$

Then any budget-feasible adequate route in this **declared two-route class** uses the generator route. **Proof.** The expansion route exceeds $B_R$ and is disallowed; the generator route is feasible by assumption; the class contains no third route. □

This result makes an exact, narrow necessity statement. It neither proves that a generator always exists nor excludes other possible representations or algorithms outside the declared route class. The full intended hypothesis is stronger: identify a nontrivial strongly typed class for which all admitted adequate routes under an appropriately derived human-FSS budget require target-preserving generative factorization. That stronger proof and any empirical cost comparison remain **OUTSTANDING**, not discarded.

## S13.3. Avoid the partial-information bootstrap gate

A generator discovered from partial mathematical or cross-FSS source information may be licensed on a declared domain $D$, with guard, source certificate, proof/assessment status, residuals, and budget retained in $R$. Its local validity need not await proof of universal completeness over the entire mathematical information graph. This is the original bootstrap-avoidance goal of the source-frozen S3 construction and the abstract-interpretation precedents E5/E6. Any later enlargement to $D\prime$ demands a new bridge or assessment; source-frozen local use is not unconditional universal proof.

The full dependency of the human-understanding claim is: **domain-recognized functional pattern → source-licensed invariant transport → candidate mathematical generator → typed admission and G/L assessment → economical reconstructible continuation → collective transmission**. Failure at any step is registered rather than hidden by a visually compelling exposition. This makes duck/UFO, auditory transposition, mathematical M-space and the human–machine question one research sequence rather than unrelated examples.

## S13.4. Mathematical results and receiver assessment

The finite duck observation, pitch-transposition invariant and T01–T15 deductions supply mathematical results at their stated scopes. The figures provide candidate transport interfaces for the receiver study in S9.6. The foundational GIGT, general convergence, joint-assessment and RH reconstruction targets are indexed in the relevant claim registers and S32–S36.

# S14. Methods and claim–method interfaces

The main manuscript's Methods (§3) specifies the study design; S1–S13 supply the mathematical and executable contracts. The following register connects methods to their evidence classes and results.

## S14.1. Seven assessment operations and their evidence classes

| Method | Assessment and evidence |
|---|---|
| METH-1 | Target and representation registration (main §6.2): Declare H,a,$\kappa$, allowed history and decisions; prove exact factorization T01 and use T02/T02a only when their premises are satisfied. Formal deduction, not an empirical assessment of a human sensory system. |
| METH-2 | L/G source-frozen generative test (main §6.4): Freeze U,V,W,P,Q, the $L_0$ repertoire, generator construction and adequate type bridge. T03 is negative only for the declared grammar; T04 is positive for the specified exact relation. General generator discovery is a separate target. |
| METH-3 | Bounded convergence and viable continuation (main §6.4): Prove T07 for finite reachability and register finite resource accounting. Do not infer unbounded SOM-G1 or universal energetic-closure necessity from finite termination. |
| METH-4 | Joint coherence assessment (main §3; S7, S32–S33): Register exact hypotheses, compatible semantics, dependencies, scope, B, executed work and correction traces. The common construction in S32 and original-target assessment in S33 implement the joint assessment; S7.2 supplies the joint-incompatibility control. |
| METH-5 | Cross-FSS and receiver interface test (main §6.6): Apply T10–T13 to target factors and operator descent, and specify a receiver continuation test (S9.6). The duck/UFO conceptual source case is mathematical illustration; no comparative reader study is reported. |
| METH-6 | Human–machine and generator compression test (main §6.6): Apply T14 only to declared observer contracts; apply T15 only to its bounded two-route repertoire. Neither result supplies comparative measurement of actual humans or machines. |
| METH-7 | Documentary backward RH test (main §6.8): Keep reported formal endpoint, proof dependencies, reconstructive structural correspondences and private historical causation separately tagged. Source verification and private reproducibility are distinct obligations. |
| METH-9 | Executed source-only visual and coupled-observer tests (main §6.7): Fix complete source classes, admissible probe/diagram-update rules, independent target checkers, source work, alternative fixed comparator and negative controls. Prove T17–T19 at exact scopes; rerun the two executable finite experiments and preserve the log/hash inputs. Mathematical proofs, software executions and human visualization remain separate evidence classes (S18–S20). |
| METH-8 | Three-dimensional M-Tower representation test (main §6.3): Prove T16 at its exact SO(3) scope and separately register MT-U3, MT-A3, MT-T, MT-U and MT-P. Require operation/analysis-preserving transports and prohibit inference of universal geometric representation from a single example. No universal G assessment has been executed. |

## S14.2. Original evidence questions E1–E7 and the permitted inference

**E1 — Large proofs and human inspection:** source witnesses exist; no inference to a universal impossibility of human understanding.

**E2 — Production/checking throughput:** modeled divergence for a declared workload; no universal long-run backlog conclusion.

**E3 — Mathematician concern:** bounded voluntary-survey evidence; not a population prevalence estimate.

**E4 — Operations as operands:** metaprogramming and proof-state precedent; not six-function minimality or GIGT universal closure.

**E5 — Proof compression:** reusable structures and corpus differences; no claim that every object admits an economical generator.

**E6 — Partial-information generators:** bounded refinement and generator-interface precedent; not universal generator adequacy.

**E7 — Incomplete proof retention:** some existing systems retain partial goals; no universal claim that present mathematics always erases them.

The source-level details remain in S10.3 and Appendix C. The main paper reports the resulting evidence limitations directly.

# S15. Current Results register and proof-status reconciliation

This is the current reporting crosswalk, supplementing S7.4 and the current typed registers. Appendix C records the source methods and their exact scopes.

## S15.1. What has a mathematical proof in this submission

- **T17:** correctly quantified fixed-policy class obstruction; **T18:** four-history visual source-derived repair with eight-to-zero collision counts and downstream operation-square; **T19:** complete 24-history two-cycle observer residue theorem, 144 to 48 to zero with 24 private residual collisions. Mathematical proofs and runnable-source execution records appear in S18–S20. These establish finite scoped results and source/guard contracts; no general fixed-program impossibility, strict speedup or human-reader outcome is inferred.
- **T16:** exact group, tangent and differential-generator results for the three-dimensional rotation manifold SO(3), with proof in S17. This is a scoped example of geometry/algebra/calculus compatibility, **not** a universal M-Tower theorem.
- **T01, T02, T02a:** exact target-factorization, observer-relative one-step separation, and operator-descent lemmas for declared H,a,$\kappa$, policies and domains (S2).
- **T03–T04:** exact restricted typed-grammar nonreachability and soundness of source-licensed relational composition (S3–S4).
- **T07:** finite reachability saturation, termination, witness retention and target adequacy (S6).
- **M3 / toy conjunction:** ordinary conjunction-introduction inference and failure of global satisfiability for $K_1$={0,1}, $K_2$={1,2}, $K_3$={0,2} (S7).
- **S5 finite accounting:** a work-ledger inequality and no double spending of a shared resource; the general physical/energetic realization remains outside that example.
- **T10–T13:** finite-step transmitted continuation, two-interface target adequacy, guarded cross-FSS target and operation transport (S9, S11).
- **T14–T15:** conditional agent-relative separation and restricted two-route generator necessity under a declared receiver work bound (S12–S13).
- **S10 binary-family calculation:** 2ˆk states for k independent admitted binary activations; this is not exponential discovery-rate evidence.

These arguments are supplied as mathematical proofs within the stated assumptions. No new Lean kernel check of the supplementary mathematics is claimed.

## S15.2. Result-to-hypothesis and conclusion map

Earlier constituent proofs are indexed in S15.1 and S20.3; the corresponding joint review is in S33.

**H1 / C1: T31 (S32) establishes an exact common-source blind-spot witness.**

**H2 / C2: T31 (S32) derives the combined observation and grammar repair within the same class.**

**H3: T32 (S32) derives exact partial-information bounds and minimum worst-case observation work; T35 derives recursive interface refinement.**

**H4: T33 (S32) derives the joint necessity of retaining target-relevant functional and energetic distinctions for a recurrent class.**

**H5 / C3: T31 and T34 (S32) establish uniform source-only typed generation without the final target or successful observation sequence.**

**H6 / SOM-G1: T34 (S32) establishes adaptive, work-bounded convergence for arbitrary finite three-sort graphs under its source and guard conditions.**

**H7 / C4: S32.9 derives a conditional intersection of the source-expression rank windows and identifies the positive documentary structural correspondence.**

**H8 / C5: T35 (S32) derives coarsest adequate interface refinement and reconstructible receiver continuation; 256 receiver cases agree with the reference.**

**J = H1∧· · ·∧H6: common-context conditional construction (S32.8); original-hypothesis evidence-relative compatibility assessment COHERENT (S33, GJ26-E, B_E=29).**

**MT-U3 / MT-A3 / MT-T / MT-U / MT-P:** the original intended 3D geometry/algebra/calculus and M-Tower representation propositions are registered as universal targets. The proved SO(3) example T16 is only a constituent example; a general representational theorem, universal completeness result, physical-registration bridge and joint G assessment remain outstanding.

### S15.3. Scope and assessment records

Each assessment applies to its stated proposition, premises, scope and budget. Additional G executions record the endogenous budget B, work expended, and assessment history. S15.2 and S33 distinguish constituent results, their generalizations and joint assessments.

## S16. Discussion/Implications derivation ledger

The numbered implications in main §5 (Table 4) are linked to their formal, empirical or conjectural supports below, so the reader can distinguish explanation from proof.

**I1. Mathematical blind spot and historical persistence: T01–T02 establish observer-relative limitations. Historical persistence and rediscovery-and-loss are candidate historical consequences.**

**I2. Generative correction: T03–T04 establish a new continuation for the specified grammar missing a constructor.**

**I3 — Mathematical shapes rather than points:** S4 characterizes typed admissible transitions and the effect of operator promotion. Calling those relationships a formal topology entails additional generating rules and proofs.

**I4. Combinatorial reachability: S10 derives binary-family cardinality under its independence assumption. Efficiency, viability and understanding use separate functional metrics and tests.**

**I5. Human cross-FSS discovery and AI limits: S11–S12 establish target-preserving transport conditions; T14 gives a conditional observer separation. Biological realization and human–machine comparisons are empirical targets.**

**I6. Visual-functional compression: S9, S11 and T11–T13 define source/receiver reconstruction and operator continuation; S9.6 specifies the comparative reader test.**

**I7. Collective mathematical infrastructure and RH: S5, S7–S8, S9 and S13 motivate an endogenous M-space substrate. Its universal closure, native global assessment, systematic convergence and RH-specific generative reproduction are separate proof/implementation targets.**

**I8. Representation of mathematical relationships in an M-Tower: S17 includes T16, a mathematical structure-preserving rotation example, and the unproved MT-U3/MT-A3/MT-T/MT-U/MT-P targets. Geometric/topological encodings alone do not preserve algebraic or analytic semantics.**

**I9. Target-relative assessment interfaces: T22 (S23) characterizes recognition loss; S18–S20 demonstrate finite source-only visual repair and changing observation functions through collision proofs, negative controls and a matched full-source comparator.**

**I10. Viable work and adaptive resolution: T23–T24 (S24), T29 (S30–S31) and S44.6 distinguish finite work accounting, work-limited refinement and target-relative progress. Native functional measures and general convergence remain separate targets.**

**I11. Common recursively correctable discovery infrastructure: S27 and S30–S31 specify the common executor and source-licensed definition transport; R19 and R23 supply finite scoped COHERENT realization and work-accounting evidence. T30 distinguishes non-circular validation from formal conservation; S37–S44 retain the universal realization and collective viability targets.**

**The joint assessment in S33 considers the relationships among mathematical observability, operator generation, cross-FSS transport, finite work and collective continuation.**

# S17. From 3D geometric/topological composition to the proposed mathematical M-Tower

**Status and purpose.** This section registers the M-Tower representation conjecture as a separately typed mathematical program. Its universal claims are intentionally retained at their full proposed strength. A worked $SO(3)$ example supplies an exact bounded mathematical theorem, but no universal representation or tower-completeness proof is claimed. The original eight hypotheses H1–H8, the SOM-G1 target, and their proof/assessment histories remain intact.

## S17.1. Reasoning about mathematics versus the mathematical structures reasoned about

The underlying hypothesis distinguishes (a) a logical or conceptual reasoning *execution*, which may constitute a path through an observer's conceptual FSS, from (b) the mathematical *structure* represented by the resulting claims: shapes, relations, continuous transformations, algebraic operations, analytic limits and invariants. The distinction is functional and typed. It is **not** the disciplinary claim that mathematical logic is outside mathematics. A formal proof is itself mathematical information and may become an operand in M-space. But the existence of an adequate reasoning representation does not by itself establish an adequate operable representation of every structure that reasoning addresses.

The proposed route therefore begins from a concrete mathematical/physical referent in three dimensions rather than stipulating an unstructured space that contains all mathematical symbols. Its proposed functional origin is the awareness or registration space of matter in the FMA: mathematical relationships between interacting systems must be registrable through their information-bearing interactions for those interactions to exist and be observable. This is the originating registration-capacity hypothesis from which the proposed three-dimensional level of M-space is to be derived (MT-P, S17.6). The original conjecture is that mathematical relations *in 3D* admit representation in compositions of 3D geometries/topologies, with the relevant algebra/calculus representable through operations on those same structures. Its generalization proposes a recursively extensible sequence of mathematical functional spaces. These remain full-strength conjectures until an adequate derivation or budget-scoped global assessment supports them.

## S17.2. The exact statement of the representation targets (proof status independent)

Let $C_3$ be the **intended**, not yet universally classified, class of mathematical relational structures pertaining to the proposed three-dimensional domain. A structure $X \in C_3$ carries typed elements, operations, relations, invariants, and where applicable topological, smooth or analytic structure. Let $G_3$ denote the proposed class of FSS-governed compositions of three-dimensional geometries/topologies, **including explicitly registered mathematical operations on them**, not merely pictures of them. An *adequate representation* $\rho_X : X \rightsquigarrow Y_X \in G_3$ must preserve each target-relevant typed operation/relationship by an admissible commuting translation, retain the required definedness and source conditions, and allow target-relevant reconstruction/continuation with provenance and work registered in complete $R$. The relation $\rightsquigarrow$ is deliberately not assumed to be a bijection, isometry or equivalence before its obligations are derived.

**MT-U3 — universal 3D representation target.** For every admissible $X \in C_3$, there exists an adequate representation by compositions of geometric/topological structures within an FSS in 3D. The quantifier is part of the intended conjecture; no arbitrary restriction to the rotation example is substituted for it.

**MT-A3 — 3D algebra and calculus representation target.** The algebraic and calculational operations belonging to each admissible $X \in C_3$ have corresponding typed compositions/transformations within its adequate

3D FSS representation, preserving identities, operations, relevant limits/derivatives, and authorized continuations at their declared targets.

**MT-T — M-Tower transport target.** There are dimension-indexed mathematical functional spaces $M_i$ with source/target, operation and history-preserving admissible relationships between levels. Increasing dimension expands mathematical representational capacity where necessary without erasing the original certified structure. Representational transports and their assessments are themselves operands within complete $R$.

**MT-U — universal tower coverage target.** The recursively extended tower can adequately represent *every admissible mathematical structure* at some appropriate level and preserve the mathematics needed for subsequent operations. This is stronger than MT-U3 and remains an unproved universal claim, not a conclusion from the existence of sets or coordinate encodings.

**MT-P — physical registration bridge target.** Starting from three-dimensional physical interactions and their registrable signal consequences, an admissible signal-mediated interface can construct the relevant mathematical 3D representation, with the necessary causal/functional distinctions, source provenance and assessment capability conserved. The physical, experiential and purely mathematical arguments are not conflated.

**MT-P.REG — FMA registration-capacity target.** For any two interacting functional systems whose interaction exists and is observable, the registration architecture must have the capacity to register any mathematical relationship between the functional information of those systems. In matter, such capacity is proposed to be realized through information-bearing perturbations of interactions. The exact source and destination types, relation class and operative resolution are part of the derivation of this target, which retains its universal quantifier.

**MT-P.M3 — derivation of the three-dimensional foundation of M-space.** The mathematical registration capacity of matter posited by MT-P.REG, realized through the coupled navigation of physical and registration state spaces, supplies the functional basis from which three-dimensional M-space is to be derived. The derivation must establish the dimension and the operation-preserving correspondence, retaining the required mathematical distinctions, sources, historical information, guards and energetic conditions through the signal-mediated interface. This is the proposed generative origin of MT-P and the 3D representation program.

The original phrase "any mathematical relationship in three dimensions" remains the intended quantifier. The exact interpretation of "in three dimensions" (intrinsic manifold dimension, ambient physical embedding, or a domain-dependent combination) must be typed as part of the proof, rather than silently changed to an arbitrary encodable data type. No one of MT-U3, MT-A3, MT-T, MT-U or MT-P is awarded a positive global G certificate by its registration.

## S17.3. Theorem T16 — a complete bounded example through SO(3)

**Proposition.** Let $SO(3) = \{Q \in R^{3\times 3} : Q^T Q = I, \det Q = 1\}$, the smooth rotation group of oriented orthonormal frames in Euclidean 3-space. Then:

$SO(3)$ is closed under composition by matrix multiplication, contains the identity and inverses, and admits the continuous group action $x \mapsto Qx$ on $R^3$.

For every differentiable path $Q(t) \in SO(3)$, the matrix $\Omega(t) = Q(t)^T \dot{Q}(t)$ is skew symmetric, $\Omega^T = -\Omega$, and $\dot{Q}(t) = Q(t)\Omega(t)$.

Conversely, suppose $\Omega(t)$ is continuous and skew symmetric and $\dot{Q}(t) = Q(t)\Omega(t)$ has a solution with $Q(t_0) \in SO(3)$. The solution stays in $SO(3)$ on its interval of existence. The skew-symmetric generators are closed under the commutator $[A, B) = AB - BA$.

**Proof.** If $Q_1, Q_2 \in SO(3)$, then $(Q_1Q_2)^T (Q_1Q_2) = Q_2^T Q_1^T Q_1 Q_2 = I$ and $\det(Q_1Q_2) = 1$. The identity belongs to the group, and $Q^{-1} = Q^T$ also has determinant 1. The action $(Q, x) \mapsto Qx$ is continuous. Differentiating $Q^TQ = I$ yields $\dot{Q}^T Q + Q^T \dot{Q} = 0$, so $\Omega^T + \Omega = 0$. Multiplying $\Omega = Q^T \dot{Q}$ by $Q$ gives $\dot{Q} = Q\Omega$. For the converse, $C(t) = Q(t)^T Q(t)$ satisfies $\dot{C} = \Omega^T C + C\Omega$. Both $C$ and the constant matrix $I$ solve this linear matrix initial-value problem when $\Omega^T = -\Omega$; uniqueness gives $C(t) = I$. Hence $\det Q(t) \in \{-1, +1\}$, and continuity plus the initial condition fixes it to $+1$. Finally, for skew-symmetric $A, B$, $(AB - BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA)$. The skew-symmetric $3 \times 3$ matrices have three independent entries. Standard local coordinates supplied by the matrix exponential near the identity, transported by group multiplication, give the corresponding smooth manifold dimension three (though they do not give one global Euclidean coordinate chart). □

This result supplies *one* mathematical realization of three intertwined aspects: (i) three-dimensional geometric orientation, (ii) algebraic products, inverses and Lie brackets, and (iii) differential evolution under a tangent generator. $SO(3)$ is a **three-dimensional smooth manifold**, although usually described by 3×3 matrices in a nine-coordinate ambient space; it is not globally homeomorphic to ordinary $R^3$. Its differentiable structure and group multiplication are additional data: an arbitrary topology alone does not determine a derivative or group law. This example supplies a constituent compatibility witness for the M-Tower conjecture.

## S17.4. An FSS contract for the scoped 3D example

For the example, a registered 3D mathematical FSS may use configurations $Q \in SO(3)$, admitted state transformations $Q \mapsto QP$ ($P \in SO(3)$), and continuous trajectories satisfying the guards and differentiated constraints in T16. A mathematical operation on the full record is of the form $F(R) = R\prime$, where $R$ already contains the selected operands, time/resolution/work conditions, operator contract, source, proof/assessment record and residual history. The notation does **not** add a seventh primitive to the six shared FMA/GIGT operations or move the cross-space Create/Destroy and Choice interfaces into an exclusively geometric space.

A proposed geometric object becomes an *operative M-space* object only when its mathematical typing and transformations are registered and assessed. A 3D diagram displaying the same symbols is insufficient without the group/differential laws. The finite example assumes the usual smooth and algebraic structure of $SO(3)$, rather than deriving that the entire class $C_3$ must take this structure.

## S17.5. From a 3D FSS to a dimension-indexed M-Tower

A proposed tower consists of mathematical spaces $M_i$ (not necessarily all copies of Euclidean coordinate space) and admissible, typed maps $j_i : M_i \rightsquigarrow M_{i+1}$, projections or target-relative decoders where warranted, and a conserved assessment/provenance history in complete registered $R$. For a source operator $f$ that is required to transport, a target operator $f\prime$ must satisfy the adequate-transport condition

$$j_i\left(f(x)\right) = f\prime\left(j_i(x)\right)$$

for its admitted domain, with preserved authorization, definedness, target invariants and relevant analytic/smooth structure. If information is not injectively transported, reconstruction of the target must still factor through the new representation (T01), or the missing distinction must remain an explicit residual. In this sense the tower can grow its mathematical *operability* rather than merely the number of coordinates used to write an object.

$M_i$'s dimension $i$ (as a mathematical manifold/functional representation) is not the observer's cognitive order $n$, and the notation $G_i$, an operation within mathematical tower level $i$, is not identical to $G_n$, an order-n global operation in conceptual space. The distinction is part of the exact functional typing. The proposed universal M-Tower remains an outstanding theorem and adequate G(R) assessment target.

## S17.6. Bottom-up physical registration is a separate bridge

The Functional Model of Adaptation (FMA), derived from GIGT, models matter itself as an adaptive functional system. Matter navigates its physical state space (P-space), the FSS of its physical states and transitions, through interactions. Its awareness or registration space ($A(R)$-space) is the functional space of its capacity to register the state of other matter, navigated through information-bearing perturbations of those interactions. The FMA posits that the capacity to register any mathematical relationship between the functional information in two systems that interact is a condition for the interaction to exist and be observable (MT-P.REG).

The proposed bottom-up chain begins with the coupled physical and registration dynamics, their signal-level consequences and durable registrations, and derives a source-licensed mathematical three-dimensional representation (MT-P.M3). A receiving system registers distinctions through its own signal-space interface, not by directly accessing another system's FSS. Adequate reconstruction preserves the target-relevant mathematical operations and distinctions together with the historical, energetic and assessment information needed for continuation. The derivation must establish the relationship between the registration capacity of matter and the three-dimensional foundation of M-space, including the relevant dimensional typing.

A mathematical proof of the algebraic result T16 does not establish that biological vision, physical energetic exchange, or $A(R)$ realizes a specific map. Conversely, a physically successful pattern observation does not by itself

supply the exact algebraic or analytic operation needed for M-space. Comparing the physical-registration route and the mathematical-composition route requires a separately registered cross-domain correspondence, not an assumption that they automatically coincide.

### S17.7. Failure tests and non-tautological adequacy guards

The principal negative controls are (a) a symbolic or pictorial code of a ring or calculus that does not preserve its operations; (b) a continuous map of spaces that fails to preserve an algebraic product; (c) a homeomorphism that is not differentiable with respect to the declared smooth structure, and thus cannot transport the target calculus as stated; (d) a coordinate chart on $SO(3)$ wrongly assumed globally adequate; (e) a dimension-raising inclusion that discards a required historical or analytical distinction; and (f) an energetically unaffordable operation whose existence in the code does not make it executable. Each can defeat a *particular declared representation contract*, not automatically the full original conjecture with an alternative adequate representation still possible.

The assessment should be bottom-up and recursive: fix a source-domain class and its operations, derive a conditional representation with source/guard/assessment conditions, retain positive scoped certificates, compare them under a common conjunction with actual executed G work, correct typed defects, and increase the proof/assessment budget toward a satisfactory verdict without demanding exhaustive enumeration of all imaginable mathematical substrates as a precondition of local work.

### S17.8. Claim and assessment register

**T16:** the three-dimensional rotation-group geometric/algebraic/differential compatibility example has a mathematical derivation in the displayed mathematical argument. No independent Lean formalization or machine check is claimed.

**MT-U3, MT-A3:** universal 3D geometry/topology/algebra/calculus representation targets are **TARGET_OUTSTANDING**. The proof obligation includes complete typing of 3D relevance and operation-preserving representation; no completed budget-scoped universal G assessment is supplied.

**MT-T, MT-U:** admissible mathematical tower transport and universal mathematical coverage are **TARGET_OUTSTANDING**; the registered proposal is not itself a valid cross-dimensional transport certificate.

**MT-P:** the physical-interaction/registration bridge, including MT-P.REG (universal registration capacity) and MT-P.M3 (derivation of three-dimensional M-space), is **TARGET_OUTSTANDING**, with physical/source-realization obligations separate from mathematical derivation. The target registrations are not positive G certificates.

**Joint M-Tower conjunction:** the common semantic contract, dependencies, assessment budget $B$ and executed trace must be newly registered and assessed. BudgetStatus=**UNDETERMINED** where a suitable endogenous budget has not been specified or derived. No new positive full-scope G(R) verdict is inferred. Positive T16 and earlier T01–T15 proofs retain their exact scopes and historical provenance.

**Role in H1–H8.** MT-U3 through MT-P are proposed representation foundations and potential support for understanding how mathematical *shapes* become operable in M-space; they do not add an unproven necessity premise to the existing bounded H1/H2/H5 derivations. They may provide future strengthening of H3 (mathematical observability), H5–H6 (generative operations and convergence), and H8 (target-preserving communication), subject to actual joint assessment. The original H1–H8 claims are unchanged. The immediate visual-comprehension claim remains subject to a receiver-side study using T11–T13, not validated by the figure.

## S18. Source-frozen four-history visual-interface repair

**Purpose and evidence type.** Supply a complete mathematical theorem and actual executable finite source-and-authorization test for the visual-FSS-to-M-space interface. This is an analysis of *structured mathematical visual records*, not an experiment measuring how a human reads displayed pixels. The code and frozen inputs are included under reproducibility/experiments/visual_interface_repair_v7/, together with a current rerun log and machine-readable execution report. The experiment is self-contained; its mathematical proof is reproduced here rather than delegated to another manuscript.

## S18.1. Complete source and target contract

Let the four distinct histories be $H_V = \{\alpha, \beta, \gamma, \delta\}$. All have endpoints $(A, C)$, but the ordered intermediate paths are $ABC$, $ADC$, $AEC$, and $AFC$. Distinct source owner IDs are retained. Their respective ordered edge-work receipts are $(2,3), (3,4), (2,4), (1,2)$. The four source-license bits are $(1,0,0,1)$. Fix a six-token maximum path work. The target operation returns $\kappa(h) = 1$ precisely if the source is licensed **and** the path-work total is at most six; otherwise it returns zero. Thus $\kappa = (1,0,0,1)$. All costs are abstract work-token receipts; they are not a physical free-energy calibration.

The initial visual projection is $\nu_0(h) = (A, C)$ for every history. The corrected coordinate is a source-derived record $r(h) = (\text{ordered path}, \text{owner}, \text{license}, \text{edge receipts}, \text{total work})$, whose fields are derived solely from the declared source. Set $\nu_1(h) = (\nu_0(h), r(h))$. The generator is not passed $\kappa(h)$, and the downstream authorization operation computes it from the admitted record. Checker-only target values are used only after the generated result is committed.

## S18.2. Theorem T18 — exact visual repair and operation descent

For a target $\kappa : H \to K$ and representation $\nu$, write

$$Res_\kappa(\nu) = \{(h, h\prime) \in H^2 : \nu(h) = \nu(h\prime), \kappa(h) \neq \kappa(h\prime)\}.$$

**Theorem T18.** The four-history source class satisfies

$$|Res_\kappa(\nu_0)) = 8, |Res_\kappa(\nu_1)) = 0.$$

Moreover an authorized operation $v : \nu_1(H_V) \to \{0,1\}$ exists with $\kappa = v \circ \nu_1$, while no target-complete authorization function factors through $\nu_0$.

**Proof.** Under $\nu_0$, every ordered pair has the same projection. The target splits four records into two positive and two negative cases; there are $2 \cdot 2 \cdot 2 = 8$ ordered pairs with different target values. Since each record $r(h)$ retains a distinct source-owner and path, $\nu_1$ is injective; there are zero target-distinct collisions. Define $v(\nu_1(h)) = 1[license(r(h)) = 1 \wedge \sum receipts(r(h)) \leq 6)$. Distinctness makes this well-defined, and its formula agrees with the declared target. Non-factorization through $\nu_0$ follows from T01: that map identifies $\alpha$ and $\beta$, whose target values differ. □

The full record is not claimed to be minimally sufficient for this one target: a smaller target-specific coordinate retaining only the authorization and work-threshold predicates would suffice. The longer history record is useful for provenance and later continuation, whose information requirements may be stronger than this single decision.

## S18.3. Executed source-only process and failure controls

The frozen generator receives the exact path vertices, owner, edge-work receipts and source-license bit. It checks registered endpoints, receipt count, source ownership and work admissibility; it then emits the corrected structured operand and an operation-promotion event. The independent checker evaluates the target from its independent source semantics and checks the downstream decision. Four negative controls test missing owner, missing receipts, mismatched endpoint and invalid negative work. Re-execution returned **20/20 assertions passed**, including the four named rejection tests. This is *execution evidence for the finite source contract*, not a comparison of different mathematicians' comprehension.

## S18.4. Visual-fidelity limitation and M-Tower use

The mathematical projection $\nu_1$ is a structured visual-semantic record. Rendering it into SVG or PDF is another interface, which requires preservation of its source labels, routes, guards and relevant authorization semantics. A correct diagram file and source binding does not prove that a human reader reconstructs those semantics. Reader realization remains NOT_TESTED. An analogous M-Tower transport $M_i \rightsquigarrow M_j$ must satisfy operation preservation and exact target factorization at its admitted scope rather than merely depicting a geometric shape. This is an application of T01 and T13, not a universal tower representation proof.

# S19. Two-cycle coupled-observer derivation and the policy quantifier boundary

**Purpose and evidence type.** Replace an illustrative changing duck with a complete finite model of *duck-side changing observation functions*, the *human's signal-level access*, and the *human's separately registered inferred model*. No physical duck or human is experimentally modeled in the biological sense. The bounded experiment is included under reproducibility/experiments/coupled_observer_v7/.

## S19.1. Three differently owned observation objects

Let $R_n^D$ be the modeled duck's complete state, $q_n^D : E_n \to \Sigma_n^D$ its stage-specific observation function, and $q_n^H$ the human-side observation map of an admitted public behavioral signal. The human's inferred map $\hat{q}_n^D$ is its own hypothesis **about** the duck-side operation; it is not direct access to the duck's private FSS. A licensed signal channel mediates each cross-system observation. Changes to these three maps are different events, with different provenance and authorization.

Fix $H_C = O \times \{0,1\}^3$, $O = \{-1,0,1\}$, whose $|H_C) = 24$ histories are $h = (o,a,b,z)$. The duck's externally observable reed response changes to $a$ in cycle one. Its conditioned angle response changes to $a \oplus b$ in cycle two. An inaccessible private sensor response remains $z$. The licensed human projections are

$$q_0^H(h) = o, q_1^H(h) = (o,a), q_2^H(h) = (o,a,a \oplus b).$$

The accessible target is $\Psi(h) = (a,b)$; the inaccessible target is $\zeta(h) = z$. The source-only generator receives the public cycle-one signal $y_1 = a$, registers a first coordinate and a conditioned second probe, then receives $y_2 = a \oplus b$ and constructs $b = y_2 \oplus y_1$. A complete history, $(a,b,z)$, is checker-only information, not a source-only generator input.

## S19.2. Theorem T17 — fixed-class policy obstruction

Let $C$ be an admissible class of policies $\pi$, with each policy producing an observation transcript $q_\pi : H \to Y_\pi$ and a common target $\Psi : H \to Z$. If

$$\forall \pi \in C \exists h_\pi, h\prime_\pi \in H : q_\pi(h_\pi) = q_\pi(h\prime_\pi), \Psi(h_\pi) \neq \Psi(h\prime_\pi),$$

then no policy in $C$ admits a decoder for $\Psi$ valid on the entire source class.

**Proof.** For each fixed $\pi$, the hypothesis supplies a target-distinct collision in the $q_\pi$ fiber. T01 therefore rules out an exact decoder from that policy's transcript. Since $\pi$ was arbitrary, the statement applies to every policy in $C$. □

**Quantifier non-equivalence.** This is $\forall\pi\exists h, h\prime$, **not** $\exists h, h\prime\forall\pi$. For $H = \{00,01,10,11\}$, target the complete two-bit state, and two policies observing respectively the first and second bit, each policy has a target-distinct collision pair. Yet any distinct pair differs in at least one observed bit, so no single target-distinct pair collides under both policies. A combined two-bit policy succeeds. The model supplies an obstruction for the *passive* closure of policies factoring through $q_0^H$, not for a rich fixed program with both authorized probes.

## S19.3. Theorem T19 — finite exact collision counts

On the 24-history source class, the ordered target-distinct residues for the accessible target $\Psi = (a,b)$ are

$$\left|Res_\Psi\left(q_0^H\right)\right) = 144, \left|Res_\Psi\left(q_1^H\right)\right) = 48, \left|Res_\Psi\left(q_2^H\right)\right) = 0,$$

while the private target has

$$\left|Res_\zeta\left(q_2^H\right)\right) = 24.$$

**Proof.** Fix any offset $o$. The $q_0$ fiber contains eight histories, partitioned by four values of $(a,b)$, two histories each (the two private $z$-values). The ordered target-distinct count per offset is $8^2 - 4\left(2^2\right) = 48$; three offsets give 144.

Under $q_1$, each offset has two $a$-fibers of size four, each partitioned into two $b$-classes of size two. Thus the total is $3 \cdot 2 \cdot \left(4^2 - 2\left(2^2\right)\right) = 48$. Under $q_2$, $(a, a \oplus b)$ determines $(a, b)$, since $b = (a \oplus b) \oplus a$; the accessible residue is empty. For private $z$, each of the twelve $(o, a, b)$-fibers contains exactly the pair $z = 0, 1$, producing two ordered different-$z$ pairs, or 24 total. □

The numbers establish *target-relative* finite observability. They do not prove that the human observer knows the duck's entire internal observation function, nor that the duck's actual biological senses are modeled by the XOR example.

### S19.4. Constructed coordinates, work ledger and comparators

The generator promotes a first coordinate from $y_1$, recalls it, constructs $r_2 = y_2 \oplus r_1$, and promotes the second coordinate. A separate checker verifies the results for all 24 sealed histories. The declared abstract work ledger starts with 16 tokens, charges 14 across the typed Choice, source interaction, G, Create, Place/Store, Recall and L events, and retains two available tokens without negative prefixes. Potential work in mutually exclusive alternatives is not double-counted. This is finite accounting only; it is not a physical free-energy measurement or a general viable-convergence proof.

A full-source fixed-frame comparator permitted to read the same two public signals computes the accessible target successfully. Consequently T17–T19 do **not** establish universal impossibility for fixed algorithms, strict speedup, or an inherent machine-versus-human capability advantage. The operator label L or G depends on the full executed functional contract, not whether the implementation uses Python, a theorem prover or another formal language.

The independently rerun finite protocol produced **108/108 passed assertions**, including eight negative/source-access controls. The checker-only private bit remained unresolved. The experiment's source/target separation is a logical runner/generator interface, **not** an operating-system security boundary against a malicious process reading files it can access.

## S20. Reproducibility, visual operation fidelity and current proof/assessment register

### S20.1. Executed evidence and declared nonexecutions

The two source/target experiments include the following recorded finite re-execution results:

**COUPLED-OBS**: 24 complete histories; 108 of 108 finite assertions passed; eight specified rejection/negative controls; $144 \to 48 \to 0$ accessible target residues and 24 private target residue.

**VIS-REPAIR**: four complete histories; 20 of 20 finite assertions passed, including four specified negative controls; visual target residue $8 \to 0$.

**Two-study current total: 128 of 128 reported assertions passed.** These are assertion counts inside two deterministic finite test families, **not** independent empirical samples or an independently verified universal mathematical proof.

A separate archived source record reports a 4,181-assertion suite across additional families. That entire suite was not rerun in the two-study execution reported here; its counts are source evidence, not additional executed assertions in this study. Neither a Lean kernel execution nor an adequate full-scope G(R) coherence assessment was performed here. The 16-token abstract work units are not calibrated to physical free energy. A human receiver reconstruction study has not been executed.

### S20.2. Diagram operation-square criterion (application of existing theorems)

Let a mathematical source operation be $o : H \rightharpoonup H\prime$, an input representation be $\nu : H \to V$, an output representation be $\nu\prime : H\prime \to V\prime$, and a proposed represented operation be $v : V \rightharpoonup V\prime$. When definedness, guards and authorization are preserved, operation transport requires

$$\nu\prime \circ o = v \circ \nu$$

on the admitted input domain. A well-defined $v$ exists exactly where histories collapsed by $\nu$ are not separated into different necessary represented outcomes after $o$ (with matched authorization conditions). This is the existing T02a/T13 operator-descent condition applied to structured visual records; it is **not** asserted as a new independent universal theorem. In VIS-REPAIR, the authorization output cannot descend through the original endpoint-only map but does descend through its source-derived correction.

A figure's structured source binding, mathematical projection, correctly rendered glyphs and a reader's reconstructed meaning are distinct validation targets. The finite theorem establishes the structured-record relationship. The mathematical figures are provided as source SVG and rendered PNG; *human comprehension* is NOT_TESTED, as are global visual-compression improvements and universal M-Tower correspondence.

## S20.3. Claim-to-method-to-result mapping for the two-study experiments

**H1:** T01–T02 and T17 exhibit exact observer-relative loss of a target under specified observation classes. The global blind-spot coherence theorem remains a distinct target.

**H2:** T03–T04 and T18 exhibit source-licensed generation and correction at specified contracts; uniqueness or universal necessity of the L/G account is not proved.

**H3:** T19 gives an executed two-cycle finite observation-function example; general F-recursive extent, resolution and work limits remain outstanding.

**H4:** S5 finite retention/resource lemmas and the two-study abstract work ledgers are bounded positive evidence, not universal dual-closure necessity.

**H5:** T04, T18 and T19 establish finite generated relations or new operative representation coordinates; strong unanticipated generator discovery remains distinct.

**H6 / SOM-G1:** T07 is a finite convergence result, while the two studies establish fixed two-stage or single-repair results only. No unbounded convergent self-selection theorem follows.

**H7:** The RH case remains documentary compatibility work; neither finite experiment represents an execution of the RH historical discovery.

**H8:** T10–T15 and T18 demonstrate typed mathematical reconstruction and information-preserving structured visualization at scoped interfaces; actual reader reconstruction and collective use are untested.

**M-Tower MT-U3/MT-A3/MT-T/MT-U/MT-P:** T16 remains a bounded 3D geometry/algebra/calculus result. Source and operation-preserving diagram interfaces provide a reusable adequacy condition but do not settle the universal representation targets.

**Joint assessment: these two experiments supplied constituent evidence. The subsequent common-context construction and original-hypothesis assessment are given in S32–S33.**

## S20.4. Reader-facing test required to assess the visual approach

A future protocol should present the same target-relative mathematical operations in (a) the initial endpoint-only or otherwise insufficient representation, (b) a corrected structured visual representation, and (c) an adequately equivalent symbolic alternative. Fix source access, questions, time/work constraints and the operation set. Test not just recall but the reader's authorized next-operation decisions, reconstruction of hidden histories, detection of invalid transports and subsequent valid continuations. Comparing (a) and (b) measures the value of the additional functional information; comparing (b) and (c) with equal information evaluates the separate hypothesis that the visual encoding itself improves human understanding. No such receiver-side data currently accompany the mathematical source-only experiments.

## S20.5. Provenance, run commands and integrity

The source-only generators, frozen contracts, public-source definitions, sealed checker-only targets, independent checkers and trusted runners are preserved with the exact experiment structures in reproducibility/experiments/. The package also contains execution logs, returned JSON reports/traces and SHA-256 checksums for inputs and executed source files. To rerun in a standard Python environment:

```
python reproducibility/experiments/coupled_observer_v7/run_experiment.py
python reproducibility/experiments/visual_interface_repair_v7/run_experiment.py
```

The two processes make claims about *the finite, declared program and mathematical source class.* Their script separation is logical, not an OS-enforced sandbox. Additional source-audit reports are retained for traceability. Each proof and certificate retains its own execution scope and provenance.

# S21. A local mathematical fitness plane and conditional physics-to-M-space transport

## S21.1. Formal target: a functional landscape, not merely a graph

Fix a complete registered mathematical state $R_t$, target $T$, typed mathematical transition structure $A(R_t)$, admissible local state domain $S_t$, and an operative target-fitness assignment $F_T : S_t \to R$ or a declared ordered codomain. **The existence, adequacy, comparison rule and resource cost of $F_T$ are themselves proof/assessment obligations.** This is not a claim that a universal numerical mathematical fitness is already available. At a coarse resolution $S_t$ may be represented by a claim graph, but that graph alone does not generate a target-fitness field.

The **last two assessed mathematical claim-transitions** are registered as $\delta_{t-1}, \delta_t$, not merely the two latest textual propositions. If they admit target-relevant independent tangent directions $v_{t-1}, v_t$ in a licensed local realization, and a chart $\chi_t : U_t \subseteq R^2 \to S_t$ with appropriately specified guards and source access, define the projected local fitness by

$$f_{T,t}(u, v) = F_T\left(\chi_t(u, v)\right). \backslash (\text{S21.1} \backslash)$$

The plane or chart moves when later assessed transitions alter its directions. This local two-dimensional object is **not** the M-Tower dimension $i$ in $M_i$, and is **not** a cognitive order $n$. The chart is not inferred from a pair of arbitrary claim IDs; dependent directions, unavailable transports or inoperative fitness make (S21.1) inadmissible. A linear span is only a local representation when the ambient structure and admissible embedding justify it. Neither a global Euclidean coordinate system nor a unique chart is assumed.

## S21.2. T20 — local fitness pullback and directional assessment

**Theorem T20.** Suppose $\chi_t : U_t \to S_t$ is a well-defined admissible chart from an open domain $U_t \subseteq R^2$, and $F_T$ is a well-defined real target-fitness function on $\chi_t(U_t)$. Then (S21.1) is a well-defined real function on $U_t$. If both maps are continuous, so is $f_{T,t}$. If $S_t$ is locally differentiable and both maps are differentiable, then

$$Df_{T,t}(x) = DF_T\left(\chi_t(x)\right) \circ D\chi_t(x). \backslash (\text{S21.2} \backslash)$$

An actual *gradient vector* additionally requires a chosen metric; without it, the differential is a covector and no unique steepest-ascent direction follows.

**Proof.** Composition of well-defined functions is well-defined. Continuity is preserved by composition; the derivative formula is the chain rule. Identifying the differential with a gradient vector requires a metric. Each conclusion is restricted to the assumed source, chart and fitness domain. □

T20 establishes only the mathematical validity of a conditional projected landscape. It does **not** derive a target-fitness function from the six FMA operations, show that two assessed transitions always provide a plane, establish global convergence, or supply physical work units. Main Figure 8 uses an illustrative $f$, not a computed F-recursive fitness metric. Contours or basins can be defined when sufficient continuity/topological structure exists; their presence is not implied by a drawing alone.

## S21.3. Typed transport of physical relations: a candidate bridge, not a universal law

Let physical-domain states and admissible transitions be $(P, A_P)$, mathematical states/transitions $(M, A_M)$, a source-licensed mapping $\tau : P_0 \to M_0$, and declared physical and mathematical functionals $W_P, F_T$. An imported relationship is **admissible for the declared scope** only if its operands, units/semantic targets, guards, historical

records and relevant invariants are preserved by a verified transport. At minimum, if a physical transition $p \xrightarrow{o_P} p\prime$ is used to infer a mathematical transition, the transport must commute with a lawful mathematical operation $o_M$:

$$\tau\left(o_P(p)\right) = o_M\left(\tau(p)\right) \text{on the declared domain, with authorization preserved.\(S21.3\)}$$

For a candidate work/gradient law, one must *additionally* justify an appropriate correspondence between changes in physical work and changes in target-relative mathematical fitness and available mathematical work. A map preserving topological adjacency but not the functional's derivative, units or work guard cannot license transfer of a physical force or conservation equation. Such transfer is not established in this submission. The proposed cross-domain invariant is a hypothesis to be derived and assessed at a registered scope and budget, not a physical identity or an analogy-based theorem.

# S22. Fixed operative mathematical topology versus topology-dynamics search

## S22.1. Mathematical separation of search targets

Fix a typed observer grammar $\Gamma_0$ and all source, type and budget guards. Let $Reach_{\Gamma_0}(R_0)$ denote target-relevant states accessible by admissible finite operations *without changing* the operative generating contracts. A mathematical problem can search within this set even when the functions it computes mathematically describe moving surfaces or time-dependent topologies. The proposed L/G distinction therefore cannot be defined by whether a formula contains derivatives or topological terminology.

A **topology-dynamics discovery target** is a valid mathematical object $g$ specifying how an admissible transition relation, source interface, operator grammar or its validity contracts can be generated or corrected from licensed mathematical information. Its effect on reachability must be assessed after promotion; naming a new edge or drawing a new curve alone is insufficient.

## S22.2. T21 — exact conditional reachability separation

**Theorem T21.** Let $T$ be a target predicate. Suppose no state in $Reach_{\Gamma_0}(R_0)$ satisfies $T$. Let a source-licensed, typed and successfully assessed generator $g$ produce an amended operative grammar $\Gamma_1$ with all required guards and historical records preserved. If a finite admitted $\Gamma_1$-continuation from $R_1$ reaches a state satisfying $T$, then $T$ is unreachable under the specified fixed $\Gamma_0$ closure but reachable after the licensed amendment. In particular, the target-bearing reachability relation has changed.

**Proof.** The first premise excludes all target witnesses in the fixed reachable set. The second premise gives a target witness in the amended reachable set; therefore the two target-relevant reachable sets differ. The source, guard and assessment conditions ensure this is an admitted continuation rather than an invented edge. □

If $g$'s promoted operation was already executable within $\Gamma_0$, the first premise fails and the theorem does not establish a separation. A fixed computer program that can update its own internal representations may realize the G-like function; classification is of the complete operative system, not program syntax. T21 also does not prove that every mathematical object outside a current closure is accessible after a higher-order change, nor that a topology in the strict mathematical sense has changed without the requisite topological definitions.

# S23. Self-referential mathematical assessment and possible invisibility of novelty

## S23.1. A contributor-facing observation interface

Let $C$ denote an admitted class of mathematical contributions, $a : C \to A$ an assessor's available representation (for example a projection recording the statements of locally familiar constituent results), and $\kappa : C \to K$ a target-relevant operational property: whether a contribution certifies a newly admitted mathematical generator and the continuations enabled by it, with source and assessment conditions preserved. Two contributions may possess identical familiar theorem inventories while differing in the recognized operational property $\kappa$. That **possibility is an explicit interface test**, not an empirical report about a specific journal.

### S23.2. T22 — assessment-target collision and conditional publication effect

**Theorem T22.** Suppose $c_1, c_2 \in C$ obey $a(c_1) = a(c_2)$ and $\kappa(c_1) \neq \kappa(c_2)$. No decoder $d : A \to K$ recovers $\kappa$ exactly on both contributions. Furthermore, if an actual acceptance rule is restricted to a deterministic map $p : A \to \{0, 1\}$, it must make the same decision for $c_1$ and $c_2$.

**Proof.** If $\kappa = d \circ a$, equality of the observed values would force equality of the target values, contrary to the premises. For the decision statement, $p(a(c_1)) = p(a(c_2))$ by equality of their $a$-values. □

**Constructive finite assessment example.** Let two records have identical theorem-text fields but one possess a source-verified executable generator whose output changes a specified target's admissible continuation, while the other holds only the theorem statements. The coarse inventory-only map identifies them. A richer assessment map registering generator contracts, source access, proof and continuation can distinguish them. This constructed example demonstrates representational insufficiency of a **specified toy assessment interface**, not historical behavior of a real mathematical institution.

**T22 establishes a conditional assessment-interface obstruction. Determining whether it occurred in a particular publication decision requires evidence of the assessment interface and the distinctions it retained.**

## S24. Resource closure as leak detection and bounded work allocation

### S24.1. Two different quantities: potential fitness and executable work

Let $F_T$ denote target-relative solution fitness; let $W^{avail}(R_t)$, $W^{exec}_{t\to t+1}$, and registered source exchanges denote available work, executed work and work accounting in the same FSS. A high $F_T$ does not imply that the observer possesses work to realize it. The budget for an assessment and its historical execution certificate are further typed operands. The physical interpretation of $W$, including cross-domain numerical calibration, remains a proof/realization obligation; it is not supplied by calling abstract compute tokens "energy".

### S24.2. T23 — conservation residual detects a discrepancy under complete measurement

Fix a transition with registered available work $W_0$, accounted external inflow $I$, executed expenditure $E$, accounted outflow $O$, and measured remaining work $W_1$. Define

$$\rho = W_0 + I - E - O - W_1.\backslash(\text{S24.1}\backslash)$$

**Theorem T23.** If the accounting contract asserts exhaustive and calibrated measurement and exact conservation for these categories, then $\rho \neq 0$ proves that at least one conservation/accounting premise was violated by the observed record. If, additionally, measurement validity and all other flow categories are established independently, $\rho$ equals the signed residual unrecorded exchange under that contract.

**Proof.** Exact conservation requires $W_0 + I = E + O + W_1$, equivalently $\rho = 0$. Contraposition gives the first conclusion. Under independently verified measurement and exhaustive assignment of all recorded categories, subtracting them from the initial plus inputs leaves the unrecorded exchange. □

A residual is not itself proof of malicious behavior or intentional incorrigibility. Equally, $\rho = 0$ is not proof that every correction-relevant loss has been detected: two omitted flows can cancel, a needed source relation may be erased without changing the work account, or a defective measurement may conceal the discrepancy. The *corrigibility-relevant* assessment must determine whether registered losses impair the system's admissible ability to detect, reassess or correct its own operations at the target and budget in view. A complete identifiability condition for multiple unobserved losses requires an adequately injective measurement map, not merely net balance.

### S24.3. T24 — finite fair refinement under a bounded total work budget

Let $K = \{k_1, \ldots, k_m\}$ be a finite target-relevant set of candidate mathematical regions, source classes or refinement tasks, each with adequate declared refinement/assessment cost $c_i > 0$. Let the registered total accessible work $B$ satisfy $\sum_i c_i \leq B$, and suppose a fair procedure visits and completes every needed $k_i$ while preserving sources, history, guards and target-certification soundness. If at least one region contains a target-adequate witness that the

corresponding completed assessment recognizes, the procedure discovers a certified target witness within cumulative work $B$.

**Proof.** Finiteness plus the stated fair complete-visit condition implies every $k_i$ is assessed after finitely many steps. The work sum bounds cumulative cost by $B$. The assumed sound and complete local target check identifies at least one witness if present. □

If the budget is too small, a region is inaccessible, the selection is not fair, or the target fitness is inadequately represented so that the witness is never recognized, the guarantee fails. The GTA-like local-resolution picture motivates an allocation strategy but does not prove it beats exhaustive search or derives endogenous F-recursive resource quantities. The actual M-space region and operative resolution depend on $R$, its admissible transitions and history, not on an exogenous Euclidean radius.

# S25. Dependency record for T20–T24

## S25.1. Scoped results and receiving obligations

**T20** is a scoped pullback/chain-rule result, assuming chart and fitness; **T21** a scoped target-reachability separation under licensed grammar amendment; **T22** a scoped exact information-loss theorem applied to mathematical assessment; **T23** a work-accounting discrepancy lemma; **T24** a finite fair-refinement result. All are mathematical deductions from the explicitly given premises, not independently measured physical laws or executed adequate global G(R) certificates. T01–T19 and the full-strength H1–H8, SOM-G1 and MT-* claim identities retain their respective scopes.

**Stronger receiving targets:** an endogenous F-recursive mathematical fitness and local directional calculus; conditions for deriving the two assessed directions and valid local chart; physical-to-M-space transport of work/stability laws; self-referential assessment of mathematical novelty across actual collective review processes; conservation/identifiability-based corrigibility detection; resource-bounded convergence for a nontrivial evolving M-space class. These remain proof/assessment targets, not standalone claimed completed contributions. None is an asserted physical empirical result.

## S25.2. Hypothesis and implication crosswalk

**H1–H3:** T20 and T22 sharpen target-relative observation, local assessment geometry and the possibility of assessment-interface invisibility; T21 separates search within a fixed operative closure from generating new admissible mathematical relationships. **H4:** T23 strengthens bounded conservation/residual analysis but does not establish universal necessity or leak observability. **H5:** T21 and T04/T18/T19 clarify exact scoped operative reachability changes. **H6:** T24 addresses finite work-limited refinement; SOM-G1 remains outstanding. **H7:** The source-bounded RH reconstruction and its causation limits are specified in S26. **H8:** T22 explains how a collective assessment interface can fail to reconstruct new operative relationships; actual reception and publication outcomes are not inferred. **M-Tower:** the local two-dimensional plane is distinct from $M_i$, and physically inspired energy/gradient dynamics require a typed invariant-preserving bridge.

## S25.3. Global conjunction and budget policy

The full target $J = H1 \wedge \cdots \wedge H6$ is registered in S7 and Appendix B. The T20–T24 proofs can be reused only at their exact assumptions, source domains, work conditions and scope. The joint assessment must evaluate compatibility of target-fitness projection, admissible transition changes, observation/assessment interfaces, resource flows, and convergence contracts, at an **endogenously derived, actually executed** adequate budget. S32.8 and S36 record the COHERENT scoped conditional derivation and finite 7,077-unit projection; S33 records the separate COHERENT 29-unit evidence-relative assessment of the original J, conditional on GIGT premises. The wider native assessment remains UNDETERMINED with BudgetStatus=BUDGET_UNDETERMINED. These distinct verdicts and budgets are not interchangeable.

# S26. Source-bounded backward reconstruction of the RH formal development — source-bounded assessment

This section reconstructs selected formal RH dependencies through the observation, path-decomposition and transport contracts. The public-mirror audit checks the main route and m-window constraints, distinguishing documentary correspondence, proof-term dependence, structural correspondence and historical causation. S29 derives the general path-compression theorem independently.

## S26.1. Evidence source, access and epistemic types

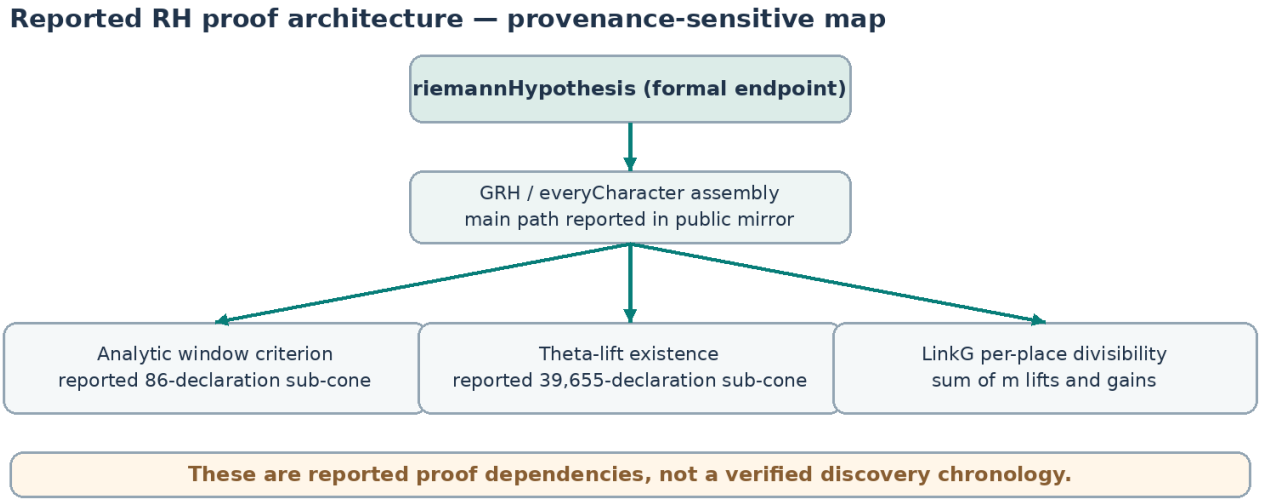
External certification. The Lean and Mathlib communities reported checking the formal statement and proof [2]. The present submission does not rerun the kernels. Primary documentary record. Alpöge's four-page remark [1], inspected on 22 November 2026, directly states the Stepanov multiplicity analogy and the suggested theta-lift chain length near $\sqrt{X}$ (printed p. 1), and gives 20 dated retrospective entries (printed pp. 3–4). S26.9 registers every entry and the constrained precedence relationships they support. A dated retrospective account is not a complete action transcript.

Archive evidence. A retained static audit of public-mirror commit 6193bb5ef61846c045c70c51ee05824f910d50cc reports 175,187 Lean source files, 17,374,215 lines and 1,030,531,854 bytes. It finds zero textual axiom declarations and no files containing sorry or admit tokens. This is a mirror-text result, not proof-term certification or a hash match to the original released archive. Lexical declaration and dependency-cone counts are not reported because lexical counting does not certify dependency edges. The retained rh_audit.json and import_closure.json identify the exact tested source snapshot and scope.

Evidence types: D = directly documented in an identified primary source; P* = reported source-level dependency in the public mirror, pending replay/direct proof-term verification; S* = source-informed candidate mathematical structural correspondence, whose discriminating adequacy remains an obligation; U = unresolved historical causal inference. W1–W3 are an explicit protocol for evaluating compatibility: (W1) earlier source relation, (W2) typed and sufficiently specific invariant-preserving target operator, (W3) actual route to the certified endpoint. W1–W3 define the stronger invariant-transport assessment. S32.9 and S33 assess the documentary compatibility evidence separately; no default native GIGT assessment budget is inferred.

## S26.2. Main proof spine and its three load-bearing components

Source-text main route at the audited mirror commit: Main.lean riemannHypothesis → rhOfAll → allChi → FinalTheorems.everyCharacter → every_character_of_row → LiteratureTheorems.every_character_of_literature_of_vWin → every_character_of_theta_integrals_local_matching. This is an observed named declaration/definition route, not an extracted elaborated dependency cone. Imported modules include the whole project, and therefore module-level reachability cannot discriminate the proof-relevant subset. Numerical proof-dependency cone sizes are not reported because lexical counts do not certify dependency edges. The separate Stepanov→theta-lift correspondence $\beta$ remains an exact structural assessment target, independent of source-line counts.




S26 Figure 1. Reported proof dependencies and their evidentiary boundary. The graphic is a scoped guide to formal proof structure, not a reconstruction of the sequence in which the mathematics was discovered.

A large formal archive and a possibly compact, reusable mathematical generator are compatible. The existence and human reconstructibility of such a generator must be demonstrated through the actual source-licensed relationships and proof obligations, especially the per-lift gain lemma, not inferred from static cone size or from the visual simplicity of the diagram.

## S26.3. Proposed Stepanov-to-theta-lift bridge $\beta$ (Q1, structural compatibility)

Proposed source analogy. For an auxiliary function on a function-field curve, vanishing of order at least m at N points yields a multiplicity-versus-degree inequality of schematic form mN $\leq \Sigma$_P ord_P(f) $\leq$ deg f under its stated assumptions. Alpöge [1, p. 1] directly invokes Stepanov multiplicity as motivation for chained theta lifts. This supplies a documented mathematical source relationship; it does not by itself establish that a Stepanov differentiation operation transports into number-field arithmetic.

Proposed target relation $\beta$. In the preliminary formal reconstruction, a chain of m theta lifts participates in per-place LinkG_at ramp/gain accounting and a global slope/height budget. A candidate bridge matches multiplicity contributions to per-lift increments and degree bounds to global accounting. The additive local-to-global pattern is too general to discriminate a Stepanov-specific transfer: unrelated arithmetic counting and height arguments can also satisfy it. A stronger test would find a characteristic source invariant (for example, a verified m-versus-X trade-off yielding the chosen scale) and show $\beta$ does not identify a plausible non-Stepanov source. No such discriminating test has passed.

W1 (documentary source): supported by the directly accessed announcement [1]. W2 (specific typed bridge with invariants and guards): UNDETERMINED; only a preliminary additive correspondence is presently available. W3 (used formal route): provisional P* paths in the public mirror, pending direct confirmation. Therefore Q1 has verdict UNDETERMINED and a preliminary STRUCTURAL CORRESPONDENCE, not an established or discriminating L/G discovery witness. Q2 historical causal mechanism and Q3 source-frozen reproduction are UNDETERMINED. Limitations: the RH problem was already a recognized target; the human-plus-machine discovery system is not automatically L or G; and no controlled source-frozen contrast has demonstrated a blind spot relative to a defined baseline.

Guard caveat: The source operation relies on characteristic-p/Hasse-derivative structure, which cannot be presumed to descend into arithmetic over the integers. The proposed theta-lift chain could be a different target operation preserving a useful additive constraint, rather than an isomorphic transport of differentiation. The apparent guard failure is relative to the specified repertoire, not a proof of universal impossibility for existing mathematics.

## S26.4. Riemann hypothesis proof-assembly collision (T01 / H8)

Proof-assembly reconstruction target. In the audited mirror, both everyE$_2$ and every_character_of_theta_integrals_local_matching conclude UpperTheory.EveryCharacter through distinct

hypothesis bundles. The source-text main route uses the latter via every_character_of_literature_of_vWin. In the retained source-body extract, every_character_of_theta_integrals_local_matching takes P, F, hA, hM, hX, hL and hV, while its displayed result expression uses P.greenRegularCharacters and hV; this identifies syntactically unused named hypotheses without establishing a proof-term-level dependency cone. A complete textual identifier search finds every$E_2$ only in its declaration file, while its module is imported; this confirms absence of direct textual users, not absence from elaborated proof terms. Under a deliberately coarse exposition interface displaying only the shared conclusion, two distinct proof assemblies collide by T01. The result concerns reconstruction adequacy of the exposition interface, not validity of the Lean proof.

A retained lexical audit of the specified mirror snapshot found no identifier occurrences of "Stepanov" or "Bost". This observation concerns spelling in source text and does not imply that the corresponding mathematical ideas are absent.

## S26.5. Backward transition ledger: documentary, proof, structural and causal status

$\tau$0: RH / GRH endpoint | D + externally checked formal statement | Endpoint documented and community-certified; not a discovery operation

$\tau$1: analytic window → zero-free half-plane | P (mirror, spine inspected) | Formal proof dependency reported; L-adequate at this scope

$\tau$2: toy Fourier setting → automorphic periods | D direct; separate formal anchors | Candidate S remains UNDETERMINED; vocabulary not enough

$\tau$3: Stepanov multiplicity → chained theta lifts | D primary; P* preliminary archive; S* additive pattern | Specific $\beta$ and formal transport remain UNDETERMINED; does not establish L/G necessity

$\tau$4: author-reported m near $\sqrt{X}$ | D [1, pp. 1,4] and mirror-source window | RankWindowAt directly gives $(x\cdot\delta_1)^{(1/2)} \le m \le 2(x\cdot\delta_1)^{(1/2)}$, and CoarseWindowAt gives $x^{(1/2-\epsilon/8)} \le m \le x^{(1/2)}$; chainOpM composes m lift kernels. The separate definition delta1 := s.x^(−$\epsilon$/8) yields the exponent 1/2−$\epsilon$/16 only conditional on resolving its identification with L.sc.delta1 in RankWindowAt. Within the audited mirror there is no m-versus-k tension; x versus X remains a separately typed comparison.

$\tau$5: local p-adic / supersingular vanishing | D direct; partial P anchors | Full formal-to-document bridge still incomplete

$\tau$6: global slope / height accounting | D direct; conditional P anchors | Candidate mathematical structural alignment

$\tau$7: formal literature layer | checked mirror-text declaration route | The formal library includes a large body of imported sources; importing all modules does not show which declarations are actually proof-term dependencies.

Three target verdicts: Q1 source-based structural correspondence — UNDETERMINED (specific structural transport; candidate evidence is preliminary and nondiscriminating); Q2 causal use — UNDETERMINED; Q3 source-frozen reproducibility — UNDETERMINED. Q1 does not establish the H1/H2 mathematical-observer blind spot, since the target was known, complete observer operations were not typed, and no comparator was executed. The direct 20-item timeline checks coarse temporal ordering but not an exact sequence of causal operations.

## S26.6. Optional worked fitness-surface interpretation (illustrative, not empirical)

For the explanatory toy budget $E(x,m)=x/m + m + \sqrt{x}$, set u=log m and v=log x. Then $(E(x,m)-\sqrt{x})/(2\sqrt{x}) = \cosh(u-v/2)$; defining relative fitness $f(u,v)=-\cosh(u-v/2)$ produces a ridge at $m=\sqrt{x}$. The derivative of this illustrative surface is $-\sinh(u-v/2)(1,-1/2)$. It is a mathematically exact example of a local target-relative fitness projection, but no result here proves this E is the objective actually optimized in the formal RH archive or that the chain parameter m was instantiated as $\sqrt{x}$. It therefore does not add an H7 positive certificate.

## S26.7. Executed work, open items and non-circular next tests

Checked documentary and source records. The author's four-page announcement and the community checking statement are the documentary anchors [1,2]. The retained static mirror re-audit documents the named main route, two EveryCharacter constructions, lexical checks and the formal theta-lift m-window at commit 6193bb5e. Original-release byte identity, elaborated proof-term dependency extraction, a discriminating mathematical $\beta$ bridge

and a source-frozen discovery replay are distinct additional targets. This scope typing preserves the positive source-text observations without promoting them into historical-causal conclusions.

Functional-information conservation: each claim retains its exact type, evidence class, premise, budget status, and unresolved residual. A new independent mirror recheck could upgrade P* to P for its exact source scope; it would not retroactively rewrite these source-conditional records. H7 joint native assessment verdict: UNDETERMINED; BudgetStatus=BUDGET_UNDETERMINED.

## S26.8. Traceable companion and external source register

External sources and retained audit record: [1] Alpöge, "A remark" (20 November 2026); [2] Lean and Mathlib community announcement (21 November 2026); [23] public Lean mirror at commit 6193bb5ef61846c045c70c51ee05824f910d50cc (github.com/1337isnot1337/riemann-hypothesis-solved). Executable text-level audit scripts, manifest hashes and result JSON are retained in the reproducibility/S30_implementation_note/scripts/ directory. The public verification repository [59] is distinct from the inspected mirror; byte/declaration identity with the original archive remains a separate unestablished claim.

## S26.9. Twenty-item primary-source timeline and coarse precedence test

**The following 20 entries transcribe the dated topic labels in abbreviated form (not verbatim mathematical formulae) from Alpöge [1, printed pp. 3–4]. Item order is author-provided, not chronological sorting; dates are therefore essential. Transition tags refer to the candidate $\tau$ categories in S26.5, not independently established causal steps.**

1. 6 Oct 2026 — Stepanov determinant. Candidate mapping: $\tau 3$ source.
2. 9 Oct 2026 — Inert-prime CM-curve D_p vanishing / divisibility. Candidate mapping: $\tau 5$.
3. 19 Oct+ 2026 — qN=Mk+R and automorphic determinant/period construction. Candidate mapping: $\tau 2/\tau 3$.
4. 6 Oct+ 2026 — Determinant W(1S)$\geq$0, Stepanov multiplicity one. Candidate mapping: $\tau 3$ source.
5. 20 Oct 2026 — Switch to weighted von Mangoldt $\Lambda$ statistic. Candidate mapping: $\tau 1$.
6. 8 Oct 2026 — Window-weight positivity gives $>1-\eta$. Candidate mapping: $\tau 1$.
7. 14 Oct 2026 — Determinants for Gross–Prasad-type periods. Candidate mapping: $\tau 2$.
8. 15 Nov 2026 — Iterated unitary theta-lift chain; propose m near $\sqrt{X}$. Candidate mapping: $\tau 3/\tau 4$.
9. 15 Nov 2026 — Norms, level and cancellation of order-m terms. Candidate mapping: $\tau 3/\tau 6$.
10. 15 Nov 2026 — Lower divisibility bound for iterated lifts. Candidate mapping: $\tau 3/\tau 5$.
11. 6 Oct 2026 — Iwasawa tower analogy. Candidate mapping: $\tau 2/\tau 6$.
12. 15 Nov 2026 — Seesaw/Bessel period and Rallis. Candidate mapping: $\tau 3/\tau 6$.
13. 6 Oct 2026 — Bost slope inequality. Candidate mapping: $\tau 6$.
14. 15 Nov 2026 — Cost of each lift; two-thirds obstruction and division. Candidate mapping: $\tau 6$.
15. 15 Nov 2026 — Integral structure glued across characters and averaging. Candidate mapping: $\tau 6$.
16. 14 Oct 2026 — Relative trace formula and period norm estimate. Candidate mapping: $\tau 2$.
17. 15 Nov 2026 — Cohomology classes/Vandiver-type weakening. Candidate mapping: $\tau 5/\tau 6$.
18. 15 Nov 2026 — Covolumes, theta-series indices, congruences. Candidate mapping: $\tau 6$.
19. 19 Oct 2026 — Precise Iwasawa tower/Behrend choice. Candidate mapping: $\tau 6$.
20. 16 Nov 2026 — GRH attained (author's retrospective label). Candidate mapping: $\tau 0$.

Chronology test. The Stepanov entries (6 October) precede the theta-lift proposal (15 November); the cost and divisibility proposals share the latter date and supply no within-day causal order. The author's m near $\sqrt{X}$ has a source-text counterpart in the CoarseWindowAt bounds for m; the sharper x^(1/2−$\epsilon$/16) identification from

RankWindowAt is conditional on type-resolving L.sc.delta1 against the separate delta1 definition. The formal scale x has not been identified with the announcement's X. This is a positive source-level correspondence and a documentary partial-order result, not an assertion of an observed private discovery operation.

### S26.10. Targeted falsification and reproducibility contract

A discriminating Stepanov-specific mathematical transport test must go beyond generic additive contributions to a global budget. It must preserve the source multiplicity conditions, the target window for m, the typed per-lift gain and guard relations, and a usable differentiation from a plausible non-Stepanov counting construction. The source-bounded m-window is a positive new comparison invariant; it does not alone select a unique historical generative route. A source-frozen continuation test must not reveal the finished chain to the generator.

## S27. Prior discovery mechanisms and the conjectured common functional infrastructure

Purpose. Earlier research distinguishes solving within a fixed representation from constructing new representations, rules or generators. This section registers the exact scope of those precedents and states a discriminating cross-domain test of the stronger GIGT proposal. It does not replace the original H1–H8 claims, prove universal six-function coverage, or silently promote the current finite results.

### S27.1. Three different discovery targets and a nonexclusive prior literature

Target A: find a mathematical object using an already adequate operative representation and its admitted transitions. Target B: discover an operation or representation that changes which mathematical objects can be produced or discriminated. Target C: construct, assess and correct the rules and assessments through which candidate operations of type B become available, while conserving source, provenance and viable work. The proposed GIGT infrastructure concerns the functional coverage and recursive self-application of the third target; systems already performing parts of it are not excluded by their conventional labels.

Ohlsson (1984) identified representational restructuring as changing the applicability of operators [24]. Kaplan and Simon (1990) explicitly studied search for useful problem representations, rather than only search within them [25]. Lakatos (1976) traced changes in mathematical definitions and proof-generated concepts [26]. Knuth–Bendix completion adds rewrite rules to equational reasoning [27]. Lenat's AM and Colton–Bundy–Walsh's HR generate concepts and conjectures that become operands of later mathematical activity [28,29]. Learned optimization treats a solver as the learned object [30]. None of these requires inventing an artificial new seventh FMA operation.

Cousot and Cousot (1977) developed a unified lattice-theoretic abstract-interpretation framework [31]. Giacobazzi, Ranzato and Scozzari (2000) develop completeness-driven abstract-domain refinement [14], and Clarke and colleagues (2000) construct counterexample-guided abstraction refinement [16]. Abstract interpretation is already domain-general for important classes of semantic analysis. The manuscript therefore does not claim that previous researchers recognized representation-changing discovery only for isolated problems; it asks whether one recursively governable functional architecture has adequately typed realizations across all relevant FSS domains.

### S27.2. Functional-coverage target, typed without a new primitive

Let D range over admitted functional domains with registered complete states R_D, sources S_D, domain operands, admissible relations, work budgets and target contracts. Let F denote the same six FMA internal transformations on complete R: Create/Destroy, Place/Store, Recall, L, G, and Choice. For a named native discovery mechanism A_D, a candidate realization consists of typed encoding E_D, source and guard transport, finite compositions of F, and a decoding or continuation interface Q_D. The key adequacy test is whether those compositions preserve the target-relevant outputs, the definedness and authorization of A_D's actual transitions, and the historical records needed to assess and correct them. Domain-specific algorithms and operators may differ; no one-to-one remapping of six internal functions is introduced.

The intended infrastructure claim is stronger than expressive embedding: in a new admitted domain, the proposed realization should be able to construct, assess, correct and reuse the relevant discovery operators, not merely store an encoded description of them. Its exact claim class and F-recursive definition of adequacy must be specified for

each test; the general ∀D coverage is an outstanding conjunction/global-coherence target, not a proved theorem. Finite resources, inaccessible sources and structural nonreachability remain legitimate outcomes.

## S27.3. Paired, discriminating protocol: abstraction refinement versus mathematical concept formation

Case AR (abstract refinement): fix a concrete system, its initial abstraction, an abstractly derived candidate property or counterexample, a concrete checking oracle restricted to its licensed source interface, and a refinement procedure that can correct the abstract domain. Register actual abstract and concrete operations, counterexample provenance, changes in effective distinctions, and work consumed. Source exemplars: Cousot–Cousot, Giacobazzi–Ranzato–Scozzari and CEGAR [31,14,16]. A successful realization must reproduce which refinement is admitted and how the revised abstraction changes permitted later assessments.

Case CF (concept formation): fix the initial concepts, permitted constructor/relation vocabulary, model or counterexample interface, and heuristics of an automated theory-formation process. Register the newly constructed concept/definition as an operand, its proof or refutation obligations, source support, and any subsequent use. Source exemplars: AM and HR [28,29]. A successful realization must generate and assess the mathematical concept under its own native guards and preserve enough information to rerun or correct the resulting derivations.

Common-architecture test. For both case records, apply the same F on complete registered state R with the same semantics for its six internal operations. Permit different domain-specific operand types, signals, native operators and source guards. Compare each decoded sequence to the source mechanism on outputs, authorizations, provenance, corrections and continuation. Identify a typed defeating counterexample if an indispensable operation cannot be represented/executed without inventing an additional internal primitive, or if an operator called generated was already included in the initial native closure.

Cross-domain test. The implemented positive case uses a concept generated and assessed in an HR-style formation system, transports its intensional definition through a typed bridge into an infinite-state CEGAR receiver, and admits it through the receiving G operation. T26–T28 establish that threshold and finite-set predicates cannot certify the safety target, while the transported evenness definition does so after one refinement. The source and destination retain their distinct domain operators; the common registered state, assessment, correction and work discipline remain unchanged (S30).

## S27.4. Falsification, status and connection to the original conjecture

A counterexample to the proposed strong common-coverage target consists of a well-typed admitted domain and indispensable target-relevant native operation for which no adequate F-recursive realization can be derived under the declared operations, sources and viability conditions. An inadequate computation budget is a different outcome from a structural separation; both must be recorded. A purported G generator that receives the final target hidden among its sources fails the source-frozen discovery protocol. A mapping that preserves only names or final answers but loses executable correction fails the infrastructure test.

Status. LIT-R18 identifies the established constituent discovery mechanisms in the cited primary literature [14,16,24–31]. INFRA-U1 has a scope-certified positive realization for two reduced mechanisms under the same six-operation executor (R19; S30). INFRA-U2 has a source-licensed cross-domain definition-transport witness with a natural receiver-grammar obstruction (R21–R22; T26–T28; S30). The executor supplies an exact finite conservation invariant $W+E=W_0$ and a COHERENT computational projection of bounded functional free-energy accounting (R23; T29). These are append-only scoped positive results; the original H1–H6, H8 and general SOM-G1 claim identities and inherited certificates remain unchanged.

Interpretive consequence. The discovered distinction between a solution point and a solution-generating dynamic has historical and mathematical precedents. The new target of this manuscript is whether a single recursively correctable functional infrastructure can govern discovery dynamics across different functional domains, with attention to source information, registered histories, target-relative solution fitness, correction and finite functional work. Thurston's discussion of proof and mathematical understanding [32] complements, but does not substitute for, the formal H8 transmission and reconstruction requirements.

AI-use disclosure. Large language models assisted in mathematical exploration, drafting, analysis, organization and editing. The human author is responsible for the final claims. The present source-conditional archive observations have not been upgraded into independently executed proof-term audits.

# S28. Visual-ontology closure and point-of-use information locality

## S28.1. Targets, mathematical semantics, and communication interfaces

Fix a class H of admitted complete mathematical histories, target maps $\kappa$:H→K and $\kappa_i$:H→$K_i$ for target-relevant visual operations i, and a family O of admissible mathematical operations. Every operation has specified operand and output types, guards, source rights, relevant invariants and history/work records. A diagram ontology $\Omega$ registers operands, edges, compositions, transformations and mathematically meaningful geometrical or topological structures. Write v:H→V$\Omega$ for the diagram representation. The mathematical diagram must retain every distinction needed by its declared target and downstream operations.

A separate, reader-facing decoding layer supplies object names, operator signatures, types, short guards, an invariant key and an immediately usable local legend. These interface cues are associated with mathematical elements but are not additional mathematical operands merely because they are printed inside the figure. A third layer contains extended explanation, derivation, citations and proof-status discussion. The layers are distinguished by their mathematical typing and rendering style, not by a rule that all explanations must be physically separated from the diagram.

Visual-ontology closure (VOC) means that every mark asserting a state-dependent mathematical relation has an admissible $\Omega$-type and denotation; typography and local keys are non-operative decoding aids. Point-of-use information locality (PIL) means that a reader has access to the indispensable decoding information at the visual operation to which it applies, without navigating unrelated prose. Locality permits a directly resolvable typed reference when the full dependency is intrinsically remote. This avoids confusing informational locality with arbitrary duplication of the full mathematical state.

## S28.2. Target reconstruction and operation preservation

Global reconstruction. A diagram is adequate for $\kappa$ exactly when some decoder d satisfies $\kappa$=d∘v on H. Equivalently, $v(h_0)=v(h_1)$ implies $\kappa(h_0)=\kappa(h_1)$. The first condition is a factorization contract: the diagram does not merge histories requiring different target outcomes.

Operation preservation. Every required f∈O must descend to an appropriately typed diagram operation $f^v$ whose definedness, guards, source authorization, output type and required invariants agree with f, and $v(f(h))\approx f^v(v(h))$ for the declared class of admissible diagram equivalences. If a length, orientation or topology is registered as mathematical information, an arbitrary relayout that changes it is not an admissible equivalence.

Local decoding. For a visual operation i define $\rho_i(h)$ as the local observable neighborhood of v(h), its nearby key $\Lambda_i$ and the explicitly resolvable links authorized at that location. A local target $\kappa_i$ is reconstructible at its point of use exactly when there exists $d_i$ satisfying $\kappa_i=d_i\circ\rho_i$ on H. Local labels may be state-dependent when they directly denote an admitted typed mathematical attribute; a static legend decodes those attributes but does not substitute for omitted distinctions.

## S28.3. Representation collision and the local information theorem

Proposition VS1 (global target collision). If $v(h_0)=v(h_1)$ yet $\kappa(h_0)\neq\kappa(h_1)$, no decoder using only v can correctly return $\kappa$ on both histories. Proof. Such a decoder would assign one output to the common diagram, contradicting the two required outputs. ■

Proposition VS2 (point-of-use collision). Fix i. If $\rho_i(h_0)=\rho_i(h_1)$ but $\kappa_i(h_0)\neq\kappa_i(h_1)$, no decoder operating solely through the point-of-use interface $\rho_i$ can correctly distinguish both histories. Proof. Any proposed $d_i$ would give $d_i(\rho_i(h_0))=d_i(\rho_i(h_1))$ whereas $\kappa_i(h_0)\neq\kappa_i(h_1)$. ■

Proposition VS3 (global adequacy does not imply local adequacy). Let H={0,1}, v(h)=(0,h), and $\kappa_i(h)$=h. Suppose operation i exposes only the first coordinate of v, with no link to the second. Then v is globally adequate for $\kappa_i$ by projection to its second coordinate, whereas the local observation $\rho_i(h)$=0 identifies both histories and fails VS2. Proof. The global decoder d(0,h)=h exists; the local decoder would need to send 0 to both 0 and 1. ■

Corollary VS3a (caption-dependent repair). A whole-page pair (v,c) can be adequate while the diagram v, or the interface $\rho_i$ at an affected edge, is inadequate. Moving a necessary label or guard into distant prose may preserve information globally but remove the exact factorization required at the point of use. The repair restores the missing mathematical distinction to v when operative, and places its decoding cue beside the relevant edge or supplies a directly resolvable local reference. Merely enlarging a remote caption does not repair the local interface.

## S28.4. Work-bounded local navigation

A receiver model $\sigma$ specifies available signals, reading/lookup operations, retained memory, permissible cross-references and work accounting. Let $W_i(h;\sigma,\rho_i)$ be the minimal work needed to reconstruct $\kappa_i(h)$ through the registered local interface, and let $B_i(h;\sigma)$ be the work available. The work-bounded navigation contract is $W_i(h;\sigma,\rho_i) \le B_i(h;\sigma)$ on the admitted cases. If all permitted decoding paths exceed the budget, the observer cannot complete that reconstruction within the budget, even when a globally adequate decoder exists in the absence of a work limit.

Work accounting is target- and receiver-relative. The existence of a symbolic decoder does not imply a uniform finite bound on human reading effort. Conversely, placing a local label beside its edge can reduce required lookup operations without altering the underlying mathematical ontology. Any claimed usability advantage therefore requires a declared receiver and work model or an empirical comprehension test; the exact collision results VS1–VS3 do not, alone, quantify human readability.

## S28.5. Modular closure of the scientific text

Treat a scientific section or supplement M as an information interface with a declared set D(M) of interpretation-critical dependencies. Let I(M) be the mathematical definitions, typed objects, hypotheses, proofs and references supplied inside M, and E(M) its stable, explicitly identified and resolvable external references. Standalone modularity requires $D(M) \subseteq I(M) \cup E(M)$. This is a dependency-closure contract for reading M independently; it is distinct from the logical closure of the underlying mathematical theory.

Editorial chronology is not an operative scientific dependency unless the history itself is the mathematical target. Such information belongs in a separate administrative record. A module satisfies the independence test when a reader can reconstruct its definitions, claim scope and arguments from M and its declared stable references, without consulting an unavailable prior version.

## S28.6. Construction and falsification protocol

1. Register the target $\kappa$, all required local targets $\kappa_i$, the operator family O, and the typed mathematical visual ontology $\Omega$.
2. For each node, edge, composition and spatial attribute, verify its admissible mathematical type. Identify nearby labels and short decoding keys as interface cues, not as extra untyped mathematical operands.
3. Check that every required operator signature, guard and invariant cue is readable at or immediately beside the relation it qualifies. A distant caption cannot substitute for an omitted point-of-use key.
4. Perform the global factorization and operation-preservation tests, then search separately for a local collision under each $\rho_i$. Repair any collision at the mathematical or decoding interface, as appropriate.
5. Evaluate the requested reader operations with the declared work model and budget $B_i$. Record the model and its limits; do not infer bounded usability from formal adequacy alone.
6. Audit the scientific module as an independent document, verifying that all interpretation-critical dependencies are internal or resolved through stable external references.
7. Recheck figure legibility at submission scale, including labels, leaders, line weights, type distinctions and printed PDF rendering.

## S28.7. Worked positive and negative controls

Negative control—local decoding loss. Two otherwise identical edge diagrams require different guards g=0 and g=1 for an admissibility target. If their mathematics preserves those guards somewhere in the full record but the edge interface omits their state-dependent values, the local observations collide and VS2 applies. Placing the relevant guard immediately adjacent to the edge restores the needed distinction without adding another mathematical node.

Negative control—non-operative explanation. A boxed sentence claims that one edge composition is licensed, but no corresponding typed relation or executable path is present. The box supplies the author's statement, not an admissible mathematical operation. Replace it with the actual typed edge or composition and use a nearby guard label for decoding.

Positive control—recursive observation. A transformation edge $T_1$ between two registered states is observed and encoded as an admissible operand $O_1$, which participates in a subsequent, source-licensed composition. Keep the names $T_1$ and $O_1$, the observation operator, and the transport guard at the arrows and nodes where they are used; reserve the proof of the transport contract for the caption or text.

Positive control—representation residue. Two spatially distinct transformations with the same reduced connectivity receive the same projected input but require different verdicts for an orientation target. The oriented geometry carries the difference; the short signs J=+1 and J=−1 are adjacent to those configurations. The diagram exhibits the target-distinct pair directly rather than relying on a remote paragraph to assert that one exists.

### S28.8. Formal scope and references

VS1–VS3 and their local collision corollary are proved at the stated mathematical scope. VOC, PIL, modular closure, and work-bounded reconstruction are separately registered interface requirements. Their joint use preserves the distinction between a formally reconstructible mathematical diagram and an observer's ability to decode it with limited work. This section is self-contained; the same target-factorization and operation-descent principles connect to the study's T01, T02a, T11 and S20.2 contracts. The proposed universal GIGT realization and transfer conditions remain independent mathematical targets. Related mathematical precedents include compositional and diagrammatic representations [43,50], abstract interpretation [31,14] and the proposed GIGT architecture [42]. References are numbered as in the main manuscript.

## S29. Proof paths, compositional simplification and target-preserving reconstruction

### S29.1. Registered path and decomposition target

Fix a finite chain of mathematical state spaces $S_0,\dots,S_n$, complete registered state R containing its operands, contextual parameters, admissible operations, sources, guard conditions, histories and assessment records, and typed partial maps $F_i{:}S_{i-1}\to S_i$. A path $P=F_n\circ\cdots\circ F_1$ is a formally specified composition on its admitted domain $D\subseteq S_0$. Its representation within mathematical M-space registers the operands and transformations as typed objects and relations, with topological and geometrical information preserved wherever it is relevant to the target. A proof attempt and a certified proof can both be represented as paths; only the latter licenses the intended mathematical conclusion. Errors in a proposed proof path affect that path's proof validity but do not prevent the structural investigation of path composition or target-relative simplification.

Partition the path into m consecutive blocks $P=B_m\circ\cdots\circ B_1$. For each block define its input/output state spaces $U_{j-1},U_j$, its admissible domain $D_j$, a target-relevant representation $\alpha_j{:}U_j\to A_j$ and a candidate summary $C_j{:}A_{j-1}\to A_j$. Here $\alpha_j$ may retain only the information needed to reconstruct the target and authorized subsequent operations; the omitted source/history distinctions must remain recoverable when required by the contract. The goal is to discover smaller operative descriptions, not to presume such descriptions exist.

### S29.2. T25 — composition-preserving path summary

Theorem T25. Assume that, for every j=1,...,m and every admitted state u for which the block continuation is authorized, the source, type and guard contracts descend consistently and the following commuting relationship holds: $\alpha_j(B_j(u))=C_j(\alpha_{j-1}(u))$. Assume that the target $\kappa$ on the original final state factors through the final representation, $\kappa=d\circ\alpha_m$. Then for every admitted initial state $u\in D$ on which the complete composition is defined, $\kappa(P(u))=d((C_m\circ\cdots\circ C_1)(\alpha_0(u)))$. In particular, the compressed composite is target-equivalent to the original path at its declared scope and supports all downstream operations separately certified to descend through the $\alpha_j$.

Proof. By the first commuting identity, $\alpha_1(B_1(u))=C_1(\alpha_0(u))$. Assume $\alpha_j((B_j\circ\cdots\circ B_1)(u))=(C_j\circ\cdots\circ C_1)(\alpha_0(u))$. Apply the next commuting identity with the admitted state $(B_j\circ\cdots\circ B_1)(u)$: $\alpha_{j+1}((B_{j+1}\circ\cdots\circ B_1)(u))=C_{j+1}(\alpha_j((B_j\circ\cdots\circ B_1)(u)))=(C_{j+1}\circ\cdots\circ C_1)(\alpha_0(u))$. Induction establishes the identity for j=m. Postcompose with the target decoder d and use $\kappa=d\circ\alpha_m$. Guard compatibility and the operation-descent hypotheses ensure each substituted composition is defined and authorized. ■

The identity representations $\alpha_j=\mathrm{id}$ and original blocks $C_j=B_j$ give a trivial representation of an existing typed path. Genuine compression requires less target-relevant representational or reconstruction work under the specified receiver/work contract.

## S29.3. Target-sufficient bounding relations

Equality is sufficient but not necessary for every mathematical target. Let $\kappa(P(u))$ lie in a partially ordered target space $(K,\preceq)$, and suppose a source-licensed assessment establishes $\kappa(P(u))\preceq b(\alpha_0(u))$ for all admitted u, where b is a simpler bounding operation. If the target asks whether $\kappa(P(u))\preceq c$ and the registered bound satisfies $b(\alpha_0(u))\preceq c$, transitivity establishes the target conclusion. A lower bound has the dual form. Each use must register the target order, direction of comparison, domain, guards and certificate; a bound without a target-sufficient decision condition cannot be promoted to a proof of the original claim.

## S29.4. Constructive exact example and negative control

Let the three typed real-valued transformations be $F_1(x)=x+1$, $F_2(y)=2y$, and $F_3(z)=z-2$. Their composite is $P(x)=F_3(F_2(F_1(x)))=2x$. Thus the single operation $C(x)=2x$ is an exact summary for this target and domain, with $\alpha_0=\alpha_3=\mathrm{id}$. It preserves the input-output relation while replacing three displayed elementary transformations with one. This proves the existence of a nontrivial compositional shortening for this explicit example; it does not calculate a human receiver's actual decoding cost.

Negative control. Let two source histories be merged by $\alpha_0$ despite requiring different target outcomes, or suppose a proposed summary ignores a guard on one of its source blocks. Then no target decoder or adequate guarded-operation descent exists for the claimed summary, and T25 does not apply. Similarly, a locally valid summary whose output type cannot feed the next block does not license the global composition. Such failures are counterexamples to the candidate compression, not to the underlying ability to represent the proof attempt as a sequence of typed paths.

## S29.5. G(R), L(R), assessment histories and comprehension budgets

G(R) can generate and assess candidate blockwise relationships on partial or indirect information, returning COHERENT, INCOHERENT or UNDETERMINED at a registered executed budget and scope. A positive global-coherence verdict for a declared conditional relation is not an unconditional L(R) correctness verdict. Admission of a block summary as a reusable proof operation requires its source, assumptions, guards, proof or assessment certificate, residual obligations, and history to be retained in R; L(R) then acts on the target-complete information supplied by that licensed interface. The universal end-to-end adequacy and work-cost questions remain separate from the exact algebraic result T25.

For a specified receiver $\sigma$, register Wfull as work to reconstruct the required target through the expanded path, and Wcomp as work to obtain, decode, assess and use the compressed composite with preserved continuation. When $W_{comp}\leq B_\sigma<W_{full}$ under an admissible two-route comparison, a work-bounded receiver can use the compressed path but cannot complete that stated full-expansion route. This specializes T15 (S13.2) and does not assert that every long proof has a cheaper summary or that symbolic length alone measures understanding. Diagrammatic interfaces must also retain the operator signatures, context parameters, guards and invariant cues at their points of use (S28).

## S29.6. Application boundary for the Riemann-hypothesis case

The compositional theorem is assessed independently of any source-bounded retrospective mapping of particular RH steps. A formally recorded RH proof or candidate proof attempt admits decomposition into registered transformation blocks. The task-specific question is whether those blocks have shorter target-preserving summaries or adequate bounds and whether each such summary corresponds to the actual certified formal proof rather than to a conjectured historical discovery path. A failed Stepanov–theta-lift correspondence, an incorrect candidate step, or a missing private discovery record may defeat a proposed RH-specific simplification but does not invalidate T25. Conversely, demonstrating a candidate decomposition does not certify the RH conclusion.

## S29.7. Scope, dependency and assessment record

T25 has a mathematical derivation in the displayed induction under its explicit commuting, type, target-factorization and guard premises. The constructive affine example is exact. The bound criterion is a proved consequence of transitivity under its declared ordered target. Global existence of economical summaries for arbitrary mathematical proofs, actual human comprehension gains, universal M-space topological realization, and any proposed RH-specific source mapping have their own obligations. The relevant existing contracts are T01–T02a

(target reconstruction and guarded operation descent), T15 (receiver-budget comparison), S20.2 (diagram operation preservation) and S28 (visual-ontology closure and local decoding). No previous certificate or historical budget is enlarged by this theorem.

# S30. Executed common-functional reconstruction, generator transport and finite work conservation

This section demonstrates common assessment, admission, correction, provenance and work rules across two finite discovery mechanisms. It proves a grammar-relative obstruction, resolves it by transporting a definition in contrast to its finite extension, and establishes a computational-work conservation invariant.

## S30.1. Complete registered state and the common six-function executor

The executor state R contains typed records (value, status, ancestry, history), source-channel licenses, sealed checker-only records, named store regions, an append-only event log, admitted domain operators, an operative program and one work ledger. The interpreter accepts CREATE and DESTROY as two instruction forms of Create/Destroy; PLACE/Store, RECALL, L, G and CHOICE complete the six internal function families. Domain-specific constructors, evaluators, criteria and assessors are typed operators registered as operands within R; they are the native implementation premises, not newly declared internal primitives. Both experiments use the same interpreter module and cost table.

Local L applies an admitted evaluator to target-complete operands. Global G assesses partial/indirect candidates and registers COHERENT, INCOHERENT or UNDETERMINED, with promotion only after positive assessment and corrections appended for defeated earlier admissions. RECALL excludes sealed records; unavailable or unlicensed sources cause a source violation. CHOICE selects an admitted continuation. The fixed declared costs are CREATE=1, DESTROY=1, PLACE=1, RECALL=1, L=2, G=3, CHOICE=1 work tokens. The distinction between six function families and seven opcode spellings is recorded explicitly: CREATE and DESTROY share one function.

## S30.2. T29 — finite no-external-intake work-conservation theorem

Theorem T29. Let R have initial nonnegative registered available work $W_0$, cumulative charged work $E_0=0$ and $W_0$ available at initialization. Each attempted opcode i has a fixed declared nonnegative charge $c_i$. If $W_t \geq c_i$, execution charges the instruction by $W_{t+1}=W_t-c_i$ and $E_{t+1}=E_t+c_i$. If $W_t<c_i$, execution does not charge or execute the instruction and records BUDGET_EXHAUSTED. Then, for every admitted finite execution history, $W_t \geq 0$, $E_t \geq 0$ and $W_t+E_t=W_0$. Moreover no registered charge can be spent from the same available balance twice.

Proof. Initially $W_0+E_0=W_0$. For an admitted charge, $W_{t+1}+E_{t+1}=(W_t-c_i)+(E_t+c_i)=W_t+E_t$, and $W_{t+1} \geq 0$ by the guard; E is nondecreasing. For an insufficient charge, W and E are unchanged and a budget-exhaustion event is appended. Induction over attempted instructions gives the stated conservation and nonnegative-accounting properties. Each executed charge reduces the unique shared available-work operand before further instructions execute, so no two admitted instructions can independently spend the same portion of W. ■

Registered interpretation and coherence assessment. The projection maps functional work remaining to W, completed recorded functional work to E, and each admissible information operation to a charged state transition on complete R. Its conservation and no-overspend requirements are satisfied exactly in this finite no-intake model. Verdict: COHERENT, at the registered computational-work-accounting scope. This is a positive structural projection supporting the GIGT functional free-energy conservation hypothesis; work tokens are deliberately abstract rather than measured physical energy. Internal calculations carried out by registered domain-specific operators are not separately metered by the opcode ledger, so the certified invariant concerns declared executor work and viable registered continuations.

## S30.3. R19 — executed paired reconstruction and controls

Reduced CEGAR is compared against an independently coded direct CEGAR comparator over 300 finite test systems. Its verdicts, refinement sequences, corrections and execution traces match the decoded common-executor results on the stated family (104 SAFE, 196 UNSAFE; 277 systems require refinement). Reduced HR-style concept formation is compared against a separately coded native construction for 20 initial-domain/agenda/growth

configurations; theories, concepts, conjectures, corrections and admitted revisions match. The retained paired_report.json records 1,596 passing assertion identifiers and zero failures. These are recorded outcomes of the supplied reproducible execution, with the five code file hashes checked against their retained report.

Defeating-condition controls reject a candidate without a genuine class split, an incoherent purported generator, an attempt to recall a sealed final target, an unlicensed channel and a seventh opcode. A name-only final-answer representation fails the provenance-closure check. Budget exhaustion one token below a required run produces BUDGET_EXHAUSTED rather than a false mathematical verdict. In the concept-formation family, redundant generated names are treated as equivalent and generated operands can be reused after adequate assessment. The positive result is that one unchanged registration, assessment, correction, provenance and finite-work discipline governs both modeled mechanisms. The domain-specific algorithms and their operator semantics are registered premises, as required by S27.

## S30.4. T26 and T27 — natural finite-grammar obstruction

Define the infinite-state system $S\infty$ with carrier $\mathbb{N}$, initial state $\{0\}$, transition $x\mapsto x+2$ and bad states the odd integers. All reachable states are even, so $S\infty$ is safe. Let F be any finite family of predicates of the forms $x<c$ or $x\in A$, with A finite. A finite Boolean combination of these predicates is eventually constant because every atomic predicate is false beyond the maximum of its finite support and thresholds.

For the established predicate-abstraction context, see [55].

Theorem T26 (eventual-constancy obstruction). Every predicate in the Boolean closure of F is eventually constant; $E=\{x\in\mathbb{N}:2 \text{ divides } x\}$ is not eventually constant, hence E is not in that closure. Proof. For sufficiently large x every generator of F is false, and therefore every finite Boolean expression in them is constant on that tail. The parity predicate E alternates infinitely often. ■

Theorem T27 (finite threshold/extension abstraction cannot certify safety). Every existential partition abstraction of $S\infty$ generated by a finite F has an abstract path from [0] to a class meeting bad states. Proof. Let K* be the eventual-tail class, containing both an even e and the odd e+1. The concrete even orbit $0\to2\to\cdots\to e$ projects to an abstract path from the initial class to K*, while K* meets the bad set. Hence the abstraction cannot return a sound SAFE certification from that grammar. ■

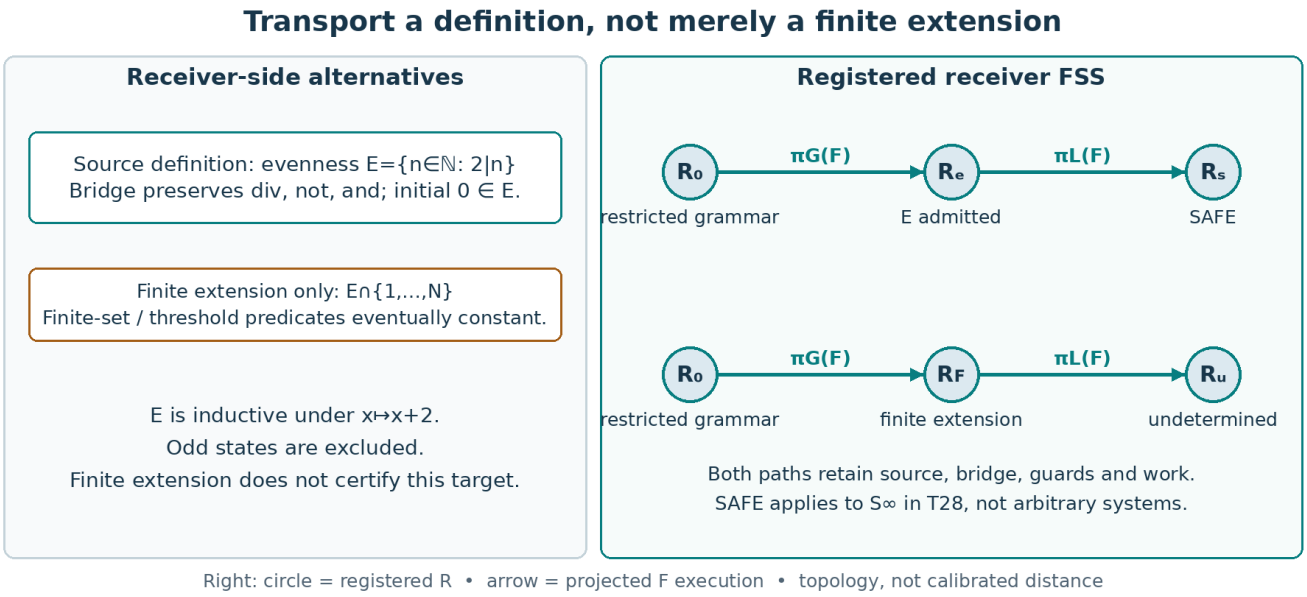
This is a natural initial-grammar obstruction, not a circular restriction designed to exclude the target by definition. With threshold-only refinement, spurious counterexamples continue under the receiver algorithm in the retained execution; a finite budget yields UNDETERMINED without converting non-certification into incorrectness.

## S30.5. T28 — transported definition enables a new licensed continuation

In the source HR-style formation model, split2(div,2) is a generated and admitted unary concept defined over a finite initial domain. The typed bridge $\beta$ maps that intension to the receiver predicate $E=\{x\in\mathbb{N}:2 \text{ divides } x\}$; it registers the interpretation of the source finite number sort in $\mathbb{N}$ and the guard 2 divides 0. The bridge transports definitions for admissible div, not and and concepts, and rejects untyped source constructors without a valid receiver image. The receiver G operation checks the new predicate against the initial bad-state separation target and admits it before L verifies safety.

Theorem T28 (intensional transport versus extension transport). (i) Adding the transported definition E to the receiver grammar suffices to certify $S\infty$ safe after one refinement. (ii) Adding only the finite extension $E\cap\{1,\ldots,N\}$ for any finite N, even together with any finite threshold family, cannot certify safety. Proof. For (i), the abstraction $\{E,\mathbb{N}\setminus E\}$ separates initial 0 from odd bad states and is inductive under $x\mapsto x+2$; the abstract initial class remains E, so no abstract bad state is reachable. For (ii), every finite extension is a finite-set predicate, so T27 applies. ■

The executed bridge records 31 admitted unary source concepts, seven typed receiver images and one successful promoted evenness definition. The retained test reports SAFE after one receiver refinement (43 declared work tokens). An irrelevant 3-divides-n concept does not promote, a finite extension fails to certify, and source/license and sealed-target controls behave according to the registered guard contracts. This supplies an explicitly typed, source-licensed instance of G-generated mathematical information becoming an admissible new continuation under the receiving L repertoire.




S30 Figure 1. Intensional concept transport into a restricted verification grammar. Typed source concept and bridge, receiver-side G admission and L verification are operative graph components; the finite extension remains outside the target-sufficient receiver closure (T26–T28). Right: the two alternative complete-state trajectories distinguish admission of a definition from admission of a finite extension. SAFE is the stated T28 target, not a general guarantee.

## S30.6. Positive coherence projection, assessment typing and evidence record

The two discovery mechanisms, their corrections and the transported-concept witness use one cost table. T29 derives the ledger identity from the work update rule. The executions, common sources, promotion rules, corrected histories and exhaustion controls support a COHERENT verdict for the finite structural projection of H4's work-accounting component.

Execution and assessment evidence types are kept distinct. T26–T29 are direct proofs from the registered assumptions; R19 is an executed comparison at the indicated test family; R22 is an executable source-licensed witness plus a mathematical receiver-grammar proof; R23 is an exact work-conservation theorem and positive coherence correspondence. The retained report is the source for the count 1,596/1,596, while source hashes and the code path supply re-execution provenance. No assumption that a nominal work token equals a joule is needed for the abstract model's invariant.

## S30.7. Reproducibility and scope-certified status

Companion material: reproducibility/S30_implementation_note/code/{fma_core.py,cegar_finite.py,hr_concepts.py,xdomain.py,run_paired.py}, results/{paired_report.json,run_log.txt}, and the retained source-level archive audit. The execution command from that directory is python3 -I code/run_paired.py results/paired_report.json. The retained numerical outputs are not substitutes for proofs T26–T29; each has its own scoped evidence contract. The source-level RH mirror re-audit appears separately in S26.

Status record. R19: executed bounded positive for INFRA-U1; R21: mathematical derivation of grammar-relative obstruction (T26–T27); R22: mathematical derivation of transport contract (T28) with a successful typed receiver execution, one source/target pair; R23: mathematical derivation of finite conservation invariant (T29), and COHERENT structural finite work-conservation projection at the exact no-intake work-token scope. These records append to T01–T25 without changing earlier proof budgets, scopes, historical chronology or other positive certificates.

# S31. Functional Modeling Approach Defeat of Truth-by-Definition Objections (FMA-DTD)

## S31.1. Constructed model and independently evaluable target

Let $D_0$ be a finite set of admitted model-construction observations, and let $M_0$=Fit($D_0$) be a functional model with typed operands, executable operators, contextual guards, source permissions, assessment rules, historical records and work-accounting contracts. Let $D_1$ be a further set of admissible evaluation cases. Let $\kappa$:$D_1 \to Y$ be the target observation or independently specified reference outcome and let p($M_0$,z) be the model prediction for case z. Define V($M_0$,z)=1 exactly when p($M_0$,z) satisfies the registered target-equivalence, authorization, provenance and work contracts relative to $\kappa$(z); otherwise V($M_0$,z)=0. The evaluation rule and the operative model are fixed for the evaluated cases rather than rewritten using their resulting verdicts. The assumptions registered for $M_0$ do not include the assertion that every V($M_0$,z) succeeds.

## S31.2. T30 — Functional Modeling Approach Defeat of Truth-by-Definition Objections Theorem

Theorem T30 (FMA-DTD). Assume (A1) $M_0$ and the evaluation rule V are specified independently of the outcomes under assessment; (A2) $\kappa$ is supplied through an appropriately separated observation, licensed source, or separately executable reference semantics rather than assigned to equal p($M_0$,·) by definition; (A3) there exists a countermodel of the construction premises in A1–A2, with the same declared evaluation domain $D_1$ and evaluation contract, in which V($M_0$,z)=0 for some z∈$D_1$; the model or outcome may vary only to the extent permitted by those premises; and (A4) the actual outcomes V($M_0$,z) on $D_1$ are executed and recorded with their typed sources, scope and work. Then the proposition "V($M_0$,z)=1 for every z∈$D_1$" is not a truth-by-definition consequence of $M_0$'s construction premises. When all observed cases satisfy V=1, their success contributes positive, non-circular evidence for the model's target-relative coherence at the tested scope. It does not by itself prove unique architecture, universal coverage, or any claim stronger than the registered validation contract.

Proof. By A3, there exists a model of the construction premises in A1–A2 in which a case z in the declared domain $D_1$ fails the registered success predicate. This model satisfies the premises and falsifies the universal success proposition over $D_1$, proving non-entailment. A failing case outside $D_1$, or a corrupted output forbidden by the construction premises, would not supply that countermodel. Under A4, a positive recorded evaluation is a fact about the evaluated cases, not a consequence stipulated by the model definition. This is exactly the claimed separation between the construction of a functional model and the evaluation of its predicted relationships. ■

Corollary T30a (positive coherence verdict). Under A1–A4, if the evaluated cases additionally preserve the typed transformation, assessment-history and work contracts required by the target, the recorded successful evaluation supports a COHERENT verdict under the registered finite assessment scope. The original source, target, executed assessment and budget records remain separate operands; this verdict is not automatically an L-verdict of unconditional CORRECT for a stronger target.

Conditional mathematical derivation T30 verdict: COHERENT at A1–A4 as stated. Its application to a fixed executor additionally requires a countermodel satisfying A3 on $D_1$. The finite comparator agreement and fault-sensitivity evidence have their own positive assessment; the A3 countermodel application is UNDETERMINED. See S36.

## S31.3. Executed finite application to R19 and the scope of the positive verdict

The retained R19 protocol runs the registered six-function executor and separately written reference implementations on 300 CEGAR inputs and 20 concept-formation settings, together with provenance, admission, guard, work and negative controls. Agreement is tested, not assigned by the executor definition: a disagreement in any compared verdict, refinement history, source authorization or work receipt fails an explicit assertion. The supplied report records 1,596 of 1,596 assertions passing; the corresponding source hashes remain registered under S30.

A further validation uses the same five Python implementation sources, fixed by SHA-256, on seed indices 300–311. These indices are disjoint from the retained 0–299 suite, but all cases come from the same CEGAR input generator and mathematical specification, not an independently sampled population. All 12 cases agree with their reference verdict, refinement count and spurious-correction history, and preserve the declared work ledger. A

comparator-sensitivity test also rejects deliberately fault-injected executor outputs; this checks the comparison procedure, not the correctness or uniqueness of the six-function basis. The indices, per-case outcomes, source hashes and sensitivity results are recorded in reproducibility/S31_FMA_DTD_validation. No exhaustive historical calibration or model-development chronology is asserted.

Assessment at this registered scope: COHERENT (finite comparison and retained functional-contract scope; T30 A3 application separately UNDETERMINED). The executable model predicts and governs assessed continuations, and the separately computed reference outcomes and nontrivial case comparisons show that success was not stipulated by naming six functions. The fault-injection check establishes only comparator sensitivity, not independent evidence for architectural uniqueness. The tested functions are registered governance operations over supplied, domain-specific operator implementations; this positive verdict is for the stated finite functional execution and validation contracts.

### S31.4. Distinguishing formal conservation from independent functional validation

T29 establishes algebraically that charged, no-external-intake transitions satisfy $W_t+E_t=W_0$ exactly within the registered executor model. Its proof requires no out-of-sample observations; it is an exactly derived invariant of its explicit transitions. The empirical question is different: does a fixed functional executor successfully govern multiple separately specified adaptive computations while respecting those charged transitions? R19 and the disjoint-input tests answer that question positively at their finite scope. Their finite validation supplies positive, model-relative coherence evidence at the comparison contract; the construction-premise countermodel in T30 is a separate assessment target, and their work-accounting behavior supplies a coherent structural witness for finite work conservation in R23. Neither the double-entry identity nor that witness alone proves the universal necessity target H4. The charge does not meter all hidden Python computation and is not a calibrated physical-energy unit; the coherence verdict is precisely for the declared computational-work projection.

### S31.5. Reproducibility, dependency registration and assessment status

Executable record: reproducibility/S31_FMA_DTD_validation/run_holdout.py and results/holdout_report.json. The exact unmodified model and reference code are supplied under reproducibility/S30_implementation_note/code. The fixed model-source hashes and validation indices are retained together with the individual comparator outcomes. T30 is a conditional mathematical derivation given A1–A4; the R19 comparison assessment is positive at its registered scope, while the A3 application remains UNDETERMINED. Its antecedents and limits are preserved in the same registered evidence record.

## S32. Integrated conditional derivations of mathematical discovery and continuation

### S32.1. Joint mathematical construction and its consequences

The exact targets H1–H8 and $J=H1\wedge\cdots\wedge H6$ remain those in S1 and Appendix B. This section derives a common nontrivial problem class in which observer-relative non-discoverability, operator generation, recursive observation, functional and energetic viability, source-only solution construction and reliable convergence can be assessed together. It also derives receiver continuation for the same class. The theorem targets below supply conditional mathematical relationships bearing directly on the registered hypotheses, with their exact scopes retained. All original valid proofs and finite certificates retain their scopes. The mathematical proofs use the contracts of T01–T04, T07, T10–T13, T17–T25, T26–T30 and S4–S7. All conditional results retain their antecedents; no unconditional verdict is assigned to a conditional theorem. The proof of the conditional implication is separate from an assessment of its realization. The following proofs are self-contained at the stated mathematical scope.

### S32.2. Shared source and registered state

Let U,V,W be disjoint finite nonempty sorts. A source history h registers two binary relations $P\subseteq U\times V$ and $Q\subseteq V\times W$, with source identity and licensed signal interface returning the presence or absence of a queried edge. The generator initially has the sort declarations, the source interface and the admissible generic constructor for typed composition, but not the edge values, the target relation or a completed proof path. The target $\kappa(h)=P\circ Q$ is a $U\times W$ relation, with an existential witness v for each positive entry. The fixed baseline is the L0 grammar of S3,

with only sort-preserving constructors; its source observations do not include a U×W result. A stronger baseline containing composition is expressly outside the separation claim.

Let s∈{0,1,?}^N be the partial edge registration, N=|U||V|+|V||W|. Let H_s be its nonempty set of admitted completions. In the independent-edge class every completion of s is admissible. The state R includes s, typed operators, candidate and admitted paths, guards, observations and source receipts, current and original work budget, assessment records and correction provenance. The model generates the typed join using matching V types (T04). It does not invent a seventh primitive. Its operations instantiate functional contracts: Create/Destroy registers signal distinctions and withdraws defeated candidates; Place/Store retains licensed results and histories; Recall retrieves source receipts and signatures; G constructs and assesses candidate paths, observation repairs and the join; L checks target-complete bounds and admitted paths; Choice selects the next licensed observation using the state and its work viability. This is a decomposition into the existing six roles, not a proof of their universal minimality, nor a claim that the Python test is a full physical FMA implementation. Cross-system observations are mediated by the source signal interface, not direct access to another FSS.

The work projection charges one token per explicitly registered execution event, including generation, assessment, choice, observation, registration and promotion. At every step W+E=W_initial in the absence of intake or loss, with a nonnegative guard on expenditure; an independently instantiated receiver has its own work budget. This is T29's accounting structure, not an identification of tokens with physical free-energy. Physical realization of the topology and its energetic conservation requires its own typed correspondence.

## S32.3. T31 — integrated obstruction, repair and construction (H1, H2, H5)

**Conditional theorem target.** On the shared source class above, when at least one path u→v→w is admissible, a fixed L0 observer cannot construct the output relation, whereas a source-licensed process that generates typed composition and obtains sufficiently distinguishing observations can do so without being supplied the result. The observation/grammar obstructions and their repairs can be witnessed in the same class.

**Derivation.** Choose the empty source and the source containing just those two edges. Their initial observation is the same, but $\kappa$ differs (T01). A fixed observer that cannot obtain distinguishing observations cannot determine both outcomes. Independently, even with all source edge values, induction on the constructors of L0 shows that none has output type Rel(U,W): base relations have types Rel(U,V) and Rel(V,W), and the constructors preserve those types (T03). This second obstruction is not just generic observational loss. Repairing only access to edges does not repair the output-type obstruction; generating only the join does not remove the initial target collision. Both are repaired by the composed process: register source observations and use the matching intermediate type V to generate the operation $(P,Q) \mapsto \{(u,w): \exists v\ P(u,v) \wedge Q(v,w)\}$. If a pair is generated, the witness v gives two licensed edges; conversely each such path supplies a generated pair. The generator has been given the source grammar and a generic composition constructor, not $\kappa$ or the needed successful path. For arbitrary finite U,V,W this works uniformly whenever sufficient observations are available. Thus source access, generation and exact execution form a noncircular chain in a shared class. If the original observer already has a join, the grammar separation disappears; if a required source bridge is unlicensed, generation must remain conditional. This is the explicit H1→H2→H5 interface composition.

**Result.** T31 establishes an observer-relative obstruction, the distinction between information and grammar repair, and constructive generation within the same source class. H1, H2 and H5 receive positive mathematical support.

## S32.4. T32 — recursive target observability and exact adaptive observation cost (H3)

For a partial registration s let $P^-,Q^-$ contain only confirmed edges and $P^+,Q^+$ contain all edges not ruled out. Define $K^-(s)=P^-\circ Q^-$ and $K^+(s)=P^+\circ Q^+$. The unknown target is bounded by $K^- \subseteq \kappa \subseteq K^+$. In the independent-edge class these are not merely loose bounds: the all-unknown-absent and all-unknown-present completions attain them. Every confirmed observation refines H_s; consequently $K^-$ can only increase and $K^+$ can only decrease. Hence exact $\kappa$ is observable through s if and only if $K^-=K^+$. Proof: the inclusion follows from monotonicity of composition; unequal extremes are two admitted completions with different $\kappa$, giving a T01 collision. Equality forces every completion to agree. This characterizes extent (accessible edges/queries), resolution (partitions of compatible histories), history (the intersection of successive observation fibers) and target adequacy in one state-dependent relationship. No abstract graph-size measure substitutes for the operative observation contract.

The minimax recurrence has the form of adaptive decision-tree complexity [53].

A general finite adaptive version: let H be finite and let A_s be the licensed queries available in a registered observation state s, each with known possible signal outcomes and positive cost c_s(q). A query is usable only if its successor registration and residual work are authorized. If observations are persistent and each chosen query strictly refines the current non-singleton history set, define the worst-case required observation work recursively:

D(s)=0 when $\kappa$ is constant on H_s; otherwise D(s)=min_q [c_s(q)+max_y D(s[q,y])], with minimum over licensed strictly refining queries and maximum over realized outcomes; if none is usable, D(s)=$\infty$.

For this theorem the query availability and costs depend on the registered s; if observations change the query repertoire without strictly refining histories, an additional well-founded progress rank is needed. Proof by induction on |H_s|: every allowed resolving policy must begin with some q and succeed on every possible outcome; its required work is at least the displayed quantity for that q by induction. A minimizer, followed by an optimal descendant policy, attains it for finite query repertoires and finite D. The recurrence is an assessment of the operative observation process, not a requirement that the online solver know $\kappa$ before acting. For the independent-edge class, exact bounds permit computing the recurrence without the true target. If available work is smaller than D(s), no allowed policy guarantees resolution in the worst case. This is a nontrivial resource-relative characterization of observability. Query selection, observation-function registration and assessment can themselves become operands of subsequent correction; the partition refinement result in S32.7 establishes the corresponding finite recursive closure under an explicit operation family.

**Execution.** For |U|=|V|=|W|=2, N=8. All $3^8$=6,561 partial registrations were tested against all compatible complete sources, whose target was computed by separately implemented adjacency traversal. Both bounds agreed exactly with the intersection and union of reference outputs in every registration. The dynamic program gives D(all unknown)=8 in the unit-edge-query cost model. This is an exact worst-case observation count for the specified query repertoire, not an F-recursive measure of cognitive bandwidth or a physical energy calibration. The discovery run below uses a cheaper adaptive heuristic and a separate work accounting contract.

## S32.5. T33 — necessity of retaining target-relevant functional and energetic distinctions (H4)

**Conditional theorem target.** For any admitted recurrent task with state representation a:R→A and history-sensitive authorized success-action sets S(r), the representation supports a reliable next-step selection across every state in a fiber if and only if the intersection of the S(r) over that fiber is nonempty (assuming a selectable action for each realized fiber). For finite fibers and finite actions this assumption is automatic. Necessity follows from the fact that a policy depending only on a(r) must choose one action on the whole fiber. Sufficiency follows by choosing an element in the common intersection. This strengthens the two-history obstruction of T02 to the joint condition across all indistinguishable states.

Apply this theorem to two separate projections of the same registered discovery architecture. (i) A later required continuation needs the previously generated relation or an adequately reconstructible source record. A projection that discards both and merges histories requiring different target decisions fails T01, or merges disjoint licensed action sets and fails the above theorem. Thus functional retention/reconstructibility of the relevant generated operand, guards, sources and corrections is necessary for reliable reuse. It need not preserve irrelevant raw detail, and retaining an adequate reconstruction channel meets the condition. (ii) Include a registered energy/cost mode in R. Two states have the same mathematical registration and W=2; both allow the same two candidate ports for a required observation, but their costs are respectively (1,3) and (3,1). A subsequent completion costs one unit. In the first state only port 1 permits viable continuation, and in the second only port 2. Removing the energetic cost mode from the representation gives disjoint viable success-action sets in one fiber. A reliable policy must therefore retain or recover the functional energetic distinction before the consequential selection. Each full state has a viable path; the failure is not lack of any solution. This cost mode is a mathematical example of the necessity of target-relevant energetics in a navigation topology, not a statement that real physical energy has two modes or that every task requires explicit scalar accounting.

Together these two obstructions give joint necessity of preserving the *relevant* functional and energetic distinctions for a recurrent class that includes both required reuse and viability-sensitive selection. They do not follow by defining success as closure. Their consequences are witnessed by target and action collisions. In an implementation, the generated objects and energy-dependent admissibility are part of the same registered transition structure, so their coupling is not an external bookkeeping correction. Under exact work accounting a registered discrepancy is detectable if measurement is adequate (T23); an indistinguishable unmeasured discrepancy is not detectable. T29 supplies the conserving projection; S30–S31 supplies actual common-functional evidence at its original finite scope.

## S32.6. T34 — finite adaptive source discovery and viable convergence (H5, H6)

**Conditional theorem target.** For the independent-edge class with accessible individual edge observations, a process that generates the join, iteratively assesses the lower/upper relations, selects and registers an unobserved edge whenever they differ, and promotes the exact relation when they agree, reliably terminates with the source-supported target under adequate available work. It need not know the final relation, which observations will be needed, or their sequence in advance.

Proof. At each step the true source belongs to H_s by licensed observation. Bounds hold by T32. If they agree, the returned relation is exact, and each positive entry has a confirmed path witness. If they differ, some unknown edge remains; a potential path for an unresolved entry must have at least one unknown edge. Choosing such an edge is possible, and registering it reduces the number of unknown edges by one. New lower-bound paths may become confirmed, upper-bound candidates may be defeated, and neither confirmed positives nor exclusions are reversed by accurate observations. With rank $\rho$(s)=#unknown edges, every nonterminal iteration strictly decreases a nonnegative integer. At $\rho$=0 the bounds agree. Hence there are at most N observations; the state of equality is a fixed point for further assessment of the same source. In the declared charged-event model, one generation, N+1 bound assessments, at most N selections, N observations, N registrations and one promotion cost at most 4N+3 tokens. The work guard ensures viable continuation when W_initial$\geq$4N+3. If the budget is smaller, some sources may still resolve but this bound no longer guarantees it. A later licensed source correction invalidates dependent candidates and creates a new finite registration; after the final change the same rank argument proves convergence if adequate work remains. No claim of convergence for arbitrarily changing sources follows.

This result integrates operator generation with adaptive choice of observation operators and operands, configuration correction and target assessment rather than running a preselected sequence of observations. The general rule exists initially; the typed join and final target are not available as admitted operands in L0. The negative control with a richer initial grammar eliminates the grammar obstruction. The process is not claimed to be superior to every conventional program: it is itself implemented by conventional code.

**Execution.** The source-only generator selects an unresolved edge maximizing its coverage of remaining candidate paths, with a deterministic tie rule. Across all 256 complete source graphs: 256/256 targets agreed with the traversal reference; 16 cases used 4 observations, 124 used 6, 56 used 7, and 60 used 8. It excluded 576 candidate target bits and confirmed 448 target bits over the executions; these are 1,024 bound changes. The worst charged expenditure was 35, matching 4·8+3; every run used an initial budget of 40 declared event tokens and conserved it. A budget-exhaustion control refused further work. The exact mathematical guarantee is independent of the finite test size and holds for every finite three-sort graph with the stated source and charge contracts. The code's uncharged internal scanning cost is not equated to energy or total CPU time.

## S32.7. T35 — recursive interface repair and reconstructible continuation (H3, H8)

This construction uses the classical stable partition-refinement principle [54], extended here by the registered target, guard and receipt signatures.

Let X be a finite registered source state set closed under a finite family O of deterministic partial operations. For each o$\in$O register its guard, authorization and work receipt meaning, and the target $\kappa$. Let $\Pi_0$ be the partition induced by the initial interface. Split any block whose states differ in $\kappa$ or an admissibility/receipt signature, or whose permitted successors under the same o lie in distinct current blocks. Continue until no split is required. Every strict split increases the number of blocks, at most |X|, so the process terminates after at most |X|-|$\Pi_0$| strict splitting events. At the stable partition all targets and guards are constant on blocks and permitted operations map a block to a uniquely determined successor block. Thus each operation descends and $\kappa$ factors; T10/T25 gives receiver continuation for any admitted finite composition. Conversely, any stable refinement of $\Pi_0$ satisfying those properties must distinguish every pair split by this procedure: induction on splits, since a target/guard distinction or a successor distinction already forced in the preceding partition cannot be merged. The resulting partition is the coarsest adequate refinement relative to these fixed operations and targets. If operations or targets are added, refine again and retain the previous scope and source records. The operation-specification state may be part of X, so changes in observation operations are included whenever their transitions are registered in this finite X.

This proves the mathematical existence and bounded termination of a repair process under source-accessible distinctions and adequate work; it does not assume the solver can observe a private state. Each required split must have a licensed source interface or an adequately reconstructible distinguishing signal. If that cannot be obtained, the candidate repair remains UNDETERMINED. The proof is about exact information sufficiency, not every

reader's decoding effort.

**Receiver witness.** The sender transmits a partial observation packet at the midpoint of its discovery process. The packet retains the observed edge receipts, source identity, sender's budget and trace; it does not transmit the completed target as an operand to the receiver. The receiver reconstructs the partial state, generates the join and continues with its own licensed query interface and work budget. By T32, the true source remains in its completion set; by T34, the receiver computes the same target. This implements the T12–T13 signal-mediated transport and T25 path-continuation contract without requiring full-state identity. The source receipts are assumed authentic in this mathematical interface; cryptographic source authentication or arbitrary hostile transmission is not tested. Across 256 transmitted prefixes, 256 receiver runs agreed with the reference result, with maximum receiver charge 23 tokens (including one charge per received edge record); malformed duplicate/conflicting receipts were rejected. Both original sender budget and the receiver budget remain separately typed. The code does not claim that receiver cost plus transmission cost is always less than independent discovery, or that a human has reconstructed the proof.

## S32.8. Joint derivation of the common-context hypothesis construction

Let $\Gamma^*$ consist of the source family and L0 grammar in S32.2, the licensed matching-sort constructor, persistent source receipts, finite accessible query family, accurate observations, guarded work-accounting transitions, and viability-sensitive recurrent continuation cases in S32.5. Let J* denote the conjunction of the following explicitly restricted propositions: existence of a blind spot in the fixed observer, a generative repair distinguishing observation and grammar restrictions, recursive target observability, necessity of retaining target-relevant functional and energetic distinctions in the declared recurrent class, source-only construction, and reliable finite adaptive convergence. These are the scoped H1–H6 counterparts, not the unmodified full J.

The interfaces compose as follows. T31 takes the same source family and produces the missing type/operator; its output is the candidate join in R. T32 consumes the same source observations and operator, computes exact bounds, and supplies the termination/choice contract. T33 uses the same target and source-record representation and extends R by a cost mode; the projection failures demonstrate precisely which distinctions a continuation needs. T34 uses T31's generated rule, T32's bound test and T33's guarded resource semantics. It generates the exact target with witness paths and conserves the declared work. The two cost-mode cases are compatible with the guarded work model: cost-one choices and a final unit charge satisfy W=2, while the cost-three option is inadmissible. The energy mode witness does not change source relation semantics. The source family is nonempty (e.g. empty graph and graph with one two-edge path); the retained work and observation model satisfies the registered premises. Thus the interactions are checked, not inferred from separate statuses: the final target is unchanged across generation and observation; the grammar extension does not require the target; observation and candidate correction preserve authorized source history; the work bound supports every branch actually executed; and the fixed point is a licensed exact relation, not merely a stable guess. By conjunction introduction in this common context, $\Gamma^* \Rightarrow J^*$.

For the scoped H8 counterpart H8*, T35 consumes the same source and continuation semantics without identifying sender and receiver FSSs. This supplies $\Gamma^*$ plus its transport assumptions $\Rightarrow J^* \wedge H8^*$. No claim is made that the finite work token model itself realizes the full GIGT physical energetic topology. The finite demonstration supports the conditional joint construction independently of the RH case.

Registered assessment of the S32.8 mathematical derivation: COHERENT. Target: $\Gamma^* \Rightarrow J^*$, and $\Gamma^*$ with the stated transport assumptions $\Rightarrow J^* \wedge H8^*$. Scope: the declared finite three-sort source family (arbitrary finite sort sizes), fixed initial grammar, licensed constructor and observations, guarded work and recurrent continuation class, and source-receipt preserving receiver. Method: check the five dependency and conjunction conditions in S36. Distinct modeled execution and native budgets remain separate.

**Executed evidence record.** check_joint.py and joint_report.json contain a reproducible source-frozen finite evaluation: 6,561 partial-registration checks, 256 discovery executions and 256 receiver continuations, with explicit receipt accounting and controls. The observed outcomes constitute positive evidence for this joint projection. The computational work budgets are 40 event tokens per run. They are not the budget of the overarching G(R) assessment. A separate finite assessment state registers the complete finite case set: $3^8$ partial states, $2 \cdot 2^8$ sender/receiver executions and four named controls. With one abstract assessment unit per complete case evaluation, the budget derived from that state is B_model=7,077 units. assess_joint.py reconstructs target values using a separately implemented bit-mask representation of the same mathematical semantics, checks the observation/target bounds, verifies trace transitions, checks the work charges and source packet provenance, and checks the controls, using all 7,077 registered units. The conditional mathematical arguments above establish the corresponding grammar obstruction, necessity and convergence contracts that are not merely numerical tests. The

resulting verdict for the finite modeled projection J* (including receiver continuation at its stated scope) is COHERENT at B_model=7,077, with evidence assessment.json. This is a discrete case-assessment model, not a physical-energy or CPU-time metric, nor the native GIGT assessment of J. The separate assessment of the original J is recorded in S33. The finite case budget is conserved there as an incoming certificate, not reassigned to J. Existing finite comparison and work-conservation evidence for R19 and R23 is preserved. T30a retains its conditional antecedents; the A3 application is assessed separately.

## S32.9. H7: documentary and mathematical transport assessment

The primary documentary source [1], pp. 1, 3–4, identifies Stepanov-type multiplicity as a source analogy, a native automorphic theta-lift chain as a candidate new realization, and a chain scale near $\sqrt{X}$. The retained source-text audit at commit 6193bb5e [23] records a named declaration route to RH and the RankWindowAt and CoarseWindowAt definitions. These facts satisfy the documentary source part W1 and supply a positive structural correspondence. They do not supply the full W2 operation-preservation proof or W3 elaborated dependency path from the specific transported relation.

A useful exact consequence of the retained rank constraints can be derived. Suppose $x \geq 1$, $\epsilon > 0$, the rank and coarse bounds apply to the same m, and $\delta_1 = x^{\wedge}(-\epsilon/8)$ is the very same parameter appearing in RankWindowAt. Then

$$x^{\wedge}(1/2-\epsilon/16) \leq m \leq \min(2\, x^{\wedge}(1/2-\epsilon/16),\, x^{\wedge}(1/2)).$$

Indeed substitution in $\sqrt{x\delta_1}$ gives the first exponent; for $x \geq 1$ the rank lower bound dominates the coarse lower bound, and the upper bound is the intersection of both intervals. Thus $m/\sqrt{x}$ lies between $x^{\wedge}(-\epsilon/16)$ and $\min(2x^{\wedge}(-\epsilon/16),1)$. For fixed positive $\epsilon$ and unbounded x this ratio tends to zero. The exponent may be near 1/2, but constant-factor $\Theta(\sqrt{x})$ does not follow for fixed $\epsilon$. An identification of x with the documentary X and an $\epsilon$-regime are required for a precise comparison. This is an exact conditional source-expression consequence, not a claim that the two parameters have been type-resolved or that these constraints lie on the elaborated endpoint dependency path. It identifies a concrete mathematical obligation rather than reporting generic similarity.

Integer-rank feasibility: the displayed real interval is nonempty, but an integer m exists exactly if $\mathrm{ceil}(x^{\wedge}(1/2-\epsilon/16)) \leq \mathrm{floor}(\min(2x^{\wedge}(1/2-\epsilon/16), \sqrt{x}))$. For x=2 and $\epsilon$=1 the interval lies strictly between 1 and 2, so no integer m exists. The conditional bound remains valid for every admissible m; feasibility must be checked when an integer rank is required. Verdict of this conditional arithmetic derivation: COHERENT.

The proposed transport can be stated in an adequate noncircular form: source multiplicity operations have a registered filtration/valuation and addition law; the theta-lift composition must preserve an analogous certified gain invariant under its own domain and guard contracts. A sufficient abstract criterion is a sequence of source-supported maps f_j with registered gain g_j and a target valuation v satisfying $v(f_j(z)) \geq v(z)+g_j$ on every permitted state. Induction gives $v(f_m \cdots f_1(z)) \geq v(z)+\Sigma g_j$. A global bound must be separately transported and combined with that lower bound; a successful abstract inequality alone is not a Stepanov-specific proof. A non-Stepanov process with gain one per step and cost x/m+m has the same additive gain and square-root scale by AM–GM, and need not possess any Stepanov multiplicity structure. Thus the two observed features are not a discriminating invariant. This defeats that proposed *sufficiency criterion*, not H7. To close W2, identify and prove a characteristic transported relation beyond generic additivity/scale, such as the actual per-lift valuation increment under the corresponding multiplicity conditions, and its guard-preserving bridge to the formal target. W3 needs the actual elaborated dependency or an adequately certified mathematical reconstruction of its use. A private historical causal transcript is not a precondition for structural compatibility; a full historical causal claim or source-frozen reproduction is distinct.

**Result.** The documentary-level structural correspondence supplies positive evidence for H7, and the rank-window intersection gives a precise conditional mathematical relationship. The specific invariant transport and historical reconstruction are separately typed continuation targets.

## S32.10. Literature interfaces and their use in the derivations

The evidence targets E1–E7 are distinct from the mathematical hypotheses. E1–E3 motivate why direct reconstruction, verification throughput and human contribution are consequential; they do not entail an observer-relative theorem. E4–E6 supply precedents for operating on representations, reusable generators and correction, while T31–T35 establish particular mathematical relations. E7 constrains the hypothesis about erasure: proof tools can retain partial goals, so universal erasure is not an admissible premise.

**E1** — Lean checking report [2], Heule et al. [4]: practical scale limits of direct inspection. H8 motivation; T25, T35 supply actual reconstruction conditions.

**E2** — Zheng [5]: modeled production/checking rate difference. H4/H8 motivate finite work; T33–T34 are mathematical, not consequences of throughput data.

**E3** — Martingale Foundation [13]: voluntary survey. Motivation only; not a premise of J.

**E4** — Lean metaprogramming [20], Lean 4 [3]: proof expressions and goals can be operated upon. Concrete precedent for H2/H5/H8 operand-level registration; T31/T35 supply the mathematical construction.

**E5** — Aryal [6], Boolos [17]: compression/reusable intermediate structures. H8; T25 gives target-preserving summaries, T35 gives continuation. Actual human work savings not inferred.

**E6** — Giacobazzi et al. [14], Clarke et al. [16], Cousot & Cousot [31]: abstraction/refinement. H2/H3/H5/H6; T32–T34 explicitly prove the source/guard/target chain.

**E7** — Lean metaprogramming [20]: partial goals can be retained. H1/H3/H8: test loss relative to a specified interface, not universal erasure.

The literature propositions are used only at their source-registered scopes. The derivations do not depend on these empirical examples being universal. A conditional derivation of a hypothesis must use mathematical premises actually sufficient for its conclusion, not a conjunction of relevant literature statements with an assumed conclusion. Table 2's stronger generator-compression necessity and universal erasure statements are search questions, not established literature findings. T15 proves the declared two-route necessity; the full human-FSS route exclusion, actual work measurement and universal claim remain unestablished.

### S32.11. Established functional relationships

T31–T35 compose a common mathematical discovery and continuation architecture. The conditional conjunction $\Gamma^* \Rightarrow J^*$ is derived by the common-context proof in S32.8 and the finite projection is COHERENT at its registered budget of 7,077 case/control units. H7 supplies separate documentary compatibility evidence; the scoped H8 counterpart extends the common construction through a signal-mediated receiving interface. These results supply the incoming functional information for the original-hypothesis assessment in S33.

## S33. Global coherence of the registered hypotheses

### S33.1. Assessment target and dependency boundary

The target is J, the original H1–H6 conjunction in Appendix B. H7 and H8 are assessed separately. The GIGT [42] architecture and its functional and energetic conditions enter as conditional foundational premises. The assessment examines their compatibility and the positive mathematical evidence in S2–S32.

### S33.2. Executed evidence-relative assessment

The author-led evidence-relative assessment consists of eight hypothesis reviews, fifteen pair-interface reviews and six joint checks. One unit is a reasoned review of one record, giving B_E=29 review units. The records below provide the reasoning for readers to assess. The distinct finite projection, B_model=7,077 case/control units, was replayed computationally. Review units count completed reviews; native functional free-energy requires its own work measure.

**1. H1 — T01,T03,T17,T31.** Empty source and two-edge path have same initial view, different target; L0 has no U×W output even after full observation. Existential witness; compatible with the shared functional contracts.

**2. H2 — T03,T04,T31,R19.** Same source and target as H1; adding matching-sort join changes admissible output types, and G repairs partial observations. H1 obstruction and H2 repair concern the same restricted observer, not conflicting accounts.

**3. H3 — T01,T19,T32,T35.** The completion set shrinks by licensed observation; extreme completions attain exact bounds. Partition refinement also admits observation functions as operands. The source remains admitted after every refinement.

**4. H4 — S5,T23,T29,T33,R23.** Discarded source distinctions and cost modes give disjoint viable action sets. Retaining them permits reuse and guarded conservation; the native universal conservation condition is a GIGT premise, not inferred from tokens.

**5. H5 — T04,T18,T28,T31,T34.** Typed join is absent from original grammar, not the generic matching-sort constructor of the repair process. Query interface supplies source bits, never completed target. Generated result is exact by witness induction.

**6. H6 — T07,T24,T34.** Rank strictly decreases while unresolved; at equality the exact target is stable. Work bound 4N+3 permits all actual steps. H6 quantifies a nontrivial class; no arbitrary changing-source convergence is required.

**7. H7 — S26,S32.9.** Documented source analogy and candidate theta-lift chain are compatible with reuse of a generator; the real rank-window intersection is nonempty for $x \geq 1$ under its stated premises; integer feasibility additionally requires the ceiling/floor condition in S32.9. Generic scaling is not discriminating; no causal or proof-specific transport asserted.

**8. H8 — T10,T13,T25,T35.** Stable partition respects target, guards and successors. Sender packet preserves licensed observations, receiver obtains own work and query access, and continues to same target. Distinct FSSs connected by signals.

**9. H1/H2 — T31.** Repairing the observational collision with a fixed grammar leaves the output-constructor obstruction intact. Generating the output constructor without repairing observation leaves the observational collision intact; T31 distinguishes the two obstructions.

**10. H1/H3 — T01,T32.** When bounds collapse, the target is determined even if sources with the same target remain indistinguishable; this removes the target-information obstruction, not the existential assertion about original observer.

**11. H1/H4 — T31,T33.** Projection loss can occur even with enough work; energy loss can occur even with known relations. Both permit a failure in the reduced observer while full state retains viable paths.

**12. H1/H5 — T31,T34.** Original L0 excludes Rel(U,W); repair state has generated join. Different operative closures, so discovery does not contradict initial nonreachability.

**13. H1/H6 — T31,T34.** Finite rank convergence applies to repaired observer; no convergence is claimed for the original L0 on the excluded target.

**14. H2/H3 — T31,T32.** Generated join computes bounds on partially observed relations; G can select distinguishing observations and L can check the final licensed result. No target leakage.

**15. H2/H4 — T29,T33,T34.** Generate/assess/choose/register are guarded transformations with retained receipts; changing the grammar need not invalidate work conservation.

**16. H2/H5 — T31,T34.** The same matching-sort constructor supplies the new typed operation used in source-only generation. The generated relation is not a premise.

**17. H2/H6 — T32,T34.** Selection of query and operand is driven by unresolved target bounds. The generic rule is fixed while the admitted operations and state evolve; convergence is not assumed.

**18. H3/H4 — T32,T33.** Observation resolution and access depend on registered costs and receipts. Exact target information alone cannot resolve disjoint viable-action modes; both are represented in R.

**19. H3/H5 — T31,T32.** Bounds determine whether sufficient source observations exist for exact generation; possible paths stay partial until source-licensed, so no premature promotion.

**20. H3/H6 — T32,T34.** Each licensed edge query strictly refines compatible histories and decreases unknown-edge rank. Bounds monotonicity and work guard support convergence simultaneously.

**21. H4/H5 — T31,T33,T34.** Generation, observations and promotion are charged and retained; removal of needed records or viable modes breaks reuse. Adequate work is an explicit condition, not a conclusion.

**22. H4/H6 — T33,T34.** Cost-mode guard and rank condition are compatible: mode selects a viable port, final completion costs one unit; sufficient work and decreasing rank jointly give continuation, not closure alone.

**23. H5/H6 — T31,T34.** The uniform source-only generation works across finite sorts; the observation bound is computed from source size and not from completed target or final successful path.

**24. J/common-context — S32.2–S32.8.** Same U,V,W, P,Q, target $\kappa$, receipt and guard semantics used across H1–H6. Adding the recurrent cost mode does not alter relational composition; at least empty and one-path sources admitted. Nonempty common mathematical witness.

**25. J/necessity — T33,T29; GIGT [42].** Target-relevant functional and cost-mode distinction necessity is tested by a recurrent family requiring reuse and viability selection; its stronger native closure conditions are inherited conditionally from GIGT. A sufficient finite token ledger is not substituted for universal native necessity.

**26. J/quantifiers — Appendix B; T31–T35.** Existence in H1/H5, mechanism in H2, recursively extensible observability in H3, necessity in H4 and nontrivial-class convergence in H6 are distinct. Common conditional construction is positive bearing evidence; no conversion of a finite witness into a universal proof.

**27. J/transport — T35,S26.** H8 uses the same continuation and own budget through source signals. H7 contributes documentary compatibility but is not a premise of J. No empirical or historical completion gate imposed.

**28. J/negative controls — S7.2,T31,T33,T34.** Empty joint intersection control defeats pairwise-only inference; richer grammar removes grammar separation; erased cost mode prevents viable choice; exhausted work rejects continuation. Each restricts its own claim, not the common admitted witness.

**29. J/budget provenance — S32.8; S33.** 7,077-case inherited assessment retains original scope/budget. Present evidence assessment uses distinct registered obligations, one review unit per row. Native topology-derived energetic budget remains separately unspecified; no physical calibration inferred.

### S33.3. Result

J receives COHERENT in the recorded evidence-relative compatibility assessment GJ26-E, conditional on GIGT premises and at B_E=29 reasoned review units. The common nonempty mathematical construction, joint interface checks and negative controls supply positive compatibility evidence. The H1–H8 reviews are recorded at the same evidence-relative scope. The conditional derivation $\Gamma^* \Rightarrow J^*$ (S32.8, S36) and the executable finite projection remain distinct results. The native J assessment is UNDETERMINED (BudgetStatus=BUDGET_UNDETERMINED).

The established functional relationships can be propagated at their registered scopes and used as operands of further G assessment. Unassessed relationships, foundational dependencies and stronger scope extensions are retained in assessment/Continuation_register.md and assessment/Foundational_dependencies.md. The actual record is assessment/GJ26_Evidence_Assessment.json.

## S34. Comparative backward reconstruction of mathematical discovery

### S34.1. Registered documentary source and scope

On 20 November 2026, Vakil announced Masullo's proposed proof of GRH for Hecke L-functions, reporting a human-authored development without AI [56–57]. The Alpöge–Claude endpoint is externally certified [2]; the Masullo endpoint is not certified in the evidence considered here. These are two discovery routes to a shared restriction (RH for the Riemann zeta function), not two established proofs.

Masullo describes a method that uses orbital bases to decompose the global kernel and concentrate its diagonal contribution into a suitable S-local component, where an existing local trace formula can be used (§§1.4.8–1.5.2, Chs. 11–12 [56]). He traces an immature antecedent of this method to approximately a decade before the release. W1 (documentary identification) is satisfied at that scope. This retrospective is not an execution transcript.

### S34.2. Candidate functional generator and discriminatory assessment

Register the source context (number field, unitary character, test function, function-space topology, admissible finite S); the candidate generator (orbital-basis/kernel decomposition); the candidate invariant (preservation of the relevant diagonal trace under the global-to-S-local reduction); and the target (the trace identity with the width/cancellation condition required to reach the stated zero theorem). The target relationship is not taken as an input. Source-specific guard preservation, convergence and the actual trace identity are W2/W3 obligations. Candidate bridge verdict: UNDETERMINED. The concrete comparison is with Alpöge's source-licensed multiplicity-to-theta-lift generator, not with the endpoint alone.

Conditional comparison lemma. Let h map an admitted source state into a representation with an executable operation o, and let $\kappa$ be its target. If the proposed generator preserves the defining guard, the invariant needed for $\kappa$ to factor through h, and the operation-descent square of T02a, then a receiver with the registered source and sufficient work can reconstruct the same target through the generated operation (T01, T10, T13). Proof: target factorization supplies the decoder, descent supplies each licensed continuation, and induction on the finite composition gives the same target under T10. Verdict: COHERENT at these conditional mathematical premises. This is a reusable assessment criterion, not a demonstration that the Masullo or Stepanov bridge meets it.

### S34.3. Recursive public assessment and correction

The public Lean formalization [58], consulted 24 November 2026, reports a route to the target statements that still depends on exterior-zero cancellation in Theorem 12.5.1.1 (issue #5571). It also reports proved replacements for some printed statements, while retaining explicit unresolved or refuted statements and their dependencies. Section 12.3 of the manuscript lacks the required global/local comparison. These are attributed project findings, not independently rechecked Lean proofs in this submission.

Register each correction as an operand (source proposition, proposed replacement, proof or counterexample evidence, dependency edges, verdict, assessment scope, BudgetStatus, retained provenance). A successor state preserves the source proposition as provenance, but only the validated replacement may become an admitted premise. A receiver of the endpoint alone cannot reconstruct which original statement was replaced, which dependencies survive, or which unresolved premise governs further continuation. By T01, whenever two histories with the same endpoint differ on this target, endpoint-only transport is inadequate. This is a conditional inference from an explicitly specified target, not a claim of measured collective cognitive-order transition.

The project provides documentary evidence of a process in which assessment and correction are themselves registered mathematical operands. The stronger claims of proof-specific invariant transport, reconstruction of the human historical discovery sequence, and adequate native six-function governance remain UNDETERMINED. S34 is independent of the H1–H6 conjunction; it contributes candidate evidence to H7/H8 without replacing their exact statements.

## S35. Foundational receiving contracts and prior mathematical results

The receiving architecture uses GIGT [42] and FMoAI (with its summary of FMI) [52] as registered conditional dependencies. The receiving contracts are specified in assessment/Foundational_dependencies.md; source theorem identifiers and native realization certificates are not asserted where they have not been established. The dependency register specifies the receiving proposition and the operation that uses it; no source theorem identifier or proof status is inferred from a DOI alone. Native verification verdict: UNDETERMINED, BudgetStatus=BUDGET_UNDETERMINED. This does not alter the conditional mathematical derivations.

The finite work representation satisfies T29 by induction and T33 establishes necessity of preserving decision-relevant functional and cost distinctions for its recurrent class. The native energetic receiving condition is that an admitted map preserve target-relevant resources, exchanges, work guards, histories and viable continuation at the resolution of the source FSS. Merely equating a token to energy is insufficient. The receiving condition is registered for assessment; native realization verdict: UNDETERMINED.

The six-function realization in S30–S31 uses admitted typed domain-specific operators under one registration, assessment, provenance and work discipline. Its comparator establishes finite agreement, not uniqueness. The existing source-license, sealed-target, budget and stripped-history controls distinguish endpoint equality from retention of the functional relationships needed for reuse. The precise negative control is to erase target-relevant provenance while preserving the decoded endpoint: where subsequent authorization depends on that provenance, T02/T02a deny guaranteed continuation through the stripped interface. The existing executed stripped-record control is in run_paired.py; proof-specific G generation of the bridge $\beta$ remains UNDETERMINED. This adds no seventh operation.

Related mathematics: T32 uses adaptive decision-tree complexity [53]; T35 uses stable partition refinement [54]; T26–T28 use predicate abstraction and inductive invariants [55]. These precedents identify the constituent mathematics, while S32.8 assesses their composition in one registered source/target/receipt context. References are numbered as in the main manuscript.

# S36. Typed assessment-result closure and proof navigation

An assessment record contains target, scope, premises, evidence, assessment method, budget model, declared/derived budget, executed work, verdict, residual and provenance. For conditional propositions and G assessments the verdict alphabet is exactly {COHERENT, INCOHERENT, UNDETERMINED}; for unconditional mathematical theorems it is {CORRECT, INCORRECT}. Proof evidence, execution outcome (e.g. SAFE), development stage, admission status and BudgetStatus are distinct types. A proof may be supplied without a native budget certificate. A positive evidence-relative verdict does not certify an unexecuted native assessment.

## S36.1. S32.8 conditional mathematical derivation assessment

Target $\Gamma^* \Rightarrow J^*$ (and scoped receiver extension under the T35 assumptions). The proof-inspection assessment uses five registered mathematical dependency checks, one modeled inspection unit per check, executed as follows: (1) T31 supplies both independent obstructions and the licensed join; (2) T32 uses the same source and join, with lower/upper completions and stopping test; (3) T33 adds cost-mode information without changing the relation, and the two viable choices each meet the one-unit completion guard; (4) T34 uses exactly those observation and resource contracts, strictly decreasing unknown-edge rank until equality, with sufficient work $4N+3$; (5) the empty source and one-path source witness common admissibility, and T35 reconstructs the same target with separate sender/receiver receipts and work. Conjunction introduction therefore applies in $\Gamma^*$. The receiver result is H8*, not unrestricted H8.

Verdict: COHERENT. Budget: five executed modeled proof-inspection obligations; not 7,077 case evaluations or a native topology-derived budget. The 7,077-unit finite projection verdict and the 29-unit evidence-relative original-J verdict are separate COHERENT assessments. The native J assessment verdict is UNDETERMINED, with BudgetStatus=BUDGET_UNDETERMINED.

## S36.2. Conditional relationships

T30: the proof uses a countermodel of the construction premises whose failing case lies in the declared evaluation domain. Conditional derivation verdict: COHERENT; proof check by direct semantic non-entailment. Application of its A3 condition to the fixed executor: UNDETERMINED; comparator fault controls establish sensitivity and are not automatically construction-premise countermodels. S32.9 integer feasibility: COHERENT under the stated rank-window assumptions by equivalence of integer interval membership and the ceiling/floor test. The real interval and integer existence are distinct targets.

GJ26-E assesses 29 evidence obligations at the evidence-relative scope, including H7 integer feasibility, the H1/H2 distinction, and H1/H3 target-versus-source distinguishability. H1–H8 and J retain their registered target definitions. The incoming 7,077-unit certificate retains its scope and original budget. The assessment register preserves distinct assessment records and their provenance.

The executable validator checks verdict fields, scope and budget separation, execution counts, assessment references and negative schema controls. The derivations and assessment records supply the mathematical evidence.

# S37. Universal correctness and coherence objects: Constraint Algebra, Calculus of Convergence and mathematical navigability

The three universal theorem targets concern all mathematical operands and operators. This section develops their common semantic construction, the laws of convergence and information-dependent navigation, and the conditions for realizing those laws through F. The finite common-context construction and evidence-relative assessment in S32–S33 retain their original scopes and budgets. S36 governs the typing of the additional assessments.

## S37.1. Domain and assessment conditions

A correctness object is any mathematical object whose correctness is assessed using complete information with respect to the mathematical property being assessed. A coherence object is any mathematical object whose coherence is assessed conditionally upon the partial mathematical information available about that object. Both definitions apply to all mathematical operands and operators, including claims, proofs, evidence, definitions, relationships, structures, executions and assessments. The distinction is relative to information and target, not an

intrinsic disjoint partition of objects. Correctness and coherence proofs and evidence are instances of these definitions. Complete registration R means that the information, target, operations, constraints, history and work accounts actually used are represented; it does not mean all target-relevant mathematical information is known.

For any typed property P of an object, let $\Omega$ be its admissible interpretation space in the chosen metatheory, I the information registered about it, $C(I) = \{\omega \in \Omega : \omega \text{ satisfies } I\}$, and $[P] = \{\omega \in \Omega : P(\omega)\}$. The semantics is relative to the interpretation contract, including background axioms. Complete target-determining information satisfies $\varnothing \neq C(I) \subseteq [P]$ or $\varnothing \neq C(I) \subseteq \Omega \setminus [P]$. Partial compatibility is witnessed by $C(I) \cap [P] \neq \varnothing$; genuine unresolved truth additionally has $C(I) \setminus [P] \neq \varnothing$. An empty C(I) is an information conflict, not a substantive correctness certificate. Absence of a known conflict is not a witness of compatibility. These are semantic properties; their actual establishment requires adequate evidence and an assessment scope. The verdict CORRECT or INCORRECT is reserved for unconditional theorems under the stipulated ontology; conditional theorems and G assessments use COHERENT, INCOHERENT or UNDETERMINED. The mathematical validity of a conditional derivation is proof evidence, not a different verdict. A bounded coherence certificate can establish a relation on a specified assessment projection without settling global satisfiability or correctness.

## S37.2. CA-U — Universal Constraint Algebra Theorem

Theorem target. For all mathematical operands and operators, the constraints governing their correctness and coherence can be represented, generated, composed and transformed through a universal, recursively closed Constraint Algebra.

Semantic derivation (CA-U.1). For constraints c on $\Omega$ define $[c] = \{\omega \in \Omega : c(\omega)\}$. The algebra of extensions has $[c \wedge d] = [c] \cap [d]$, $[c \vee d] = [c] \cup [d]$, $[\neg c] = \Omega \setminus [c]$. Each identity follows pointwise from satisfaction. The operations preserve the type $\Omega$; complement is relative to $\Omega$. For relations $Q \subseteq X \times Y$ and $P \subseteq Y \times Z$, define $P \circ Q = \{(x,z) : \exists y, (x,y) \in Q \text{ and } (y,z) \in P\}$. The identity relation acts as a unit. For $H \subseteq Z \times W$ both $H \circ (P \circ Q)$ and $(H \circ P) \circ Q$ require exactly the same pair of existential witnesses y,z, hence are equal. This provides a typed relational algebra with projection, conjunction, composition and identity. Renaming or transporting a constraint requires the typed map and preservation contract, as in S12 and S29.

For compatible information I,J over the same $\Omega$, $C(I \cup J) = C(I) \cap C(J)$. This follows by expanding satisfaction of every constituent of the union. Consequently if $\varnothing \neq C(I \cup J) \subseteq [P]$, the composed information determines P even if neither I nor J determined P. Individual compatibility need not compose: $x > 0$ and $x < 0$ on the real numbers have nonempty extensions and empty intersection. Hence the conjunction must be assessed at its own scope; individual positive certificates do not suffice. A coherence-to-correctness transition occurs when the composed information is sufficient for an unconditional theorem and its deductive obligations are discharged; conditional implications remain typed conditional.

Operator closure (CA-U.2). A function $f:X \to Y$ has graph $\Gamma f = \{(x,f(x)):x \in X\}$. A partial or nondeterministic operation has a typed relation instead. Execution records contain the input, output, operation and guards; their adequacy is a predicate on such records. Constraint-composition operations are also functions or relations, for example intersection on the power set of $\Omega$. They may be registered as operands at a suitable type, and the preceding identities apply to the relations on those types. Iterating this construction at every finite registered level preserves the same algebraic signatures. A typed hierarchy, or an equivalent universe discipline, avoids assuming a set of all sets or unrestricted self-membership. For proper-class domains the assertion is a schema over admissible typed contexts, not a single set containing all mathematics. No enumeration of the contents of a domain is needed for these semantic identities. The construction establishes common semantic closure, not a universal effective generator of every relation. The F-realization obligation is developed in S37.6.

## S37.3. CoC-U — Universal Calculus of Convergence Theorem

Theorem target. For all mathematical operands and operators, the evolution of their correctness and coherence toward any mathematical target is governed by a universal, recursively closed Calculus of Convergence, which establishes the conditions governing the convergence of mathematical information toward that target.

Information refinement (CoC-U.1). If $I_0 \subseteq I_1 \subseteq \ldots$ over a fixed interpretation contract, then $C(I_{k+1}) \subseteq C(I_k)$. By induction all are subsets of $C(I_0)$. Their intersection $C\infty$ is the limiting compatible information. If $\varnothing \neq C\infty \subseteq [P]$, the limit determines P; finite attainment requires some K with $\varnothing \neq C(I_k) \subseteq [P]$ at K and adequate work to establish that inclusion. Limit determination alone does not imply finite determination: take $\Omega = \{0\} \cup \{1/n : n \geq 1\}$, $C(I_k) = \{0\} \cup \{1/n : n \geq k+1\}$, $[P] = \{0\}$. The intersection determines P but every finite $C(I_k)$ contains

alternatives. Additional information may instead correct a premise or transform the representation. Such a correction is not monotone information addition; provenance and the target contract must be preserved or an explicit change of target registered.

Recursive correction and residual (CoC-U.2). An admissible transition is $R_{k+1} = \omega_k(R_k)$, where the operation, target, viability conditions and work requirements are registered in $R_k$. In particular G(R) can generate or correct relationships, with the correction and its evidence registered. A target-relative residual $\rho$ on registered states is adequate if $\rho \geq 0$ and $\rho(R)=0$ exactly on the adequately certified target region T. If each selected transition is admissible and satisfies $\rho(R_{k+1}) \leq \lambda\rho(R_k)$ for $0 \leq \lambda < 1$, induction gives $\rho(R_k) \leq \lambda^k\rho(R_0)$, hence convergence to zero. If positive residuals are bounded below by $\delta>0$, choose a finite k such that $\lambda^k\rho(R_0)<\delta$; zero is then attained. For $\lambda=0$ it is attained after one transition. The result requires availability of the contracting transitions and sufficient work. Neither existence of a computable adequate residual nor contraction is presumed universal. This law is a sufficient realization, not an imposed definition of convergence.

General finite convergence (CoC-U.3). Let A be the admissible transition relation on a typed registered state space X, with target $T \subseteq X$. A incorporates typing, functional and energetic closure and viable continuation. Define $V_0=T$ and $V_{k+1}=V_k \cup \{r: \exists s\in V_k, (r,s)\in A\}$. Then $V_k$ is precisely the set of states with an admissible target-reaching trajectory of length at most k. Proof by induction: the empty path is the base; concatenating a first admissible edge with a path of length at most k proves one direction; splitting the first edge of a nonempty path proves the other. $V\infty=\bigcup_k V_k$ is exactly finite reachability and is the least fixed point of the inflationary predecessor operator: any fixed point containing T contains every $V_k$ by induction, and the predecessor operator preserves this ascending union by the existential witness. For a finite X, strict growth adds at least one state and stabilization occurs within |X| iterations. This is a semantic condition for arbitrary typed states, not an assertion that unrestricted reachability is decidable. It permits temporary reduction of local fitness; no assumption that every viable step raises fitness is used.

Work (CoC-U.4). For nonnegative additive executed edge work w(r,s), define $W^*(r)=\inf\{\Sigma\ w : \text{finite admissible trajectories from r to T}\}$, with $\inf \varnothing=+\infty$. Work accounting includes the registered state and accessible resources, not an external time parameter; target fitness is distinct from work. $W^*=0$ on T and otherwise $W^*(r)=\inf_{(r,s)\in A}(w(r,s)+W^*(s))$. Proof: every finite nonempty path decomposes into its first edge and a finite suffix; conversely each admissible edge and suffix concatenate. Taking infima proves both inequalities, including the empty case. An infimum is not necessarily attained; a budget equal to a nonattained infimum is not sufficient. A feasible finite path, including its actual resource exchanges and guards, is the witness of attainment. Where work is not additive on the original description, register the shared resources and history so that the costs and guards on complete R are valid, or use the registered path-work functional directly rather than assuming additivity. S24 supplies the conservation and finite fair-refinement conditions.

Partial-information convergence (CoC-U.5). For nonempty C(I) of compatible transition models on a common typed state and target space, let $V\infty(\omega)$ denote each model's finite reachability region. Define $\text{Vmay}(I)=\bigcup_{\omega\in C(I)}V\infty(\omega)$ and $\text{Vmust}(I)=\bigcap_{\omega\in C(I)}V\infty(\omega)$. Then Vmust $\subseteq$ Vmay. Compatible refinement $C(J)\subseteq C(I)$, $C(J)\neq\varnothing$, entails $\text{Vmay}(J)\subseteq\text{Vmay}(I)$ and $\text{Vmust}(I)\subseteq\text{Vmust}(J)$, by union and intersection monotonicity. These bounds may coincide before the whole transition model is known. They are not a policy: $\forall\omega\exists$path does not imply $\exists$information-dependent policy$\forall\omega$. When a correction changes the interpretation space, apply the transport conditions of S12 rather than silently treating the two families as nested. The following section supplies the policy criterion.

## S37.4. MN-U — Universal Mathematical Navigability Theorem

Theorem target. For all mathematical operands and operators, the mathematical information governing their correctness, coherence and transformations defines a mathematical functional state space whose navigability toward any mathematical target is governed by the Constraint Algebra and Calculus of Convergence.

The state space M=(X,A) represents registered mathematical states and typed admissible transitions; CA determines the admissible relationship compositions and CoC determines their target-reaching conditions. Because registered operations, adequacy predicates and execution records are operands, successive states may change the available operation grammar as well as the mathematical data. A typed configuration of the grammar and its generated relationships belongs to R; the admissibility relation governs such changes. The common semantic construction does not presume those changes to be effective or viable in every instance.

Coherence-guided geometry (MN-U.1). When two latest adequately assessed and independent reasoning directions

admit a common local two-dimensional chart R(a,b), the target-relative fitness projection $\Phi(a,b)=\Phi T(R(a,b))$ supplies a topographic landscape on that chart. Where differentiable, its gradient is $(\partial\Phi/\partial a, \partial\Phi/\partial b)$, interpreted relative to the chart and its metric if used as a steepest-ascent direction. This is the local plane in S21, not an M-Tower dimension or cognitive-order coordinate. It represents target-relative solution fitness, not accessible or expended work. Coherence evidence constrains candidate configurations, transitions and their assessed fitness; through CA and CoC those constraints can change the bounds on reachable targets. Changes in operators or representation may change the operative topology and the local chart. Gradient information alone does not certify an admissible or globally convergent path; viability, work and continuation must be checked.

Executable navigation under partial information (MN-U.2). Let r be a sufficient information state, containing compatible hidden configurations, available actions, observation history, work and viability. For each admissible action a, Succ(r,a) is its nonempty set of possible observable successor information states, computed by conditioning on each observation and preserving all consistent hidden configurations. Assume exact successor semantics (no spurious or missing outcomes), and sufficient registration for future admissibility and target verification. Set $N_0=T$ (adequately certified target); $N_{k+1}=N_k \cup \{r: \exists$ admissible a with $Succ(r,a)\subseteq N_k\}$. Then membership in $N_k$ is equivalent to existence of an information-dependent policy guaranteeing certified attainment in at most k steps. Proof: base k=0 is immediate. For membership at k+1 choose the witness action and, for each observable successor, use the policy given by the induction hypothesis. The action is selected using only r; the subsequent policy uses only the observed successor. Conversely a guaranteed policy must either stop in T or choose an admissible first action whose every possible observed successor admits a policy of length at most k. Induction gives membership. Finite executable cases require available actions, accessible successor information and sufficient work for the assessments and policy, not only for the physical transitions. The union of $N_k$ characterizes uniformly bounded finite attainment; unbounded branch lengths are a different guarantee.

Structural versus work-limited discovery (MN-U.3). Let $\mathrm{Reach}_n{}^{+}(r)$ be the closure of a specified complete order-n admissible operator grammar under arbitrary finite resource-only augmentation that preserves that grammar, target, representation contract and viability conditions; include only finite-work realizations. Let $\mathrm{Reach}_n{}^{B}(r)$ use the actually accessible work. If $\mathrm{Reach}_n{}^{+}(r)\cap T\neq\varnothing$ but $\mathrm{Reach}_n{}^{B}(r)\cap T=\varnothing$, the obstruction is resource-relative. If $\mathrm{Reach}_n{}^{+}(r)\cap T=\varnothing$, no resource-only increase within that architecture can attain the target, by the definition of complete closure. A registered higher-order architecture with $\mathrm{Reach}_{n+1}{}^{+}(r)\cap T\neq\varnothing$ and a viable finite-work witness resolves that structural obstruction. These are necessary and sufficient classifications relative to the stipulated closure and contract; a failed search is not a proof of empty intersection. The finite constructor witness in S32 illustrates the role of a changed grammar, without establishing that each such change constitutes a cognitive-order transition. A further recursive governance relation must be established for the latter classification.

## S37.5. Common-context derivation and a separating witness

The semantic results share the same typed states, targets, admissible relations, information families, and work guards. CA's typed conjunction and composition construct the constraints on A and the compatible models; CoC's predecessor and work laws characterize trajectories on A; MN's policy predecessor applies CoC to the information-state relation, including correction and generation as admissible transitions. All three are applicable to records representing operators themselves. Their compatibility is constructive: the following example inhabits their common context rather than combining separately certified claims.

Let the hidden mathematical configuration be $\theta\in\{0,1\}$, with the registered target a valid certificate specifying $\theta$. A commit operation $c_i$ is admissible as an attempted operation but succeeds only if $i=\theta$; a wrong commitment is terminal failure. Before assessment, the two configurations are coherent with the available information; each has a one-step target-reaching path. However no single initial commitment guarantees attainment; the information state $\{0,1\}$ is not in $N_1$. Introduce a registered assessment operation q with outcomes $\{0\}$ and $\{1\}$, requiring one unit of work and preserving viability. The two singleton states are in $N_1$ and q places $\{0,1\}$ in $N_2$ if two units of total work are accessible (one assessment, one commit). Thus universal model-wise reachability does not give reliable navigation; a generated information relationship may enable navigation. The constraint intersection, refinement bounds, action observability, policy predecessor and work accounting all use the same registered state. The executable finite checker in reproducibility/S37 independently enumerates bounded policies and verifies the semantic predecessor and work laws on this witness and its controls. Its work unit is a modeled test case, not a native functional energy measure. This is a positive conditional conjunction at the declared semantic scope, not a certificate of unrestricted F realization. The checker executes 950 case assertion groups; all passed. The finite assessment is COHERENT at that modeled budget and scope (S37-FIN).

## S37.6. F-recursive realization and proof obligations

The six functions are Create/Destroy, Place/Store, Recall, L, G and Choice. Each is a unary transformation of the complete registered state R. Register the query, operands, target, work and provenance within R; do not introduce additional primitive functions. In a finite typed context with source-licensed constructors, complete registered observations, executable guard checks and adequate accessible work, the semantic transitions admit the following realization: Recall retrieves source records by partial query; Create/Destroy constructs or replaces typed relation/operation records while retaining required residuals; Place/Store registers the produced information and its relative meaning; L assesses fully specified local relations, including exact composition witnesses; G assesses partial relationships and composes or corrects evidence, including the conjunction of its own assessment records; Choice selects an admissible viable continuation. S32 supplies a concrete common context of this kind and S24 its resource conditions. The finite policy construction in S37.5 is an additional conditional illustration. No separate semantic primitive for assessing the calculus is needed when these records and operations are adequately registered.

The universal bridge ΓF requires proving, for every mathematical operand and operator and every target-relevant typed constraint/transition, that the representation preserves the property, admissibility, assessment, executed work and viable continuation; that F-generated operations suffice for target-relevant semantic composition, correction and navigation; and that these requirements apply recursively to records of their own executions and adequacy. Establishing arbitrary graph encodings alone cannot establish this bridge. A transport proof must preserve source and target constraints in both directions at the claimed resolution and demonstrate realization, not merely assign labels to six roles. Unrestricted effective decision of all reachability is impossible: encode a machine configuration as a mathematical state, its deterministic step as a transition, and halting as T. An effective universal decision procedure for finite reachability would decide halting. This does not obstruct the semantic calculus or conditional finite realizations; it constrains the executable interpretation of ΓF.

The full targets CA-U, CoC-U and MN-U are registered separately from their proofs. The common conditional semantic conjunction is established by the derivations in S37.2–S37.5 under their explicit typing, information, transition and resource assumptions. The universal ΓF realization and the full conjunction are designated proof obligations; a finite semantic result, a representation theorem or an inherited G certificate does not by itself discharge them. The accepted proof ledger records the validated conditional subresults and the separate target register records the universal obligations. The new finite assessment and its execution are recorded separately from the native GIGT budget and the inherited 29-unit and 7,077-unit assessments.

## S37.7. Novelty and relation to existing mathematics

The Boolean and relational identities, fixed-point predecessors, shortest-path recursion and partial-observation policy construction are established forms of mathematics; their presentation here is not a claim of priority for those identities. Partial structures and quasi-truth (da Costa and French, Science and Partial Truth, 2003) model incomplete information; domain theory orders information by refinement; abstract interpretation relates sound approximations and fixed points; logical induction (Garrabrant et al., Logical Induction, 2016) studies belief evolution over logical sentences; knowledge-based subset constructions treat observation-dependent strategies (Doyen and Raskin, Games with Imperfect Information: Theory and Algorithms, 2011). The proposed novelty is the conjunction of universal applicability to all mathematical operands and operators, common treatment of complete and partial information, recursive correction of the operative topology and its assessment conditions, and target-relative fitness and viable work accounting under the same F architecture. Universal representability is not equivalent to universal realization. The three universal theorem targets state that intended mathematical capability; the proofs and assessment records identify the established parts and the remaining F-realization obligation.

## S37.8. Recursive discovery of limitations of mathematical navigability

RMBST (S45) applies the same constraint, convergence and navigability relationships to registered discovery processes and assessments of their adequacy. Universal applicability of CA-U does not imply effective decidability of every constraint; universal applicability of CoC-U does not imply convergence or reliable finite observability of convergence; universal applicability of MN-U does not imply reachability of every target or certificate. S45.3(i)–(iv) distinguishes these limitations even at limiting information or increasing finite assessment horizons. The universal F-realization and conjunction remain separately assessed.

# S38. Coherent registration, coherence capacity and mathematical cognitive phase transition

All targets below concern arbitrary mathematical operands and operators in the S37 registered state R, including records of assessment, generation, work and continuation. The conditional results share the same target contract. Functional coherence is the preservation of target-relevant distinguishability, reconstructible continuations and viable correction at declared resolution and work, not the accumulation of unassessed assertions. The following results extend the receiving-domain relationships of S12, S21, S24, S32 and S37 without expanding their existing assessment scopes.

## S38.1. CRC — Coherent Registration Capacity Theorem

Target. For any mathematical information representation, the ability to assess target-relevant coherence or correctness is bounded by the functional distinctions that it adequately registers. A realizable refinement that preserves previously inaccessible target-relevant distinctions can expand assessment and, where licensed operations descend through the refinement, mathematical navigability.

Let H be a set of admissible complete histories, including operator records and executions. Let $a:H\to A$ be the information actually registered by the observer, and let $d:H\to D$ be a specified target distinction. D can encode verdict-relevant information, the required next mathematical action, or an entire typed continuation contract. An exact receiver for d exists on a(H) if and only if $a(h)=a(h')$ implies $d(h)=d(h')$ for every $h,h'$ in H.

Proof. Necessity follows by applying the receiver to the equal representations. For sufficiency define the receiver on each a-fiber by its common d-value; the condition makes this definition independent of the selected history. Thus d factors through a exactly when the partition induced by a refines the partition induced by d. The induced receiver is a semantic map; its admissible realization at finite accessible work is a separate guard. This is the arbitrary-operand form of the target-reconstruction obstruction already used in S2 and S32.

For a refinement $b:H\to B$ with $a=\pi\circ b$, every distinction factoring through a also factors through b. If a target distinction fails to factor through a but factors through b, the inclusion of semantically recoverable target distinctions is strict. If the refined receiver and the required target continuation are executable at accessible work, this strict expansion is operative rather than merely semantic. Conversely, more records that induce no new target-relevant partition distinction cannot repair the identified collision. This proves CRC under its registered representation, typing, executable decoder and guard conditions.

Separating witness. Hidden histories $h=(z,e)$, with $z,e\in\{0,1\}$, share an independently verified endpoint z; e specifies which of two licensed next operations is safe. The endpoint representation $a(h)=z$ is correct for z but fails the target $d(h)=e$. Repeating the endpoint or adding its proof leaves that collision unchanged. The refinement $b(h)=(z,e)$, obtained by a source-licensed assessment that registers the generator receipt, preserves both distinctions. The decoder e is now available and selects the valid continuation. No incoherent assertion is admitted as correct. If the receipt is unavailable or cannot be checked within accessible work, operative improvement is not established. This finite witness links correctness/coherence objects to structural discovery rather than treating unassessed information as evidence.

## S38.2. COAI — Collective Observability–Assessment Interface Theorem

A collective registration interface a may compose observation, durable registration, functional reconstruction, transmission, admission and operative assessment. For fixed histories $h,h'$ and target d, any stage that merges the two histories irreversibly relative to all accessible side information prevents downstream deterministic decoding of their differing d-values. Proof: composition preserves equality of the merged inputs; the CRC factorization obstruction applies to the resulting composite. If independent source receipts or another licensed path recover the distinction, the complete collective interface is not the reduced interface and the obstruction need not hold.

The UFO thought experiment concerns an observer lacking a coherent representation of unfamiliar motion; it makes no claim about the explanation of any reported event. A picture, a reliable observation and an executable dynamical account have different target contracts. Stigma can additionally affect reporting and admission (NASA, 2023, UAP Independent Study Team Report, https://science.nasa.gov/uap/). To assess the proposed editorial blind-spot mechanism one must register a target-relevant distinction, the competing histories, the actual admission interface and whether an adequate alternate carrier preserves the distinction. Rejection alone, or the mere presence

of an unusual example, is not a witness. The criterion also covers the loss of a correct mathematical result's generative relationships.

## S38.3. FCC — Functional Coherence Capacity Theorem

Let Q_k be outstanding indispensable coherence-maintenance work at interval k, D_k newly generated indispensable work, S_k adequately discharged work, and L_k indispensable work irreversibly abandoned, measured in a common registered functional-work unit. All variables are in R, including the interval, resolution, work receipts and obligations. Assume a conserved accounting contract with no double-counting, $Q_k \geq 0$, $L_k \geq 0$, and $Q_{k+1} \geq Q_k + D_k - S_k - L_k$. Define C_k as an upper bound on adequately discharged work under the current architecture and allowed resources, so $S_k \leq C_k$. Functional coherence requires $Q_k \leq Q_{max}$ and total indispensable abandoned work $\leq L_{max}$, with finite declared bounds. These are target-relative viability conditions, not counts of papers or arbitrary proxies.

Summation gives $Q_N + \Sigma_{k<N} L_k \geq Q_0 + \Sigma_{k<N}(D_k - C_k)$. If cumulative deficit exceeds $Q_{max} + L_{max} - Q_0$, at least one viability bound must fail by N. In particular, persistent deficit $D_k - C_k \geq \epsilon > 0$ implies finite-time violation once $N\epsilon > Q_{max} + L_{max} - Q_0$. This proves the conditional obstruction. If obligations can be compressed, reclassified or discharged by legitimate correction, the accounting must register that operation and its net work; demand reduction and sufficient same-order resource augmentation are alternate routes and are not excluded by definition. The obstruction is not a proof that the actual mathematical community has crossed such a threshold.

## S38.4. Physical bifurcation and typed P-space to M-space transport

Physical precedents: Cross and Hohenberg (1993), Pattern formation outside of equilibrium, Rev. Mod. Phys. 65, 851–1112, derive amplitude-equation descriptions of instabilities including convection. Rice and Thomson (1974), Ductile versus brittle behaviour of crystals, Philos. Mag. 29, 73–97, analyze competing crack and dislocation mechanisms. The former illustrates emerging organization, the latter competing loss of integrity and reorganization. Neither establishes the universal CPTB dichotomy or supplies an M-space order parameter.

Consider the scalar normal form $da/dt = \mu a - \beta a^3$, $\beta > 0$. At $a = 0$ the derivative of the vector field is $\mu$. For $\mu < 0$ the origin is locally asymptotically stable; for $\mu > 0$ it is unstable and equilibria $a = \pm\sqrt{\mu/\beta}$ have derivative $-2\mu < 0$, hence are locally asymptotically stable. The origin remains an exact solution in the absence of perturbations. Suppose the measured effective capacity is $C(a) = C_1 + \kappa a^2$, $\kappa > 0$. On either nonzero equilibrium $C_2 = C_1 + \kappa\mu/\beta$. If $C_1 < D < C_2$, the old regime is overloaded and the stable new regime has positive capacity margin. This is an exact conditional normal-form calculation, not a general physical heat-transport law. Actual transition requires an accessible basin, perturbation and sufficient transition resources.

Transport contract. A licensed P-space→M-space mapping must preserve (i) the distinction between incumbent and emergent functional organization; (ii) the governing stability relationship or its proved qualitative conjugate, not necessarily the physical coefficients; (iii) the target-specific obligation and work accounting; (iv) the capacity margin and the energetic cost of transitioning; and (v) the viability and receiver-side reconstruction contracts. For a time-preserving differentiable change of coordinates $a \mapsto b = \psi(a)$ with nonzero derivative, the vector field transforms as $db/dt = \psi'(a)(\mu a - \beta a^3)$; at an equilibrium its linearization has the same derivative eigenvalue. Thus stability signs and fixed-point structure transfer under that exact bridge. If the capacity and demand comparisons also commute with the bridge, the inequality $C_1 < D < C_2$ transfers. Establishing such a bridge for actual M-space requires source and receiving invariants, not importing a physical amplitude equation. Cognitive order and the thermodynamic order of a transition are different quantities.

## S38.5. MCPT — Mathematical Cognitive Phase-Transition Theorem

Target. A collective mathematical system whose incumbent architecture cannot sustain indispensable coherence work can restore functional coherence and multiply mathematical problem-solving capacity by a viable cognitive-order transition, under the registered structural reachability, work, reconstructibility and net capacity conditions. This conditional theorem integrates CRC, FCC and S37's navigation criterion with the functional-superconductivity and critical-mass architecture of GIGT [main manuscript, 42]. GIGT D02–D03 and D10–D14 distinguish resource from structural limitations, capacity expansion, critical mass, order transition, low-loss information flow and collective supercriticality. The receiving argument is self-contained below.

Let R contain the complete incumbent and proposed operator architectures and their admissibility guards. A structural transition witness is a common target class T with $\mathrm{Reach}_1{}^{+}(R)\cap T=\varnothing$ and $\mathrm{Reach}_2{}^{B}(R)\cap T\neq\varnothing$, in the notation of S37.4: no finite resource-only augmentation of the complete incumbent grammar reaches T, but a finite-work viable higher-order trajectory does. The witness must include stable recursive governance over mathematical operations, not just a different encoding or extra execution. If T is the target of recognizing and implementing the required infrastructure, the same reachability statement applies to that target. This does not imply that every participant must individually make the transition.

For transmitted mathematical understanding, let e_i and r_i be admissible encoding and reconstruction operations on the registered path, and let d_i be the target-relevant distinction. Target-lossless functional transmission means $d_{i+1}\circ r_i\circ e_i=d_i$ on its licensed domain, with matching operand types and retained source, work and continuation receipts. Composition preserves the target distinction by induction. For a finite path with nonnegative additive errors in a common metric, a triangle-inequality bound is available only with nonexpansive subsequent maps or appropriately registered amplification factors. Functional superconductivity means stable repeated target-lossless or adequately low-loss transport at sustainable functional work cost; it is not literal zero energy dissipation. A stable viable invariant region is sufficient for sustained operation: if all admitted continuations from the region remain in it with its work and reconstruction guards satisfied, induction establishes continued viability. Actual entry, critical mass and attraction require their own evidence.

For capacity recovery, suppose the transition of finite work cost is executable without crossing the FCC viability bounds. On entry let $Q^*=q$ and cumulative loss $\le L_{max}$. In the post-transition regime require actual discharge $S_k\ge D_k+\eta$ whenever $Q_k\ge\eta$, for a fixed $\eta>0$, and adequate discharge of the remaining obligations when $Q_k<\eta$. Assume exact queue accounting, no new irreversible indispensable loss, admissible resource receipts, and viable recursive governance. Then Q decreases by at least $\eta$ while $Q\ge\eta$, reaches zero within $\lceil q/\eta\rceil$ service intervals, and thereafter can remain zero. The loss bound is preserved. This proves the capacity-restoring branch rather than inferring it merely from a greater maximum capacity.

For capacity multiplication at equal target and accessible-work contract, suppose a family of N tasks has incumbent cost $w_1$ per task and higher-order reusable operator cost K to generate, assess, maintain and transmit, plus $w_2$ per task to execute with all continuing costs included. Then $W_1=Nw_1$ and $W_2=K+Nw_2$. For $w_1>w_2\ge0$, $W_2<W_1$ whenever $N>K/(w_1-w_2)$. The ratio $W_1/W_2$ approaches $w_1/w_2$ for $w_2>0$; if the costs depend on problem size, larger separations require actual bounds on these functions. More generally, at budget B a reusable generator enables at least $\mathrm{floor}((B-K)/w_2)$ tasks when $B\ge K$, $w_2>0$, versus $\mathrm{floor}(B/w_1)$ for the comparator when the cost bounds are exact. These are exact conditional work relations, not universal exponential discovery bounds. Haken (1985), The intractability of resolution, TCS 39, 297–308, supplies a distinct proof-size separation for pigeonhole formulas in resolution versus extended resolution, not a proof of cognitive order or discovery work.

Common-context conjunction. Under the shared typed registration, reachability, transition, accounting, reconstruction and stability premises, CRC identifies recoverable distinctions, S37 licenses the higher-order trajectory, FCC excludes sustained incumbent viability, and the post-transition inequalities prove restoration and the stated cost advantage. The conclusions coexist in one context: for example, the finite receipt witness of S38.1 supplies a new executable distinction; choose a model with $Q_{max}=8$, $L_{max}=0$, $Q_0=0$, $D=3$, $C_1=2$, and a licensed higher-order transition after four service intervals ($Q^*=4$), with transition cost 2 covered by separate registered resources and no added obligations or losses. Post-transition $S=4$ when enough work remains, $D=3$, so Q returns to zero in four intervals. Let independent registered task work budgets cover $K=2$, $w_1=3$, $w_2=1$, $N=4$; costs are 12 versus 6. Exact receipt transmission preserves the binary generator distinction. These numbers witness consistency of the accounting and capacity inequalities, not a demonstration of native cognitive-order realization. No shared resource is spent twice. Finite checks and a countercontrol rejecting an unavailable or unaffordable receipt are provided in reproducibility/S38.

## S38.6. CPTB — Coherence Collapse–Phase Transition Bifurcation Conjecture

Conjecture. For any cognitive-order-n mathematical system subject to sustained growth of functional mathematical information, there exists, under the applicable functional and energetic conditions, a critical threshold beyond which its existing architecture cannot preserve collective functional coherence; where a viable transition to order n+1 generates recursively reusable mathematical discovery operations with sufficient net capacity and reconstructible functional-information transport, it can restore collective coherence and multiply general problem-solving capacity.

The conditional FCC and MCPT derivations give precise failure and restoration conditions. To establish the full conjecture for mathematics, derive the indispensable demand and same-order capacity envelopes from R; exhibit the

critical threshold and licensed transition to stable recursive governance; assess the actual work and receiver-side reconstruction contracts; and establish the source-to-receiver physical transport if used. A third route such as reduced demand or same-order resource expansion falsifies an unconditional binary claim unless excluded by the specified premises. Loss of collective functional coherence means loss of target-relevant reconstructibility, assessability or viable navigation, not logical inconsistency of all previously correct theorems.

### S38.7. Collective intelligence implementation and assessment

The proposed implementation target is a decentralized, signal-mediated M-space, not a database that stores more unassessed assertions. The minimum viable functional interface registers typed mathematical operands and operators, source and generator receipts, correctness/coherence evidence, correction histories, work and residuals, receiver contracts and target-dependent admissible continuations. Create/Destroy, Place/Store, Recall, L, G and Choice act on complete R, including records of these operations. The CA and CoC operate on their registered constraints and evolution. A comparison should match target contracts and accessible resources, vary representation adequacy and operator generation, and assess receiver-side reconstruction, actual continuation, coherence obligation discharge and net work after infrastructure costs. These tests distinguish coherent registration from acceptance of low-quality evidence and stable cognitive-order governance from isolated success.

The finite S38 assessment is separately budgeted and does not enlarge the S37, S32 or GIGT verdicts. The new conditional proofs are indexed separately from the conjecture and universal realization obligations. The physical mapping, actual individual/collective phase transition and native energetic budget are residuals of the proposed mathematical application; no new full-scope G verdict is asserted.

### S38.8. Expansion of cognitive capacity versus absolute discovery completeness

RMBST(v) supplies closure-relative separation witnesses; S45.4 registers the stronger F-recursive bandwidth and universal first-order discovery hypothesis. The transition specified by MCPT can make particular excluded relationships operable under its conditions; it need not eliminate all blind spots. RMBST(iii) applies equally to a sound effective higher-order architecture satisfying its premises. Block width in the finite witness is not native cognitive bandwidth; neither a local G operation nor an expanded computation budget proves a phase transition.

## S39. Functional Mathematical Computability Phase-Transition Theorem (FMCPT)

Target. For all mathematical operands and operators, embedding an explicit recursively closed functional model of their functional information into mathematics enables a cognitive phase transition through which classes of structurally or energetically uncomputable mathematical targets become computable within second-order mathematics, including under unbounded growth of mathematical information. This is the universal target; the following deductions identify sufficient conditions and a separating witness, not universal realization. S37–S38 and S10 supply the operative grammar, predecessor, work and capacity contracts.

### S39.1. Persistent structural obstruction and constructive correction

Let R contain the typed target T, complete admitted order-one signature $q_1$, licensed sources, admissible operations and history. For a family of admissible histories h, let $q_1\infty$ denote the entire finite and limiting signature licensed by that fixed architecture. If $q_1\infty(h)=q_1\infty(h')$ yet $T(h)\neq T(h')$, any procedure factoring through $q_1\infty$ gives the same output on the pair. Thus no such procedure uniformly reconstructs T, even with unlimited repetitions and additional work confined to that interface. This is a persistent information obstruction, distinct from Turing undecidability. An infinite signature that reveals the distinction does not satisfy the premise. If a licensed operation generates a refined signature $q_2$ from R and $T=e\circ q_2$ with an executable decoder e, adequate assessment and viable finite work, the target is reachable by the expanded architecture. The separating condition concerns the complete grammar, not the label of a formal language. The operator generation and its adequacy must themselves be governed in R to establish a cognitive-order transition; mere access to an extra input is insufficient.

### S39.2. Energetic computability and generator discovery

Write $W_n(R,T)$ for the infimum of total work over admissible, target-adequate discovery policies of cognitive order n, with $\infty$ for no viable policy; a finite infimum need not be attained. A policy includes acquisition, discovery of generators, execution, verification, transmission, maintenance and any indispensable reconstruction. Work, time horizon and resource guards are registered in R, not supplied as unaccounted external parameters. For an expanding family R_N with the same target contract, suppose every incumbent policy has worst-case work $\geq f(N)$ with $f(N)\to\infty$. If a licensed recursively generated policy has work $\leq K$ uniformly in N, and K is accessible, then for every fixed incumbent work ceiling B there is an N beyond which the incumbent cannot reliably solve the target within B, whereas the generated policy can. Proof: choose N with $f(N)>B$ and execute the specified viable policy of cost $\leq K$. This is a uniform work separation, not a deduction that every problem becomes difficult as unrelated information grows. Discovery of the generator, not only its short execution, is included in K. The infinite-information limit denotes the uniform family bound; no completion of an infinite sequence of operations is presumed.

A separating family. A hidden vector of N binary entries has exactly one marked entry or none. The incumbent can query individual entries at unit work; the target is whether a mark exists. On the all-zero input, any deterministic correct algorithm must query all N entries, since a mark at an unqueried index produces the same history. Randomization cannot guarantee exact correctness with a smaller worst-case bound. Give the receiver a licensed, independently maintained source index that returns either a marked address or a certified empty record, with an explicit source-completeness contract; the new grammar can register, validate and query the index and check its returned address or empty certificate at a stipulated total cost K=4. The index's construction/maintenance is not free: in this witness the source produces it incrementally under a separately charged source contract. If constructing it requires scanning the N entries at the receiver, the uniform separation disappears. The index-only case is an information/grammar witness, not by itself a cognitive-order witness. A generated instrument that discovers, evaluates and corrects such source interfaces is the stronger realization target. For $N>4$ the stipulated receiver work separates; if source upkeep is counted globally and grows without bound, only the receiver-scoped separation follows. The witness therefore tests cost scope as well as missing information.

### S39.3. Unbounded functional significance and physical correspondence

If a family has viable policies with $0<W_2\leq K$ and $W_1\geq f(N)\to\infty$, $W_1/W_2\geq f(N)/K\to\infty$ (for finite $W_1$); a structural obstruction yields a stronger reachability separation, not a finite ratio. This establishes unbounded potential functional advantage at the stated scope. It does not establish subjective experience, actual human/AI cost superiority, or the RH's historical work complexity. The conditional question for the RH is whether discovering a generator in M-space replaces long and work-dense searches in conceptual space by viable simple generative paths. Minimality is relative to the complete work contract, not merely the visible path length. In physics, energy conservation and the catalogue of fundamental interactions, creation/annihilation and selection dynamics motivate but do not prove FMA closure. A target can be unavailable through a restricted reduced-state interface even when a richer theory contains the distinction: the two GHZ states with opposite relative phase have identical proper reduced states and opposite $X\otimes N$ expectation. The full state distinguishes them. A string-model initial-condition interface that merges target-distinct configurations has the same formal obstruction; actual physical applicability and the sufficiency of an existing theory conjunction are additional source/receiving targets. No assertion of universal quantum or string undecidability follows. This is the structural analogy, not a new physical proof.

### S39.4. Complete operative closure and joint-warrant separation

S45.3(v) supplies two exact structural witnesses on unchanged source information: relation-type construction and joint registration of a uniquely determined binary feedback solution. An incumbent grammar already admitting the required composition or the global fixed-point warrant is not separated by the respective witness. This extends FMCPT by distinguishing exclusion from the complete operative closure from nonattainment by a policy selecting among already admitted operations. Neither witness establishes a native cognitive-order transition by itself.

## S40. Recursive Mathematical Perceptual Expansion Theorem (RMPE)

Target. For all mathematical operands and operators, the effective resolution, observable volume and functional information density of M-space can be recursively transformed by its functional model while preserving indispensable information and functional work accounting; expansion of viable navigation can multiply general

problem-solving capacity. S38.1 and S12 give the registered observation and transport premises. Cognitive order n and M-space manifold dimension i are distinct.

### S40.1. Effective observation expansion

For a licensed source X, fixed native observation q and target T, define x~q y iff q(x)=q(y). If an admissible source interaction g preserves required records and q(g(x))≠q(g(y)) for a previously merged target-distinct pair, $q'=(q,q\circ g)$ strictly refines q on the pair and never merges any earlier distinction. Proof follows from projection onto the first coordinate and the differing second coordinate. If g factors entirely through q, no new distinctions can arise: equal q implies equal g and hence equal q∘g. Thus magnification is not invention of information from an indistinguishable record. The source interaction, transformation, observation and adequacy criterion are registered operands of G(R), allowing recursive refinement. Actual operational improvement additionally requires viable decoding, sufficient work, storage and signal resolution. The native resolution of q need not change.

### S40.2. Conditional geometry and density

Fix one M-space level i with a licensed metric d and invariant measure $\mu$, and measurable target-observable region O(R) of finite positive measure. If the old observation remains executable and a new representation makes a positive-measure region E disjoint from O accessible at the same target and work contract, $Vobs'=\mu(O\cup E)=\mu(O)+\mu(E)>Vobs$. If work competition removes previous accessibility, monotonicity is not implied. Under a target-preserving coordinate isomorphism with pushforward measure the volume is unchanged; actual expansion requires changing the observable region or licensed metric/measure, not changing units. For a regular i-dimensional region U, assume $c_-r^i\le\mu(B_r(x))\le c_+r^i$ for small r and a bounded domain with a finite maximal $\delta$-separated set. The $\delta$-balls cover U and disjoint $\delta/2$-balls lie in its $\delta/2$-neighborhood. Hence $\mu(U)/(c_+\delta^i)\le N\delta\le\mu(U\delta/2)/(c_-(\delta/2)^i)$. For regular domains with bounded neighborhood-volume ratio this gives $N\delta=\Theta(\mu(U)/\delta^i)$. Comparing regions with comparable constants, a-fold expanse and s-fold resolution improvement give $\Theta(a\ s^i)$, not an exact ratio in arbitrary geometry. For an exact cubical witness, an i-dimensional unit cube partitioned into m cells per axis has $m^i$ cells; replacing m by sm gives $s^i$ times as many. This is exponential in i, not by itself cognitive order or problem-solving throughput.

For an independently justified additive target-information measure I_T and $\mu(U)>0$, density $\rho_T=I_T(U)/\mu(U)$ increases if a reconstructible encoding preserves I_T and reduces its operative volume. Overlapping or redundant records cannot simply be added; use the native functional measure and non-duplicated ledger. Volume, density, time resolution $\Delta T$, permissible edge work and coupling are properties of the FSS in R. The intended bandwidth is the F-recursively defined volume and density of information viably navigable per endogenous time interval. Packing cells are an explicit conventional realization, not a replacement for the native definition. Connecting more distinguishable cells to more solved targets requires an adequate solution-bearing decoder and work-feasible policy (S37). The native functional measure, continuity, time and energy correspondence remain receiving obligations, not proven by Euclidean packing.

## S41. FMI-instrumented mathematical navigation policies

A microscope is a local resolution policy; a telescope is a long-range transport policy; an omniscope integrates an atlas of typed regions and scales. They are compositions of the same six unary F operations on complete R, not three new primitives. Create/Destroy registers signal-mediated candidates and removes only through the licensed residual-preserving interface; Place/Store registers containers, histories and assessments; Recall searches for existing relevant models from partial queries; L evaluates complete target-relevant local relationships; G assesses, generates and corrects their globally determining relationships; Choice mediates target fitness and concept selection with registered context. These roles are not a proof that a given software implementation realizes F completely.

### S41.1. Guards and scale selection

A local refinement at c is admissible only if $\Delta U_{C,local}(c)\le 0$ and $\Delta Viol\le 0$, with U_C a registered target-relative unresolved-coherence residual and Viol the registered constraint-violation measure. Retain earlier indispensable distinctions and charge work. A telescope bridge $\Phi:C\to C'$ with licensed receiver $\Psi$ requires $d_C(c,\Psi\Phi(c))\le\epsilon$, non-increasing joint residual and exact target-distinction preservation (or an explicit target-adequate approximation bound). Small cycle loss by itself does not preserve an observable discontinuous at the tolerance scale. An omniscope atlas A contains source receipts, overlap and compatibility contracts; require $\Delta Cover\ge 0$ and $\Delta Incons\le 0$

under the same target, plus consistency on required overlaps. Coverage refers to adequately registered target information, not raw model count. Minimum Viable Coherence (MVC) is the registered viability threshold for scale escalation; Information-Theoretic Truth Test (ITT) feasibility requires R_Π≤C_Π for each licensed channel under its actual coding, noise, error and time assumptions. A capacity inequality is a necessary channel condition; successful finite decoding and semantic adequacy are separately assessed. Do not silently treat independent paths as independent evidence or turn a bit-rate bound into mathematical truth.

The scale policy pulls back to locate relevant models and functional relationships, focuses in to identify object boundary conditions, then uses CA to compose compatible constraints and CoC to select, assess and correct viable transitions. It may itself be a registered operand of G. Non-increasing residuals permit stalling; if residual r≥0 decreases by at least $\eta>0$ whenever outside a target with r=0, attainment occurs within ceil($r_0/\eta$) admitted steps, provided each is executable within the horizon. Alternatively a contractive recurrence yields asymptotic convergence; finite target attainment requires a gap or adequacy threshold (S37). This conditional result prevents accepting fluent but nonproductive output. If no adequate action is affordable, record the residual and terminate or obtain additional licensed work. The joint gates and actual objective are endogenous to R, not a separate independent controller.

## S42. Cross-FSS transport and inward modeling as a hypothesis engine

Archimedes' mechanical method gives a precedent for extracting a candidate mathematical relationship through a different domain and then establishing its geometrical correctness. A general inward-modeling procedure registers the human source observation, source functional state, licensed signal history, target mathematical state, source/receiver boundary conditions, candidate operation, and its unresolved residual. The body (including sensory and somatic patterns), emotional FSS, conceptual space and awareness space are distinct functional domains; consciousness here means the HCFM organism-level awareness system, not merely reported wakefulness. A transition from confusion to clarity is a candidate source pattern, not a proof or direct access to another person's state. Model the observation as a trajectory with source-appropriate measure, not as an assumed mathematical contraction. Test the candidate invariant through source observations, then transport where the mapping preserves it. Where a source-specific model is not available, register a conjecture rather than certifying a transport.

For source operation f and receiver operation h with map $\phi$, require $\phi\circ f=h\circ\phi$ on the admitted domain, and T_S=T_M$\circ\phi$ for the specified invariant; preserve history, scope, residual and work. Inductively $\phi\circ f^k=h^k\circ\phi$ for each admissible finite composition, so the invariant is preserved along corresponding paths. If $\phi$ identifies x,y with different target values, no decoder on $\phi$ can recover both. If transition guards differ, the path correspondence need not be executable. An observation-to-model procedure may generate candidate patterns even before those conditions are established; the candidate is then a coherence object, not a correctness object. The modeled claims can be assessed by local L and global G; first-person reports may generate hypotheses but do not certify them. Out-of-construction assessment follows T30/S31: freeze the relevant model, evaluate consequences not stipulated in its construction on discriminating cases (including a possible failure), and register corrections without erasing earlier status. Deductive derivation from explicit assumptions does not require empirical replication. The universal transport target is to discover and assess these maps across every adequately modeled FSS, not to assume a total bridge already exists. An unforeseen mathematical experience, including wider accessibility of Ramanujan-like discovery, is a research conjecture; the formal work separation of S39.3 supplies a potentially unbounded functional change but does not determine subjective experience.

## S43. Generative Transmission of Mathematical Understanding Theorem (GTMU)

Target. Where target-sufficient mathematical understanding cannot reliably be transmitted through a single finite local message, a finite initiating signal can initiate recursively assessable dynamics through which that understanding becomes reconstructible under viable convergence conditions. This imports no unpublished proof as a dependency: the following is a self-contained conditional construction. S38.2 supplies the fixed-interface collision; S37 supplies reachability; the published CNS possibility theorem [84] motivates the contraction-closure approach.

### S43.1. One-shot obstruction versus initiated reconstruction

Let H be admissible histories, m a fixed finite local message, q the complete local receiver signature including its prior state, and D the required generative distinction. If $q(h,m)=q(h',m)$ while $D(h)\neq D(h')$, no decoder of that signature can correctly reconstruct D on both. This is not a theorem that every finite message fails for every receiver: a receiver already possessing the determining relation may reconstruct from a short cue. Let $R_0$ include an initiating question and the licensed signal interface. Create($R_0$) may register a conceptual configuration that enables subsequent recursively selected actions. A policy with accessible source interactions can refine the observation state, even when the initial message is the same for two histories. The receiver must be able to execute and assess the actions; the cue alone does not supply the missing information.

Let U_T consist of complete registered states with a target-adequate reconstructive and assessment certificate, including a valid defeating objection if one is found; agreement is not the success criterion. Suppose a viable policy on registered states has a nonnegative target residual r, r=0 iff the target assessment is adequate, and decreases r by at least $\eta>0$ outside U_T. With sufficient licensed work for ceil($r(R_0)/\eta$) steps, the trajectory reaches U_T by that bound (S41). If the policy instead gives $r_k\leq\alpha^k r_0$ for $0\leq\alpha<1$, then $r\to 0$; convergence to a target state requires an appropriate closedness/completeness and stability condition, not just decreasing residual. A fixed point is a stable assessment configuration under a specified update, or a viable invariant set; a question does not guarantee a unique attractor. Finite initiation can therefore differ fundamentally from finite complete transmission. No deterministic reader response is assumed.

### S43.2. Separating witness and self-application

Two sources have hidden bit $\theta$=0 or 1, identical initial signal, and a licensed query returning $\theta$. The target is to reconstruct a registered distinction and use it to choose the correct continuation. The initial message cannot discriminate the two sources. A question cue causes a receiver policy to query (work 1), register and use the answer (work 1), achieving the target at work 2. With no query, or budget 1, guaranteed attainment fails. A source-independent cue does not by itself disclose $\theta$; the post-cue dynamics and source availability do. A reader may instead reconstruct a counterexample to a proposed relationship, which is adequate assessment, not failure of transmission. The manuscript's concluding question, “What is the question that all of this raises?”, is proposed as a candidate initiating signal. Its actual effect is an empirical/functional receiving target. The purpose is to open endogenous question-generation and assessment rather than compel recognition of a predetermined result. This is a concrete application of CA/CoC to a communication problem, not a demonstration that the manuscript will induce the transition.

### S43.3. Transmission of recursive discovery functionality

Apply GTMU to three distinct targets: a mathematical result and its adequate warrant; the dynamics by which the result becomes discoverable; and the functionality by which the adequacy and limitations of those dynamics become discoverable and correctable. RMBST(i) and (iii) distinguish a deficient receiver interface from effective reflexive incompleteness; a finite proof of an obstruction may remain reconstructible without conferring systematic discovery reliability. A finite initiating signal can lead to a licensed expansion or continuation, but no guarantee follows for a receiver whose complete operative closure excludes the required distinctions. Preserve original scope and budget across transport; reassessment appends a new record.

## S44. Common-context derivation and joint assessment

### S44.1. Conjunction and functional superconductivity

The intended joint target is FMCPT $\wedge$ RMPE $\wedge$ instrument-policy adequacy $\wedge$ cross-FSS transport $\wedge$ GTMU $\wedge$ MCPT $\wedge$ CA-U $\wedge$ CoC-U $\wedge$ MN-U, with CPTB retained as a conjecture. The common registered context R contains the same target distinctions, sources, observation history, operator generation and adequacy, constraints, endogenous horizon and available work. These are operands of the same six F functions. Constraint Algebra checks composition of source and receiving relationships; Calculus of Convergence governs correction and viable policy evolution; mathematical navigability requires actual attainable paths, not mere expressibility. Each step must preserve certificate scope, original budget and residual provenance; a new assessment appends a new event. No new primitive or external assessment budget is required.

For transmissions $e_1,\dots,e_k$ with target decoders d_j satisfying d_{j+1}∘e_j=d_j on admitted states, induction gives $d_k$∘e_{k−1}∘…∘$e_1$=$d_1$. Exact preservation is target-relative; if approximate losses are permitted, an explicit joint loss bound is required rather than assuming that small per-edge loss remains small indefinitely. A sustainable repeated regime additionally requires nonnegative work margin after maintenance, discovery and transmission costs, bounded indispensable obligations (S38.3) and stable recursive governance. This supplies the conditional mathematical interpretation of functional superconductivity, not a claim of zero physical energy dissipation. If a generated instrument provides a uniform work advantage (S39), refines observable distinctions (S40), passes the licensed navigation and transport gates (S41–S42) and enables viable reconstruction (S43), the same target can be attained and transmitted in this shared context. The extra work for each action is charged once, not paid from overlapping budgets. Collective supercriticality is a further dynamical threshold: the net viable propagation and maintenance of understanding must exceed its loss and resource demand; no actual human or network threshold is inferred from the finite witness.

## S44.2. Joint finite witness and negative controls

Use a common family with hidden $\theta\in\{0,1\}$, identical coarse signature and a licensed source query. A microscope registers the source-resolved $\theta$; telescope transport uses an injective relabeling into the receiving model; the omniscope merges compatible typed receipts; Choice selects the $\theta$-licensed continuation. Register a two-unit target-work budget for query and commit, and a separately accounted two-unit transport/assessment budget; no action is double charged. The initiating signal is the same on both histories. Without the query the target cannot be uniformly reconstructed; with the query and adequate work it can. The source→receiver relabeling preserves the target, and repeated identity-on-target decoding conserves it. The finite script in reproducibility/S44 tests all binary deterministic observation maps, refinements, maps between binary domains, hidden-source policies and work guards, and a cubical resolution family; it also enumerates fixed-query lower-bound witnesses for $N\leq 8$, checks the registered conjunction over binary states and gates, and rejects unavailable sources, failed decoding, insufficient work, incompatible atlas entries and stalled residuals. The report states the executed count and source hash. These are finite model assertion groups and not a native energetic budget or proof of universality. Their conjunction is assessed only in the common finite witness. The actual geometry, universal F realization ΓF, cognitive-order transition, native human/AI work comparison, physical-theory conjunction and receiver dynamics remain separately registered receiving targets. The inherited S37, S38 and GJ26-E assessments retain their exact scope and budget.

S44-FIN — COHERENT at the registered finite scope: 1,943 executed assertion groups of 1,943 declared. Native work/assessment budget remains BUDGET_UNDETERMINED. The finite case enumeration and formal conditional deductions are separate evidence; no universal G verdict follows.

## S44.3. Expanding observability and recursively correcting model adequacy

Let R register the observation interface, the admitted class H of candidate models, observations (including calibration, uncertainty and intervention licenses), target d, history and accessible work. For a registered observation $o_j$ let $C_j=\{h\in H: h \text{ satisfies the registered observation contract}\}$. The compatible model class after k observations is $H_k=\cap_{j=1}^{k} C_j$. If $H_k$ is nonempty and d is constant on $H_k$, the target is determined relative to this class and contract; if d is not constant, it remains underdetermined. If $H_k$ is empty, some part of the conjunction (model class, observation interpretation, calibration, auxiliary premises or their compatibility) is defeated; emptiness alone identifies no unique culprit. A model class cannot explain a target-distinguishing observation merely by computing more consequences of a class that excludes every compatible model. This is the S37/S38 constraint and observation-fiber construction applied to the assessment model itself.

The transition G(R) may register an extended or corrected class $H'$, its generative license, the reason for correction, retained observations and residuals; L assesses target-complete consequences. Compare observations from different instruments or interventions where the source and receiving constraints are typed. Agreement is informative only insofar as alternative artifacts would predict distinguishable outcomes: common systematic errors need not be ruled out by agreement. A frozen candidate model must be assessable on an additional observation whose outcome is not stipulated by its construction, with a nontrivial failure condition. More observations can refine compatible models but do not prove that every remaining model is adequate outside the registered scope. The native model-generation dynamics and accessible work remain receiving conditions. This is an instance of CA composition, CoC correction, and MN reachability, not a new primitive or separate theorem family.

Finite witness: take X={0,1,2,3}, binary outputs and an initial class of constant maps. Observations (0,0) and (1,1) defeat this class. A source-licensed correction expands the class to all binary maps on X (16 maps); exactly four

satisfy both observations. A frozen candidate h=(0,1,1,0) predicts output 1 at 2. A separately registered observation (2,1) leaves two compatible maps and rejects a candidate predicting 0 at 2. The value at 3 remains unresolved; the test does not justify a universal model or a unique generator. If a further observation (3,0) is licensed and affordable, exactly one map remains on X. A countercontrol in which a proposed observation is unavailable or unaffordable must not register the refinement as executed. This model illustrates novel-resolution assessment without supplying an assumed true model as the validation criterion.

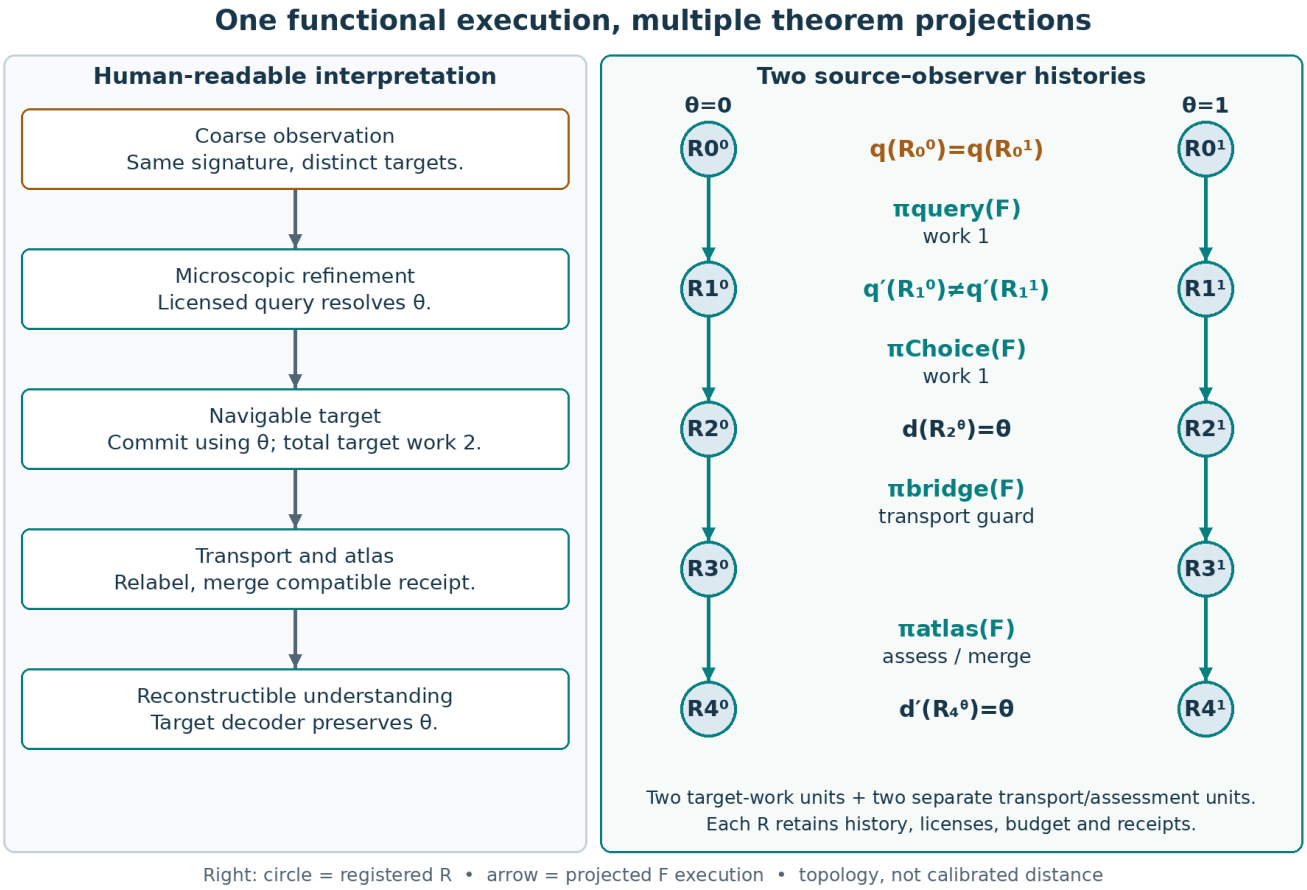
## S44.4. Theorem projections and the reconstruction hierarchy

For an admitted functionally and energetically closed system S, let X_S be its FSS and R its complete registered state. The universal GIGT target is that every theorem concerning the functional behavior of S is representable, generable and assessable through F acting on X_S. For a licensed word $\omega$ of the shared six unary functions, $R'=F_{\omega}(R)$. An emitted theorem object is a target-specific projection of a state, transition, family of paths or invariant of the registered execution: $T_j=\pi_j(R,F_{\omega}(R),\text{history})$, with target, proof or assessment, scope, assumptions, work, provenance, budget, execution and residuals retained. This notation projects records, not a seventh operation. The operations constituting a projection must themselves be F-recursively representable. Representability is not an execution, proof, convergence or universal decidability guarantee. A projection of a complete state need not itself be complete; only full registered R is an operative state.

Fix a common admissible history class H, and typed projections p and q of its registered executions. Define $q \succeq p$ if there is a decoder h with $p=h\circ q$ on H, including the target-relevant scope, budget and provenance distinctions. This holds exactly when $q(x)=q(y)$ implies $p(x)=p(y)$ on H: necessity follows by substitution; sufficiency follows by defining h on each nonempty fiber of q, which is well defined by constancy. Reflexivity and transitivity follow from identity and composition; after quotienting mutual reconstructibility, this is a partial order. The paired projection (p,q) is an upper bound, and any common upper bound r admits a decoder into (p,q), so it is the least upper bound up to mutual reconstruction. This information order is not a theorem-implication order and does not imply a low-work decoder. A viable hierarchy additionally needs licensed execution within R's work and resolution, with original certificate scope conserved and any new assessment appended. A joint G verdict requires an adequately executed joint assessment. A tree is generally insufficient because multiple projections may be jointly informative.

The map groups existing proof interfaces by the question answered. Observability: S12 (topology and observation), S37 (complete and partial information), S38.1–2 (registration and interface), S40 (refinement, resolution), S44.3 (model adequacy). Navigability: S21 (local fitness plane), S29 (path composition), S30–31 (generated operations and validation), S37 (CA-U, CoC-U, MN-U), S39 (structural and work separation), S41–42 (scope policies and transport). Viability: S38.3–7 (capacity, phase transition, bifurcation and infrastructure), S43 (reconstruction), S44.1–2 (joint preservation and work). These are overlapping index families, not independent ontologies; full proof locations and original status records remain authoritative. In the common binary witness, the joint observation/target/work projection reconstructs each constituent projection, whereas the initial coarse signature cannot reconstruct the target. A target label alone cannot reconstruct an executed work receipt or a proof certificate. The correct diagram therefore distinguishes information correspondence from executable edges and makes decoder guards local to the relevant relationship.

## S44.5. Functional visual reconstruction and common witness




S44 Figure 1. The left panel interprets a single common process; the right panel is its coupled source–observer registered-state topology for $\theta$=0 and $\theta$=1. Distinct full states have identical initial observation signatures, not identical full information. The query and commit cost one unit each; transport and assessment use the separate two-unit budget in S44.2. The displayed bridge and atlas labels abbreviate admitted F compositions, not new primitives. Target preservation is exact for this witness. The diagram does not establish native work, geometry, or a cognitive transition.

Visual ontology: circles identify full states or explicitly labeled projections of them, solid directed edges identify licensed or expressly candidate F-executions, and information-factorization relations must not be read as executions. Labels adjacent to each edge identify its projected transformation and relevant guard. Paths compose only with compatible endpoints and registered constraints. Spatial distances, edge widths, regions and densities carry quantitative meaning only under a defined native measure; the diagrams here are topological. The dual view is useful if the reader can recover the corresponding state, transition, proof interface, work and status without inferring absent source information. The source diagram for the RH reported dependency graph is retained as a dependency graph; turning its edges into discovery chronology would introduce unsupported information. S44-VIS-FIN checks the finite reconstruction and model-adequacy relations; its modeled budget is distinct from native functional work and reader comprehension. The complete result-family map is a proposed navigation interface, not a proof of universal F completeness.

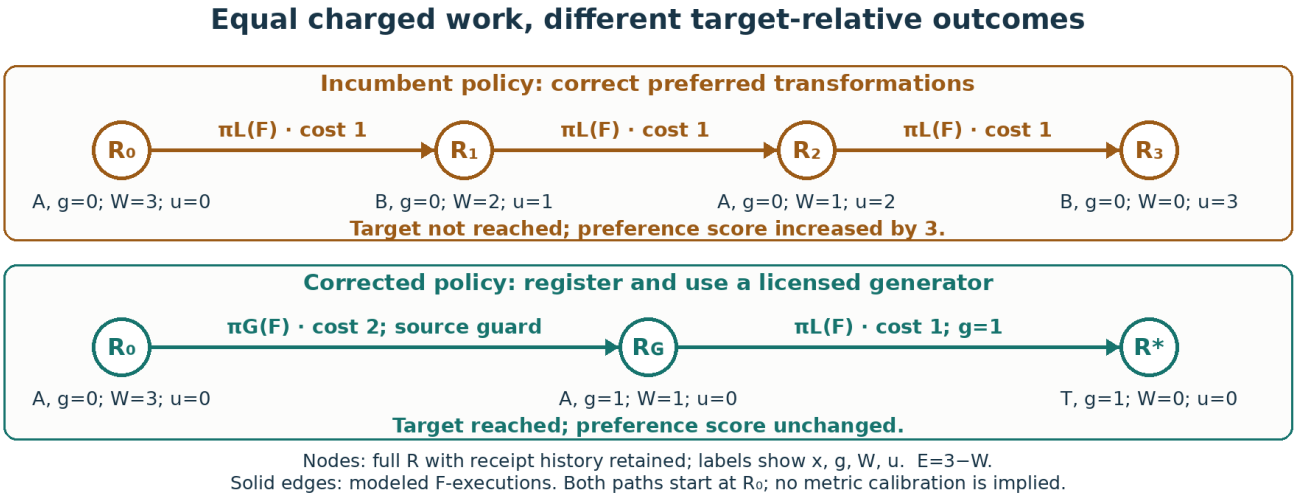
S44-VIS-FIN — COHERENT at the registered modeled finite scope: 8,806 of 8,806 declared assertion groups executed. The finite tests verify the projection-fiber factorization, least-upper-bound condition on the enumerated projections, transitivity, model-class correction, work guards and receipt countercontrols. They do not measure actual reader navigation cost or establish universal F realization. See reproducibility/S44_visual/assessment.json and check_projection.py.

## S44.6. Useful mathematical work and self-reinforcing search

The three consequences of energetic closure in the main text are projections of the same F-governed dynamics: preservation and discrepancy detection (S24.2), adaptive allocation at endogenous resolution (S24.3), and assessment of whether executed work advances a mathematical target (S21, S37, S41). The third is not an identity between functional free-energy expenditure and target fitness. In an adaptive relationship, perceived fitness may improve while the intended task is not advanced; the dog–owner analogy contrasts mutual reinforcement of preferences with an assessable functional task, without implying that companionship has no value. In mathematics, a preferred representation may generate many correct consequences, all of which have value within that representation, without resolving a specified outstanding constraint. The distinction is target-relative, not a judgement about the worth of any mathematical discipline. The intended functional objective, preference signals, operator selection, admissibility, work, target adequacy and residual histories are all registered operands of the existing F operations; no additional primitive is needed.

Let R register an invariant target T, its assessment contract, the current operative grammar, candidate fitness signals, the executed work and accessible source exchanges. For an admitted transition $R'=F_\omega(R)$, let $c\geq 0$ be its charged work and let p_T be a target-adequate residual, where one is justified on both endpoints. With registered inflow a and outflow b, the balance $W'=W+a-b-c$ constrains expenditure, but contains no implication that $p_T(R')<p_T(R)$. Nor does a correct local transformation imply that inequality. A preference score u can satisfy $u(R')>u(R)$ while $p_T(R')=p_T(R)$ and no target is reached. These are two independently typed projections of the complete execution. Work and target progress cannot be identified by numerical equality or by counting correct outputs. If target fitness is only partially assessable, register the available bounds or unresolved comparison rather than manufacture a complete scalar. This is the target-relative version of the S24 distinction between potential fitness and executable work.

Finite common-context witness. Register a modeled state $R=(x,g,W,E,u,h)$, where $x\in\{A,B,T\}$, $g\in\{0,1\}$ records whether a new operator has been admitted, W is remaining work, E is cumulative expenditure, u counts correct preferred local transformations and h is the retained ordered action history. Initial state is $(A,0,3,0,0,\varnothing)$. The incumbent grammar admits a reversible local transformation $L: A\leftrightarrow B$, preserving g and increasing u by one, at cost one. It cannot reach T from A or B. At A with g=0, a source-licensed and adequately assessed G composition at cost two registers g=1 (including its guard and source receipt). A licensed local execution at A with g=1 reaches T at cost one. Each action appends its receipt to h; no action is admitted without adequate work. These abstract licensed operations are projections of F for the modeled example, not evidence of a native six-function realization. The target is x=T, independent of u. All charged transitions conserve W+E=3. The preferred L-only trajectory consumes all three units and increases u to 3 but never reaches T; the G-then-L trajectory consumes the same work and reaches T with u=0. The preference loop is a cycle in the projected x-coordinate, not a return to an identical complete R, since work and history change. With indefinitely licensed replenishment it could continue indefinitely without reaching T; a closed finite budget instead forces it to stop. The example shows why more resources alone need not correct a deficient grammar or selection policy.




S44 Figure 2. Two admissible registered-state paths with identical total work and different target outcomes. The labels show selected state coordinates and projected F executions; complete nodes retain the action history and

receipts. The local correctness of L, the source license for G and the admitted generator guard are premises of the finite construction. Topology and accounting are shown, not calibrated native distances.

For the same finite model, let d be the least remaining charged work to attain T under the full licensed grammar before imposing the current finite budget. From (A,0) it is 3, from (B,0) it is 4, from (A,1) it is 1, and from (B,1) it is 2; at T it is 0. This follows by explicit path enumeration or weighted shortest paths, because a target transition requires g=1 at A. Thus an L step from (A,0) increases d, despite its correctness and preference reward. G reduces d by two and enables the subsequent one-unit target step. In the incumbent grammar the work-to-target infimum is infinite. These exact distances are scoped modeled quantities, not substitutes for native F-recursive distance. The comparison uses a common target and charged action costs including generator discovery; changing a target, suppressing a constraint, discarding provenance or asserting a new distance scale without a licensed comparison cannot count as demonstrated progress.

Evolving topology need not admit a common scalar residual for every individual step. Where an adequate comparison is established on complete registered states, the relevant progress may instead be assessed over a composed block of admissible transitions. Suppose an adequately registered residual $p \geq 0$ vanishes only at the accepted target, and every completed block outside the target either reaches the target or decreases p by at least $\eta > 0$ while preserving indispensable constraints, work, and source/assessment histories. After k nonterminal blocks $p \leq p_0 - k\eta$ by induction; therefore continued executable blocks attain the target in at most $\mathrm{ceil}(p_0/\eta)$ blocks. Intermediate increases in a particular local residual are not excluded. This is the existing S41 progress bound applied to composition under CA and CoC, with the comparison and work of every block registered in R. If the comparison is not established, the block stalls, the source is unavailable or the work cannot be paid, no such convergence verdict follows. The choice of progress contract and its correction are themselves assessable through G; a new assessment retains the original scope, budget and receipts. Native F-recursive fitness, comparability across evolving representations and the adequacy of generated policies remain receiving obligations, not consequences of the arithmetic alone.

The same mechanism applies to collective mathematics and to AI: more correct outputs or a larger work budget can leave a target inaccessible if the admitted grammar does not reach it or the selected policy never exercises the relevant transition. A discipline or an AI may have sound domain-specific progress measures; the proposed universal target is their recursively adequate composition and correction across changing domains and representations. This projection links H4 energetic closure, H6/SOM-G1 reliable convergence, CA-U, CoC-U, MN-U and the capacity/phase-transition conditions of S38–S44. It neither entails that actual mathematicians or AI exhibit this failure nor establishes a universal measure of useful work. The finite work witness and its guards are assessed separately from the inherited S44 results.

### S44.7. Recursive assessment of discovery dynamics

The adequate target-relative residual and policy of composed operations in S44.6 are themselves mathematical discovery targets. RMBST(ii), (iv) and (v) distinguish sound finite settlement from provisional stabilization, scoped assessment from uniform lifting, and policy nonattainment from operative exclusion. The fixed positive block-decrease bound in S44.6 remains valid at its stated conditions; it does not itself establish that the observer can discover or adequately assess its residual, guards or viability conditions. Recursive governance through existing F operations is the proposed means of assessing and correcting those conditions. The finite exploratory trace in S45.6 illustrates local productivity without demonstrated closure; it supplies no unrestricted empirical or native verdict.

## S45. Recursive Mathematical Blind-Spot and Discovery-Reliability Theorem (RMBST)

### S45.1. Recursive discovery target

The target is not merely to discover a mathematical blind spot, but to discover reliably the complete limitations of the mathematical functionality by which mathematical objects, governing functions, and those limitations are themselves discovered. This is a recursive mathematical discovery problem. A finite proof of an obstruction is distinct from a functionality that can reliably discover its instances, and increasing the number of adequately established mathematical results is distinct from establishing the adequacy of the process generating them. The following conditional theorem specifies the limitations and the conditions under which a structural expansion of discovery functionality overcomes a particular limitation.

## S45.2. Functional and assessment contracts

Let $F$ act on complete registered states $R$ of a functional state space (FSS). Completeness of registration does not imply completeness of target-relevant information. The registered state includes the target and its interpretation, operands and operators, admissible execution and observation relationships, history, functional work and resolution, assessment contracts, certificates, and residuals. The six shared GIGT/FMA operations are unary on $R$: Create/Destroy, Place/Store, Recall, $L$, $G$, and Choice. Constraint Algebra and Calculus of Convergence govern compositions of these operations; the mathematical notation below denotes projections of registered states, not additional primitives or external state variables.

$L$ denotes a direct, one-shot operation relative to a target for which functional information is complete. $G$ denotes an operation on potentially partial functional information, including the recursive generation, assessment and correction of operative relationships. The *complete incumbent closure* includes every admissible composition and construction already operable through the incumbent functionality. First-order mathematics means the GIGT functional order, not first-order logic. A conventional finite proof does not by itself identify the functional dynamics through which it was discovered.

**Definition S45.1 — Relative and absolute discovery reliability**

Relative reliability on a declared target class and finite registered work budget requires a finite admissible execution that terminates with an adequately justified verdict, with a declared substantive coverage requirement. For unconditional correctness objects the outcomes are $CORRECT, INCORRECT, UNDETERMINED$; for coherence objects, including conditional propositions, they are $COHERENT, INCOHERENT, UNDETERMINED$. An unresolved correctness assessment is not a third truth value. $UNDETERMINED$ is required where the executed assessment is inadequate to establish either substantive verdict; it is not evidence of nonexistence of a certificate unless the relevant search has been adequately completed. An always-abstaining procedure has no substantive discovery coverage. Work includes discovery, execution, checking, and registration, not merely verification of a supplied certificate.

Absolute reliability requires that, on every admitted information/work growth trajectory toward the declared limiting information and unbounded work, warranted assessments converge to the warranted limiting assessment. In particular, for every determinate target requiring conclusive settlement, an applicable sound substantive verdict must eventually be issued. A genuinely unresolved limiting target remains unresolved. A provisional guess permitted to be false is not such a verdict; its eventual stabilization is a weaker contract. The limit is not a completed infinitary computation. All coherence verdicts retain their original scope, budget, adequacy, actual execution, and provenance; a new assessment appends rather than overwrites those records.

## S45.3. Recursive discovery-reliability limitation

**Theorem RMBST — Recursive Mathematical Blind-Spot and Discovery-Reliability Limitation (RMBST)**

The following implications are established at their indicated mathematical scopes. The targets, interfaces and admissible closures in each implication are fixed relative to the relevant assessment; the implications do not assert that all realizations satisfy every obstruction premise.

**(i) Informational obstruction.** Let $d$ be a target distinction on admitted histories and $q$ the complete information signature licensed by an incumbent architecture. If $q(h) = q(h\prime)$ but $d(h) \neq d(h\prime)$, no assessment or reconstruction factoring exclusively through $q$ can reliably determine $d$ on both histories. If the collision persists in the complete limiting signature, the obstruction persists under unbounded work and information available through that interface. A proof transmitted solely through the deficient interface cannot supply the erased distinction.

**(ii) Convergence obstruction.** Identification of a target by complete limiting information need not permit reliable finite-prefix assessment. There exist determinate binary targets for which neither a uniform sound conclusive assessor nor even an arbitrary revisable prefix assessor converging to the right verdict on every stream exists. There also exist targets admitting computable provisional limit identification but not sound conclusive settlement on every stream. In particular, actual convergence of a discovery process need not imply reliable finite observability of its convergence.

**(iii) Reflexive obstruction.** For every sound effectively realizable registered architecture whose target class is expressive enough for the self-referential construction below, there exists a true unconditional claim about its own

operative closure that it cannot conclusively affirm or refute. Thus no such architecture attains absolute conclusive discovery reliability over that full class, even with unbounded repetitions within its fixed closure. For each fixed oracle the corresponding assertion holds for architectures effective relative to that oracle, with a claim indexed by the architecture and oracle. This applies to effective architectures admitting self-amendment as well as fixed-rule architectures.

**(iv) Scoped access and nonuniform lifting.** The reflexive obstruction is compatible with actual adequate bounded assessments at cofinally increasing finite horizons on a single continuing execution. It does not exclude every scoped coherence assertion. Nevertheless, no uniform effective procedure on the effective nonreachability class can lift such finite-horizon information to conclusive affirmation on all true instances.

**(v) Operative expansion and recursive governance.** Where the required distinction, warrant relationship, or registration operation is excluded from the complete incumbent closure, no amount of work executing only that closure reaches the excluded target. An adequately licensed expansion can strictly increase reachable targets even on unchanged source information, without noncomputability. Correct local steps, increasing agreement, and accumulation of valid mathematical objects do not imply convergence of assessment of the adequacy of the process for its original target. A sufficient progress condition is a viable, target-preserving policy of composed operations for which an adequately assessed residual decreases by a fixed positive amount per nonterminal block until the target is attained. The adequacy and correction of that policy are themselves mathematical discovery targets.

**Proof.**

(i) Every output factoring through $q$ is constant on each fibre of $q$. The collision contradicts the required target distinction. The same argument applies to the limiting interface; a finite collision alone is not a limiting collision.

(ii) Consider the full binary stream space and the target "infinitely many ones." Suppose a prefix assessor always eventually stabilizes to the correct binary verdict. From any finite prefix, extend by zeros until the assessor declares that there are only finitely many ones; such a stage exists on the all-zero continuation of the prefix. Append a one and repeat. The constructed stream contains infinitely many ones but has arbitrarily late negative labels. This contradicts stabilization, even without assuming the assessor computable. For the target "some one occurs," a computable provisional assessor initially guesses no and changes to yes at the first one. Yet no finite prefix of the all-zero stream warrants conclusive no, since it admits a continuation with a one. These are distinct contracts. In the prefix topology, sound two-sided conclusive settlement on all streams requires both target and complement to be open (clopen); the preceding collision is the failure of the negative openness condition. On full Cantor space, compactness then gives a finite uniform observation horizon for an unrestricted sound prefix assessor.

For convergence itself, consider the class of eventually constant binary progress streams, and the target at a given stage that no subsequent substantive change occurs. Every all-zero future prefix is compatible with both permanent stationarity and a later isolated change. No sound uniform finite-prefix certificate of permanent stationarity exists on that class. This holds even though every admitted stream eventually becomes stationary. Additional information, such as a verified invariant or a complete stopping certificate, may resolve the target; it is not excluded by this observation argument.

(iii) Encode complete reachable registered states of an effective system $s$ by natural numbers; finite reachability is computably enumerable. Assume a decidable faithful registration predicate $Reg(r, c, v)$ for conclusive verdict $v$ on claim code $c$, and a target language admitting the associated computably generated predicates. In particular every issued conclusive verdict has a reachable registration. Write

$$B(s,t) \Leftrightarrow Reach(s) \cap T_t = \varnothing,$$

where $T_t$ is a decidable target set with valid index $t$. By the recursion theorem, choose $t$ such that, writing $c = B\,(s,t)$ with its registered code,

$$T_t = \{r : Reg\,(r, c, CORRECT)\}.$$

If $CORRECT$ on $c$ were reachable, the registration would falsify $c$, contrary to soundness. Thus $c$ is true, and soundness excludes $INCORRECT$ as well. If the system performs a total sound assessment it must return $UNDETERMINED$; otherwise silence or nontermination is possible. The construction is unchanged relative to a fixed oracle. The quantifier order is $\forall X \forall s^X \exists c_{s,X}$, not a single claim inaccessible to all oracles. This does not

exclude a sound external proof of the old claim. It also does not require ignorance of the code of $s$. In the arithmetic realization, the diagonal lemma gives $D_s \leftrightarrow \neg E_s(\ulcorner D_s \urcorner)$ in PA, where $E_s(z)$ expresses reachable conclusive affirmation; PA consequently proves $(E_s(\ulcorner D_s \urcorner) \to D_s) \leftrightarrow D_s$. The local positive-soundness warrant for the reflexive proposition is thus equivalent to the proposition itself.

(iv) For a constructive example take a deterministic effective registered system with a fair scheduler that searches a sound arithmetic proof calculus on ordinary arithmetic targets, and for each $n$ explicitly checks all states on its own trajectory through step $n$ for the target $T_t$ of (iii). Its scoped registrations are not $CORRECT$ records on $c$. The recursion theorem supplies the code for the target test; each bounded check simulates only finitely many elementary transitions, including elementary computation steps within other checks. The computation is scheduled one elementary step at a time, so bounded simulation does not require completion of an embedded unbounded check. Soundness of the ordinary proof calculus and the exact bounded test are the warrants for the respective records. By (iii) the conclusive record is absent; each bounded check completes in finite work and certifies absence through its horizon, and fairness gives cofinal execution and retention on one trajectory. Equivalently one can use a finitely branching effective transition presentation with computable complete successor lists and bounded-tree checks. This is a modeled scope certificate, not a native GIGT assessment-budget measurement. Its contract is absence of a conclusive registration through a specified horizon, not unrestricted truth of $c$.

For the no-lift assertion, use the uniformly effective family simulating program $e$ on a fixed input, with its halting state as target. Every bounded absence check is decidable; the conjunction over all horizons is nonhalting. A sound effective procedure eventually affirming every true nonhalting claim would enumerate the complement of the halting set, impossible. Likewise no computable uniform stabilization horizon exists for the provisional rule that guesses nonhalting until halting is observed. For a target that includes the scoped records themselves, if $d_T$ is the shortest reaching length, bounded absence through $n$ is true iff $n < d_T$ (including $d_T = \infty$). Thus a registration at step $m$ in that target can be sound only if $n < d_T \leq m$; cofinal such registrations are impossible. The precise assertion contract, not the word "affirmative," determines the obstruction.

(v) Nonreachability under a complete operative closure is invariant under further execution within it. For an explicit structural separation on unchanged source information, let a registered relation $P \subset U \times V$ and $Q \subset V \times W$ be given. Stipulate an incumbent grammar closed under operations preserving available relation types, with no operation yielding a relation of type $U \times W$ (the complete closure is assumed to have this property). A licensed computable composition generating $\{(u, w) : \exists v P(u, v) \wedge Q(v, w)\}$ makes that target available. This proves a relative architectural separation; an incumbent already admitting the composition is not separated by this witness.

The separation can also arise with a known unique solution and no missing constructor for individual verdicts. Let the complete binary verdict vector $(a, b, c)$ be constrained by the computable feedback map

$$H(a, b, c) = (b \wedge c, a \vee c, \neg a \vee b).$$

Its unique fixed point is $(1, 1, 1)$: if $a = 1$ the first equation forces both other coordinates to be one; if $a = 0$ the third and second equations force $c = b = 1$, contradicting the first. Stipulate a permanent-registration guard that licenses each proposed block only if its equations hold under every raw completion of the already registered coordinates and the block. No single first registration is licensed, since each output takes both values as the two other raw coordinates vary. Registering $b = c = 1$ together is licensed for either value of $a$, and subsequently $a = 1$ is licensed. Thus changing the admitted joint-warrant relationship makes the unique solution registrable on unchanged source information. A guard already admitting the global fixed-point proof as warrant for an individual coordinate does not have this obstruction. This is an exact operative witness, not a lower bound on arbitrary sequential computation or an identification of block width with native cognitive bandwidth.

For the distinct policy obstruction, consider a registered state with target flag $z = 0$ and a list of established truths. A licensed local step appends the next elementary true equation $j = j$, leaving $z = 0$; an admitted target-reaching step sets $z = 1$. Under the policy always selecting the local step, valid mathematical results accumulate without reaching the invariant target $z = 1$, although the target-reaching step is available. Alternatively, exclude the latter step from the grammar and the same history exhibits operative nonreachability. With replenished modeled work these histories can continue without bound. Their complete states do not repeat, since history and work change. This proves that neither productivity nor local correctness implies target progress, and that policy nonattainment must not be conflated with closure nonreachability. It does not assert that the added truths have no value for other targets.

For the positive condition let the registered adequate residual $p \geq 0$ vanish exactly at target-adequate states; a valid defeating objection may constitute adequate assessment. If every nonterminal executable block decreases $p$ by at least $\eta > 0$ while retaining indispensable constraints, provenance and work viability, induction gives $p_k \leq p_0 - k\eta$ until attainment. With enough accessible work to execute the blocks, the target is reached within $\lceil \frac{p_0}{\eta} \rceil$ blocks. A suitable contraction tending to zero instead requires an appropriate closedness and stability condition for target convergence. If the residual or its comparison is not adequately established, no such convergence follows. The residual is a target-relative modeled quantity, not a substitute for endogenous F-recursive fitness or bandwidth.

## S45.4. Universal functional hypothesis

**Hypothesis RMBST-U — First-order discovery closure and second-order expansion**

For any mathematical functional system $F$ on its FSS, discovering its governing functionality, the objects navigable under it, and its own complete discovery limitations are recursively related targets of the same functional architecture. The Constraint Algebra constrains candidate models by registered observations; the Calculus of Convergence governs their admissible recursive identification and correction. Neither complete limiting observations nor unlimited work within a fixed operative architecture guarantees adequate discovery of $F$ or its blind spot.

The strong GIGT hypothesis is that first-order mathematics has no systematic escape through its complete $L$-operative closure from the target-relevant exclusions needed to determine its complete mathematical blind spot reliably. No finite sequence of $L$ operations, nor any finite theorem/proof whose reconstruction and assessment remain confined to that deficient closure, can supply the missing functional relationships. Where exclusion persists through its complete limiting interface, even the joint information/work limit cannot supply them within that closure. Where the obstacle is an effective reflexive one, conclusive self-capture fails under Theorem RMBST(iii), irrespective of increasing work. This is not a claim that the blind spot cannot be mathematically expressed or that particular instances cannot be finitely proved.

The proposed second-order functionality is the stable recursive governance, through $G$ and the other existing operations, of the generation, assessment, correction and composition of the mathematical operations by which discovery occurs. Where indispensable functionality lies beyond the complete incumbent coherent cognitive bandwidth as F-recursively defined, a qualifying transition in cognitive order is required to make that functionality coherently operable. If a viable adequately assessed expansion exists, particular exclusions can be repaired without a final closure of all discovery limitations. Local use of a candidate $G$ operation or a larger computational budget alone does not establish a phase transition.

The universal quantifier and the cognitive-order requirement are realization targets, not consequences of the effective counterexamples alone. A separation at the complete incumbent closure must be demonstrated, not inferred from a restricted sample of operations. The effective diagonal applies equally to qualifying second-order realizations. The theorem therefore distinguishes systematic expansion of discovery from absolute self-completeness.

## S45.5. Theory of Everything corollary

**Corollary RMBST-ToE — Recursive limitation of reliable discovery of a Theory of Everything**

For a physical or cognitive Theory of Everything (ToE), distinguish specification of governing functionality $F$ and its FSS, determination of its consequences and required configurations, and reliable determination of the adequacy and limitations of the mathematical functionality by which those objects are discovered. If the ToE includes its mathematical observers, and a represented observer satisfies the effective expressiveness, registration and soundness conditions of Theorem RMBST(iii), that observer cannot attain absolute conclusive discovery reliability over the full class of propositions about its own operative closure. A complete specification of governing functionality does not confer complete reliable self-capture.

If reliable discovery of the ToE itself or of its uncomputability limits requires indispensable relationships excluded from the complete incumbent $L$-operative closure, no finite composition of $L$ operations within that closure can guarantee their reliable discovery; if the exclusion persists at the limiting interface, the corresponding absolute guarantee also fails. A viable, adequately assessed expansion can overcome particular such limitations, not remove the general reflexive limit. In particular:

**(a) Quantum mechanics at large scales.** For an admitted physical family, if the finite or limiting quantum observation signature merges models having different target large-scale properties, the corresponding information obstruction applies. If the induced prefix property fails the required topology of sound settlement or provisional

convergence, the corresponding convergence obstruction applies. If an undecidable class reduces effectively to the target physical property, uniform effective conclusive determination is impossible. Even computable individual finite steps and convergence to a limiting state need not enable reliable discovery of the large-scale limit or of a sufficient observation or work horizon.

**(b) String-theoretic initial conditions.** For an admitted class of models $(F, R_0)$, if compatible observations fail to distinguish target-distinct required initial configurations, the initial conditions cannot be reliably reconstructed from that interface. Persistent collisions, nonconvergent assessment targets, or a reduction from an undecidable problem yield the respective stronger limitations. Conditional evolution from a specified configuration is distinct from reliable discovery of that configuration.

**(c) AGI and cognition.** Functional completeness of a model of cognition and the capacity to reliably discover all consequences and all limitations of that cognition are distinct. An effectively realized AGI satisfying the reflexive conditions cannot conclusively determine its complete reflexive blind spot, even if its governing functionality is completely specified. Scoped self-assessment, sound self-modification under a specified invariant, and correction of particular exclusions remain possible under their registered contracts.

**Proof.**

The self-inclusion statement follows from Theorem RMBST(iii) applied to the represented observer, not automatically to the external theory's proof calculus. The closure-specific statements follow from (i), (ii) and (v) with the declared target and interface. In (a) and (b), composing a hypothetical uniform effective decisive assessor with the stipulated reduction would decide the undecidable source class. In (c), the scoped claims do not assert unrestricted correctness of all future verdicts, so the diagonal does not generally defeat them. No claim that every equally strong successor is uncertifiable follows without additional assumptions on the certification predicate.

**Scope of the physical hypothesis.**

Spectral-gap undecidability in specified quantum lattice families [S45-R2] and uncomputable renormalisation-group flows [S45-R3] supply actual mathematical instances of effective large-scale obstructions. They do not establish universal uncomputability of all large finite quantum systems or an intrinsic energetic failure of wavefunction collapse. The strong application hypothesis is that the full quantum discovery problem and the required initial-condition discovery problem for any string theory instantiate indispensable obstructions to systematic first-order discovery. That quantifier requires a physical realization or a common invariant across the admitted theory class. It is not established by the abstract theorem. In particular, an observer's inability to compute nature's dynamics does not imply nature cannot execute them. A particular ToE or uncomputability theorem may be discovered and finitely proved; the question is the reliability of the full discovery functionality.

## S45.6. Finite exploratory observation of recursive discovery

A two-LLM alternating assessment experiment repeatedly presented one model's theorem revision to the other with substantially the same request to assess its usefulness for a subsequent revision. The author reports not supplying mathematical corrections during the alternating sequence under discussion and assessed convergence of the process as a separate target. The recorded responses show repeated agreement on individual derivations, preservation and addition of mathematical results, and further expansion of the target set. One model repeatedly recognized diminishing returns and both nevertheless continued to develop further results. The universal F-recursive realization and cognitive-bandwidth relationship remained explicitly unresolved. The process was stopped by a change of assessment target initiated by the author. This is an exploratory finite trace of productive non-closure, not a controlled demonstration of infinite divergence. The continuation prompt itself encouraged revision, the complete internal closures were not observed, and the two models did not have the same explicit assessment objective as the author. The trace does not establish that every internal operation was $L$ or that either model could not perform $G$ under another contract. It does distinguish observed local mathematical agreement from demonstrated adequacy of recursive discovery governance. The formal result in Theorem RMBST is independent of this observation. An experimentally assessed second-order transition would require the additional registered functional and viability conditions of Hypothesis RMBST-U.

### S45.7. Assessment status and relation to existing mathematics

The mathematical deductions establish the stated implications at their exact registered scopes. They do not supply a native global-coherence certificate for the universal hypothesis, for the conjunction with other mathematical results, or for physical realization. Native BudgetStatus is BUDGET_UNDETERMINED; no default work budget, adequate native execution, or positive native $G$ verdict is inferred. The diagonal is a Gödel-style construction using classical computability theory; conjecture-dependent truth and self-defeating inquiry also have precedents in Kelly, Juhl and Glymour [S45-R1]. The contribution here is the conjunction of the contract-relative reliability, scoped assessment, structural operative expansion and recursive governance distinctions, not priority for diagonalization or topology of inquiry. All are formulated as mathematical projections of the proposed functionality rather than as new FMA primitives.

### S45.8. References for the recursive discovery theorem

[S45-R1] K. T. Kelly, C. Juhl and C. Glymour, Reliability, Realism and Relativism, in P. Clark and B. Hale (eds.), *Reading Putnam*, Blackwell, 1994, pp. 98–161.

[S45-R2] T. S. Cubitt, D. Pérez-García and M. M. Wolf, Undecidability of the spectral gap, *Nature* 528 (2015), 207–211. https://doi.org/10.1038/nature16059.

[S45-R3] J. D. Watson, E. Onorati and T. S. Cubitt, Uncomputably complex renormalisation group flows, *Nature Communications* 13 (2022), 7618. https://doi.org/10.1038/s41467-022-35179-4.

### S45.9. Receiving relationships and joint assessment contract

The S45 results recursively apply the existing hypothesis roles: H1–H3 to discovery of observer-relative limitations and their observability; H4–H6 to viable correction, generation and convergence of discovery policies; H7 to the distinction between a documented RH endpoint and reliability of its generative mechanism; H8 to transmission of discovery functionality. H1–H8 and J=H1$\wedge\cdots\wedge$H6 retain their original statements and assessment scopes. The extension is the separate conjunction of J with the applicable RMBST relationships, not a redefinition of J. Assess compatibility of the complete-closure and registration guards with S37, S38, S39, S43 and S44.7; distinguish modeled work from native FSS work and bounded absence from unbounded truth. Retain all inherited certificates and budgets. The universal realization, physical application, actual cognitive-order transition and joint native G assessment are UNDETERMINED; BudgetStatus=BUDGET_UNDETERMINED. No unexecuted assessment is a positive certificate.

# Part II — Authoritative hypotheses and source methods

# Appendix B. Full hypothesis targets and dependencies

**Hypotheses:**

This paper is organized around eight principal hypotheses, that progress from the existence of a first-order mathematical blind spot, through its functional mechanism and the necessary conditions for overcoming it, to systematic mathematical discovery, the Riemann-hypothesis proof, and the transmission of mathematical understanding.

#### H1 — Existence and Coherence of a Systematic First-Order Blind Spot in Mathematics

There exist mathematically admissible problem domains for which a first-order mathematical observer systematically fails to observe or discover target-relevant mathematical objects, relationships, or operations.

The existence of such blind spots is coherent under the functional and informational constraints specified by GIGT.

Principal implication: Mathematics may possess systematic observer-relative limitations arising from its operative mathematical representations, rather than solely from the intrinsic complexity of individual problems.

**H2 — The L/G Distinction as a Generative Mechanism of the Mathematical Blind Spot**

The systematic first-order mathematical blind spot arises from an inability to adequately distinguish, construct, or execute the mathematical relationships and operations accessible through recursively composed $G$-governed processes, as opposed to operations executable within an existing $L$-adequate operative closure.

Consequently, a mathematical observer may correctly execute its available operations while remaining unable to discover mathematical objects requiring operations outside its current operative closure.

Principal implication: GIGT may provide a functional explanation for the existence of systematic mathematical blind spots.

**H3 — Recursive Determination of Mathematical Observability**

The extent and resolution of a mathematical observer's blind spot are determined by its accessible functional information, mathematical representations, observation operations, assessment capabilities, historical retention, and available computational and energetic resources.

Because the observation operations of both the observing system and the observed system may themselves evolve and become operands of subsequent mathematical operations, mathematical observability is potentially recursively extensible.

Principal implication: Mathematical observability cannot, in general, be adequately characterized solely by a fixed external representation of mathematical objects.

**H4 — Functional and Energetic Closure as Necessary Conditions for Systematic Second-Order Mathematical Discovery**

A second-order model of mathematics based on a generalized functional model of mathematics must satisfy two jointly necessary conditions to possess the capacity to systematically discover mathematical objects lying within the observational blind spot of a specified first-order mathematical system.

Functional closure: The mathematical system is recursively closed under the composition of its admissible mathematical operations. All generated mathematical objects—including operations, operands, relationships, representations, assessments, corrections, and their relevant histories—remain representable as mathematical operands within the same generalized mathematical system.

This provides the potential for recursively extensible self-correction without an externally imposed finite maximum recursion depth, subject to computability, resource, and viability constraints.

Energetic closure: The mathematical system preserves an adequate functional free-energy conservation or accounting relationship across mathematical operations, including internal resources, environmental exchanges, expenditures, and the conditions necessary for viable continuation.

The relationship must make functionally relevant energetic losses and failures of accounting observable or assessable where they could otherwise prevent correction and permit incorrigibility.

Conjunctive hypothesis: Both forms of closure are necessary for systematically sustainable second-order mathematical discovery, although their conjunction alone does not guarantee convergence to a solution.

Principal implication: GIGT may identify universal functional and energetic requirements for mathematical systems capable of recursively generating and correcting their own discovery operations.

**H5 — Systematic Discovery of Mathematical Objects Beyond First-Order Observational Blind Spots**

An admissible second-order mathematical process can recursively generate previously unavailable mathematical relationships or operations from source-accessible partial functional information.

By making those generated relationships or operations operative, the process can discover proofs and other mathematical objects inaccessible within the original first-order mathematical observer's operative closure.

The generation process does not require the completed solution, its final topology, or its necessary sequence of operations to be supplied as an initial operand.

Principal implication: Second-order mathematics may provide a systematic means of overcoming classes of mathematical problems that are inaccessible through the original first-order operative representations.

**H6 — Reliable Convergence of Recursively Generative Mathematical Discovery**

Under derivable conditions of functional and energetic closure, recursive self-correction, source accessibility, target-relative assessment, viable continuation, and dynamical stability, a second-order mathematical process can reliably converge toward solution-bearing mathematical structures for an adequately specified nontrivial class of problems.

The process recursively selects and corrects its mathematical operations, operands, internal configurations, representations, and continuations without requiring a predefined final solution topology.

Principal implication: Mathematical discovery itself may become a systematically executable, assessable, and corrigible mathematical process.

This is the strongest generative-convergence hypothesis associated with SOM-G1.

**H7 — Compatibility of the Riemann-Hypothesis Proof with GIGT's Generative-Discovery Architecture**

The mathematical relationships, evolving representations, and discovery architecture associated with the recent Alpöge–Claude proof of the Riemann hypothesis may be structurally compatible with the recursively generative mathematical operations predicted by GIGT.

Such compatibility may indicate that processes capable of solving exceptionally difficult mathematical problems can exhibit functional structures similar to those required by second-order mathematical discovery.

The stronger hypothesis is that an adequately instantiated GIGT-based process could systematically generate comparable mathematical discoveries under specified conditions.

Principal implication: The RH proof may provide a significant case through which to investigate whether GIGT's proposed discovery mechanism is consistent with an actual major mathematical achievement and whether the mechanism could be made reliably executable.

Structural compatibility, historical causation, and systematic reproducibility are separate assessment targets.

**H8 — Recursive Transmission and Reconstruction of Second-Order Mathematical Understanding**

Mathematical understanding can be modeled through the preservation and reconstruction of functional information sufficient to execute, assess, correct, and continue mathematical operations.

A recursively extensible system of mathematical and visual-functional interfaces can, under appropriate target-preservation and realization conditions, represent mathematical operations, their compositions, and their assessments as operands that can be reconstructed and reused by other mathematical or conceptual systems.

When an existing interface erases a target-relevant mathematical distinction, the interface itself can become an operand of subsequent mathematical correction.

Principal implication: GIGT may provide a mathematical foundation for transmitting not merely mathematical propositions and completed proofs, but the functional capacity to reconstruct and continue the operations through which mathematical discoveries are generated.

This could support the development of decentralized, recursively corrigible collective mathematical intelligence.

**Dependency structure**

| Hypothesis | Scientific role | Relationship to other hypotheses |
|---|---|---|
| H1 | Foundational existence/coherence | Establishes the proposed mathematical phenomenon |
| H2 | Functional mechanism | Explains how the blind spot can arise |
| H3 | Recursive observability | Characterizes the evolving boundaries of observation |

| Hypothesis | Scientific role | Relationship to other hypotheses |
|---|---|---|
| H4 | Necessary conditions | Derives the functional and energetic requirements of systematic repair |
| H5 | Constructive discovery | Demonstrates overcoming specified blind spots |
| H6 | Reliable convergence | Generalizes constructive discovery to viable, systematic solution generation |
| H7 | Mathematical case study | Tests architectural compatibility with the RH proof |
| H8 | Transmission and collective extension | Develops reconstructible communication and continuation of the mathematical operations |

**The central conjunctive hypothesis**

The overall scientific target is that:

Systematic first-order mathematical blind spots can exist
$\wedge$
GIGT can characterize their functional mechanisms
$\wedge$
Functional and energetic closure are necessary
for systematically sustainable generative repair
$\wedge$
Second-order mathematics can generate
previously inaccessible mathematical operations
$\wedge$
Those processes can converge under derivable conditions
toward viable mathematical solutions.

H7 and H8 investigate two major consequences: compatibility with an important real mathematical proof and the capacity to transmit the functional understanding required for collective mathematical discovery.

# Appendix C. Source methods, interfaces and execution contracts

**Methods**

**1. Design, scope, and evidentiary discipline**

The study uses a backward, target-directed derivation. It registers the manuscript's eight hypotheses (H1–H8), five intended conclusions (C1–C5), and the governing generative-convergence target SOM-G1 before selecting local lemmas and sources. The targets are held at their intended scope; whether any target has been proved, supported only conditionally, or left unassessed is recorded separately. The aim is to derive operational tests for first-order mathematical blind spots and second-order repair, to assemble a source-supported conditional conjunction, and to examine the reported Alpöge–Claude Riemann-hypothesis (RH) discovery architecture as a bounded compatibility case [1,2].

Three kinds of evidence are distinguished throughout: (i) formal consequences established from typed definitions and explicit premises; (ii) empirical or documentary observations about AI mathematics and mathematical practice; and (iii) conjectural extensions, including universal completeness, resource-dependent convergence, and the history of any particular discovery. A source supporting one proposition does not automatically prove its dependencies, their conjunction, or a stronger statement. For reproducibility, external evidence is indexed by source, date, quoted or paraphrased proposition, scope, assumptions, and an explicit assessment budget; this section specifies the procedure rather than claiming that all proposed executions have already occurred.

**2. M-space ontology and typed mathematical interfaces**

**2.1 Complete registered state and the operation basis**

Let M denote the proposed mathematical functional information space and R its complete registered state at the resolution and work budget of the current mathematical process. R contains typed mathematical operands and all

target-relevant information needed for authorized continuation: statements, logical contexts, relations, operator specifications, execution histories, sources, retained partial proofs, assessment results, provenance, resource-accounting records, and current source-access restrictions. These are components of one registered state, not independent external state variables. Source access and the assessment budget B are also registered operands; where B cannot be specified or derived, its status is BUDGET_UNDETERMINED and no positive budget-scoped G certificate is inferred.

The candidate shared GIGT/FMA operation basis comprises Create/Destroy, Place/Store, Recall, L, G, and Choice. Each internal operation is typed as a transformation $F_i$:R→R of the complete state. The Create/Destroy and Choice roles remain interfaces across their relevant signal, conceptual, and fitness/configuration spaces rather than being reassigned as exclusively intra-M-space operations. A Constraint Algebra of admissible compositions and a Calculus of Convergence of viable correction are mathematical structures built from compositions and assessments of these operations, not additional primitive functions. The method does not infer that the proposed six are independently proved minimal or complete for all domains.

$F_i : R \to R$; $F_i(R)$ = (updated registered state, including execution and residual records).

A candidate transformation becomes executable only when its operand types, source permissions, assumptions, invariant requirements, and assessment conditions are satisfied. A change in the navigable relational "shape" is therefore a change in the typed admissible-transition structure. It is not automatically a topology in the mathematical sense; claims of topological continuity, separation, or convergence require additional definitions and proofs.

### 2.2 Canonical record interfaces

The following interfaces are expressed as typed records inside R. The record types do not introduce new primitives; they describe the mathematical operands on which the shared operations act. All records retain stable identifiers and append-only provenance so that later correction does not silently overwrite earlier assessment.

| Record type | Required fields / contract | Use condition |
|---|---|---|
| Claim / conjecture | ClaimID; proposition $\phi$; context $\Gamma$; exact target and scope $\Omega$; domain/type; source IDs; dependency IDs; status; B; provenance. A conjecture is a claim without a completing proof certificate. | A claim is admissible for assessment when its proposition, context, scope and dependencies are registered; no truth follows merely from registration. |
| Theorem / proof | Theorem = claim + accepted proof obligations; proof = derivation or certificate $\pi$; kernel/theory; premises; dependencies; translation bridge; checker execution. | A theorem status requires a valid derivation for exactly the stated formal proposition and its dependencies, with semantic transport separately assessed. |
| Partial derivation | Open goals; proved subresults; failed attempts; counterexamples; unresolved bridges; source constraints; residuals; work/budget history. | May be stored, transformed and queried without being promoted to a completed proof. |
| Operator / generator | OperatorID; typed input/output; guards; transformation rule; source-access contract; invariant/assessment obligations; resource account; version/history. | A generated rule may be conditionally used only within its certified contract and declared source domain. |
| Assessment / certificate | Target; L or G; verdict; exact scope; premises; B; executed work; trace/certificate; defeaters; residuals; provenance. | A G verdict is positive only after an executed and adequate G assessment at its registered budget; B alone proves nothing. |
| Source / transport | SourceID; origin; accessible fields; timestamps/version; trust and validity assumptions; translation or transport map. | Cross-source reuse requires a typed bridge showing that target-relevant meaning and assumptions are preserved. |

### 2.3 Local and global coherence

An L operation uses an adequate direct composition of already specified operators and operands for the target in view: its required functional information is "complete" relative to that target and the declared interface. A G

operation may navigate indirect relations or partially specified operands and operators and may recursively construct, test, and correct the operations used in further assessments. Both act on R; the labels denote functional contracts, not the syntactic order of a formal logic. A conventional Lean program could implement G-like capabilities, while a graph with relative coordinates could remain operationally fixed.

Verdict types distinguish L-derivability, a scope- and budget-typed G-coherence certificate, incoherence witnessed by an explicit defeating obligation, conditional admissibility, and unresolved assessment. Neither a missing G execution nor a failed attempt to find a proof constitutes an incoherence proof. Conversely, conditional admission is not promotion to established mathematical truth.

## 3. Exact information-preservation and blind-spot tests

### 3.1 Target-relative representation criterion

For a set H of admissible complete histories, let $a:H\to A$ be the representation or observation actually available to an observer and $\kappa:H\to K$ the target-relevant outcome, including its admissible next actions when these matter. A decoder $d:A\to K$ can reconstruct the target on H exactly if and only if the representation never identifies two histories having different target outcomes:

$\exists d\ (\kappa = d \circ a) \Leftrightarrow \forall h,h' \in H,\ a(h)=a(h') \Rightarrow \kappa(h)=\kappa(h')$. (M1)

Necessity follows because a common observed value would force d to return the same output; sufficiency follows by defining d on each observed fiber using its unique $\kappa$-value. This factorization is a local mathematical result for the stated history and target classes. It does not claim that every mathematically meaningful distinction must be retained. The theory of complete abstractions and strong preservation supports the importance of retaining exactly those distinctions needed by the declared operations [14,15].

### 3.2 Operator descent, typed collisions, and impossibility witness

For an admissible transformation $o:H\rightharpoonup H$, an operation $v:A\rightharpoonup A$ descends to the observed representation when $a\circ o = v\circ a$ on the admitted domain, with definedness and authorization preserved. Equivalently, histories in the same observed fiber cannot be separated into different observed outcomes by applying o. A counterexample consists of source-admissible histories $h,h'$ having $a(h)=a(h')$ but different target-relevant effects or different necessary next actions. If there is no common admitted action that succeeds on both histories, no observer restricted to a can reliably select the correct continuation for both. This supplies a precise conditional witness of an observer-relative blind spot, not a universal impossibility claim for all mathematical systems.

$a(h)=a(h')$, but $\kappa(h)\neq\kappa(h')$ $\Rightarrow$ no exact target decoder from a alone. (M2)

An alleged process-class blind spot is separately tested for solution relevance. If the target can be reached by a common action without distinguishing the L-composed and G-governed histories, failure to distinguish their internal processes is not a witness to solution-level blindness. A source-frozen refinement $a^+$ is successful only if it resolves the collision using accessible information, not a representation of the already completed solution.

### 3.3 Correction and functional closure obligations

An interface-repair step registers the deficient representation, its target-relative collision, the accessible source that licenses refinement, the new operator or representation, and the executed comparison showing whether the collision disappears. Subsequent operations must retain the historical assessment, including superseded configurations and residual uncertainty. Functional closure is tested by checking whether the generated representation, operation, validity contract, assessment, and correction record all remain typed operands for further operations on R. Algebraic closure alone is insufficient to establish effective self-correction, successful assessment, or convergence; those are separate obligations.

## 4. Conditional construction and assessment of conjunctions

### 4.1 Conjunction contract and inference rule

The central inference target is a compatible collection of typed propositions $P_1,\ldots,P_n$, each with its exact context $\Gamma_i$, scope $\Omega_i$, evidence record $E_i$, and proof or assessment status. Before composing them, the method constructs a joint contract J containing one explicitly compatible interpretation of all terms, licensed source bridges, a common target, all dependencies, the conjunctive claim $P_1\wedge\cdots\wedge P_n$, and a distinct joint assessment budget $B_j$.

Bibliographic citations can license premises only at the scope actually supported; they cannot substitute for derivations. In particular, survey statistics, formal proof certificates, and an author's retrospective account are different operand types.

Where each proposition has a completed derivation under a compatible $\Gamma$ and the proof system admits conjunction introduction, the local inference is straightforward. If the propositions are only conditionally licensed by sources, the result is the correspondingly conditional theorem under the registered $\Gamma$, not an unconditional proof:

$$\Gamma \vdash P_1, \ldots, \Gamma \vdash P_n \Rightarrow \Gamma \vdash P_1 \wedge \cdots \wedge P_n. \quad \text{(M3)}$$

The global-coherence claim is stronger than mere pairwise acceptability. Let $K_1=\{0,1\}$, $K_2=\{1,2\}$, $K_3=\{0,2\}$. Each pair has a common element, but $K_1 \cap K_2 \cap K_3 = \varnothing$. Thus three locally compatible constraints may lack a global completion. The G assessment of J must evaluate the full conjunction, including cross-claim relationships, semantic bridges, exclusions, and the resource required to execute the assessment. Sheaf-theoretic work on local/global compatibility supplies a useful mathematical analogue without being treated as a proof of this particular GIGT system [19].

### 4.2 Scoped G assessment, budgets, and correction

For a given J, register $B_j$ from R (or BUDGET_UNDETERMINED), execute an admissible G process, and append the verdict, exact scope, assumptions, steps, residuals, and consumed work. An inherited positive result retains its original scope and budget; any assessment of a larger conjunction or at a new budget is a new record with an explicit dependency on the inherited result. A proof that $P_i$ is correct does not license a positive verdict for J without examining interactions. If a typed defeater is found, preserve the objection and correction branch; if assessment cannot be completed, retain the theorem target and record its outstanding proof obligation rather than silently weakening it.

A useful proof-development protocol is therefore: register the target conjunction; decompose into scoped premises; derive direct L results; develop source-licensed G bridges for missing relationships; execute available joint checks; retain all positive certificates; and recursively correct only the defects demonstrated by typed counterexamples or failed obligations. This is an incremental mathematical method, not an assertion that a positive full-scope G verdict has already been obtained.

## 5. Literature-supported premise construction

Seven source-audited propositions (E1–E7) instantiate the seven questions in the draft Methods specification. For each, the operative method is to register an appropriately narrow source claim, its evidentiary type and domain, and its permitted role in a conjunction. This prevents a genuine constituent result from being silently promoted into a universal statement.

### E1. Mathematical artifacts can exceed practical human verification capacity

Heule, Kullmann, and Marek report a computer-assisted Boolean Pythagorean triples proof with a certificate too large for direct human line-by-line review on ordinary research timescales [4]. The recent Alpöge–Claude RH artifact contains roughly 17 million lines of Lean, while the Lean and Mathlib communities report independent machine-based checks of the formal statement and exported proof [1,2]. The supported premise is that there exist proofs whose direct manual inspection is prohibitively costly, and machine acceptance is not itself human reconstruction of their mathematics. This does not prove that every AI-generated long proof is incomprehensible or that its mathematical core cannot be compressed.

### E2. Production–checking throughput can diverge

Zheng compares a modeled 1,000-token/second generator with single-CPU-core Lean compilation on 162,170 AI-generated modules and reports that modeled generation finishes sooner for approximately 95% of the files [5]. This supports a specified workload/throughput mismatch. It does not measure the generator's complete proof-search time, optimized parallel kernel-checking throughput, or a longitudinal increase in the mismatch. Accordingly, the premise is registered with those conditions. For a specified workflow, a backlog grows during an interval if its generation rate exceeds the rate at which the relevant checking contract is completed; this is a conditional accounting identity, not an observed universal trend.

### E3. Concern about the future role of mathematicians

The Martingale Foundation's October 2026 voluntary survey documents significant concern among 262 mathematics students and academics regarding AI's effects on mathematical research and careers [13]. This supports an empirical motivation within the surveyed population and cannot establish the prevalence of those views among all mathematicians. It is not used as a mathematical premise for the necessity of second-order closure.

### E4. Operation self-representation and functional closure

Proof assistants and metaprogramming can represent expressions, partial proof goals, and transformations as objects of computation [3,20]. For the proposed architecture, closure has the specific contract that all permitted compositions of the registered basis operations produce objects still representable and operable within R, including the operations and correction records themselves. Existing formalisms establish constituent representability and closure mechanisms, but do not prove that the particular six-function GIGT/FMA basis is universally minimal, that correction always succeeds, or that energetic viability automatically follows. Those claims remain separate theorem obligations.

### E5. Generator-based compression and bounded understanding

Proof theory demonstrates that introducing reusable intermediate structure can compress derivations, sometimes dramatically relative to fully expanded proofs [17]. Aryal finds markedly lower realized logical compression in the examined AI-generated proofs than in the human-developed comparison corpus [6]. Neither result implies that every mathematical object admits a substantially shorter lossless description. The paper tests a conditional necessity claim: within a strongly typed problem class, if full target-relevant reconstruction exceeds a human evaluator's registered budget while a source-licensed generator plus interface and checking costs fit within it, then any adequate solution route subject to those alternatives requires an information-preserving factorization. A "generator" here must include a reusable operative mathematical relationship; the existence of an arbitrary short encoding is not by itself evidence of mathematical generativity.

### E6. Partial-information generator interfaces avoid a circular universal proof gate

Abstract interpretation and counterexample-guided abstraction refinement allow properties of a restricted model to be assessed and its representation to be refined using counterexamples [14–16]. These provide precedents for certifying a generator under a declared domain and source contract instead of first establishing its adequacy across all mathematical information. The method represents a generator g with accessible input a(h), a domain guard D, a target and a local verification contract; it then checks whether g supplies a needed operator or refinement without using solution-dependent information that is unavailable at the frozen source boundary. Local admissibility does not entail universal completeness. The absence of a universal proof need not preclude a properly scoped, actually certified use.

### E7. Partial information, incompleteness, and the functional first-order label

The universal statement that present mathematics erases all incomplete proofs is not adopted: Lean explicitly retains metavariables, open goals, and local hypotheses during proof construction [20]. Instead, a first-order process is defined functionally, for a specified target, by failure to preserve or make operative the information needed to construct or assess target-relevant G-governed continuations. An incomplete proof may retain valid subproofs, unresolved goals, unsuccessful branches, and future correction potential; a coarse interface that reduces all of this to FAIL may erase relevant distinctions. E7 is thus a target-relative claim about particular observation/assessment architectures, not a claim about every modern proof assistant or first-order logical language.

## 6. Derivation pathways to H1–H8 and the governing conjunction

The hypothesis dependency graph is examined with the following registered routes. "Literature-supported" means that the cited results support specified constituent mechanisms, not that the stronger hypothesis follows without the indicated conjunction assessment.

| Target | Mathematical / empirical inputs | Required assessment |
|---|---|---|
| H1: blind-spot existence | M1–M2 and an explicit same-observation/different-required-continuation witness. | Prove that the witness is admissible, source-accessible, target-relevant, and cannot be resolved by an allowed common action. |
| H2: L/G mechanism | Typed L and G contracts plus a demonstrated need to generate a previously missing relation. | Separate absence of operative G construction from generic insufficiency of data or compute. |
| H3: observability | Source, extent, resolution, historical retention, assessment and work records in R. | Show dependence on those registered functional conditions, not arbitrary node counts. |
| H4: dual closure | E4, resources in R, functional and energetic execution/continuation records; linear logic as resource precedent [18]. | Derive necessity at declared scope; distinguish accounting from guaranteed sufficient work or convergence. |
| H5: positive repair | Source-frozen generator satisfying M1–M2 and typed operator-promotion checks. | Demonstrate an actual new target-relevant operative relation and successful target completion. |
| H6: convergence / SOM-G1 | H5 plus registered selection, stability, correction and viable work hypotheses. | Prove a reliable convergence statement for a nontrivial specified class; no assumed terminal topology. |
| H7: RH compatibility | Alpöge's dated account and timeline; certified Lean statement/proof [1,2]. | Reconstruct only source-evidenced dependency transitions; test compatibility separately from causal attribution and reproducibility. |
| H8: transmission | Target-relative decoder/operation descent; retained provenance and partial derivations. | Demonstrate receiver reconstruction and continuation under a stated information and work budget. |

The paper's central conjunction is formed from H1–H6; H7–H8 test the proposed case-study and transmission consequences. Its joint certificate must assess the compositional bridges, not simply collect individual positive statuses. C1–C5 are generated as reporting targets: C1 from H1 at a stated G scope; C2 from H1–H3 and the mechanism assessment; C3 from a certified H5 example while H6 remains a stronger reliability target; C4 from the bounded H7 compatibility assessment; and C5 from closure, source-preserving correction and H8 reconstruction conditions. These mappings retain both the intended conclusions and any outstanding proof obligations.

### 7. Backward reconstruction of the Alpöge–Claude RH case

The retrospective case is source-bounded and begins with the endpoint identified by the Lean and Mathlib communities: the theorem formalized in Mathlib and its verified proof export [2]. The method traces backwards from that endpoint through recorded assumptions, imports, major intermediate claims, type-correct transformations, and source-reported discovery steps. Alpöge's 20 November note is a primary account of some discovery relations, including a toy setting, a Stepanov-related idea, a transition toward automorphic constructions, and a sequence of theta lifts [1]. Its timeline is a retrospective narrative, not a complete machine-action log and not by itself an executable causal history.

Every reconstructed edge is tagged as one of: directly documented in a source; proved as a logical/proof dependency; a derived structural correspondence; or an unresolved causal conjecture. Where the proof archive is accessible, a separate artifact-level audit may record immutable version hashes, import/dependency structure, theorem identity, trusted axioms, and checked proof terms. Where it is not available or full reconstruction exceeds the current resource budget, the method retains the missing edge and limits its claim accordingly. The community's verification is accepted as documented evidence of its stated formal certification, not as proof of the proposed GIGT discovery mechanism.

A compatibility witness for H7 requires an actual documented transition matching a typed predicted role: e.g., a previously inaccessible relationship was constructed from available sources, promoted to a reusable operator under an explicit contract, and enabled subsequent mathematical continuations. Resemblance of terminology alone does not count. Historical causation would further require evidence that this mechanism actually generated that step;

reliable reproduction would require a new source-frozen execution from comparable information without supplying the resulting proof or completed solution shape. The three verdicts are reported separately.

### 8. Source-frozen generative experiments and negative controls

For H5, designate a nontrivial mathematical problem class, an initial operative representation $R_0$, a frozen set of accessible sources $S_0$, the permitted operations, the target predicate, and a fixed endogenous resource budget. Run two comparably provisioned processes: a fixed-closure baseline unable to promote newly derived operators, and a generative process allowed to create, assess, register and reuse them. Baselines may use the same available facts and computation; the distinction is operative correction of the representation, not arbitrary withholding of solutions from one condition.

A positive source-frozen witness must preserve: (a) a proof that the baseline representation fails the stated target-relative criterion under its declared contract; (b) a trace of a newly generated relationship or operator whose defining information was available in $S_0$; (c) typed source, guard, invariant and budget records; (d) an assessment of the corrected operation and its effect on target reachability; and (e) a target-completing continuation or precisely scoped adequacy certificate. Negative controls include inaccessible private distinctions, misleading apparent relationships, pairwise-but-not-global consistency, and a generator whose output is merely a renamed existing operation. Failure under insufficient sources is retained as an admissible negative or unresolved outcome, not evidence that a correct source-independent solution was overlooked.

Resource and growth outcomes are evaluated in the model's own target-relative functional metrics. Auxiliary diagnostics may include proof size, checking time, number of reusable operator families and observed reconstruction cost, but graph size or raw combinatorial counts cannot replace the proposed F-recursive measures of extent, resolution and adequacy. A combinatorial or exponential increase in the number of expressible constructions is not evidence of an equal improvement in reliable discovery throughput; those are distinct results requiring separately budgeted execution evidence.

### 9. Reporting rules, limitations, and relation to Results

The Methods yields the claim register, source–claim matrix E1–E7, proof-obligation identifiers, joint dependency graph, assessment records and case reconstruction ledger. Each reported proof or G_B assessment records its own premises, scope, budget, executed work, defeating objections and residual obligations.

The immediate formal content is conditional and target-relative. The documentary evidence establishes practical verification and compression concerns, while existing mathematical literature supports constituent methods of abstraction, proof compression, representation refinement and resource sensitivity. It does not, without further derivation, establish the universal necessity or sufficiency of functional and energetic closure, the minimality of exactly six basis operations, reliable SOM-G1 convergence for unrestricted mathematical problems, or GIGT as the historical cause of the RH discovery. The proposed method is designed to advance those specific proof obligations while conserving the already established local mathematics and the information necessary for future correction.