MONOCHROMATIC OR RAINBOW TRIANGLES IN THREE DIMENSIONS

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Abstract. We prove that every coloring of three-dimensional Euclidean space contains, for every prescribed nondegenerate triangle, a congruent copy whose vertices are either all of the same color or all of different colors. The set of colors is arbitrary and can be infinite. The proof is elementary. Its two geometric ingredients are an odd closed sequence obtained by moving among three concentric circles, and a finite sequence of equal-length steps joining any two points of a sphere.

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1. Introduction

1.1. **Main result.** For a positive integer d, write $\mathbb{R}^d$ for d-dimensional Euclidean space. For $x = (x_1, \ldots, x_d)$ and $y = (y_1, \ldots, y_d)$, write

$$x \cdot y = \sum_{j=1}^{d} x_j y_j, \qquad \|x\| = \sqrt{x \cdot x}, \qquad |XY| = \|X - Y\|.$$

Thus $|XY|$ denotes the Euclidean distance between points X and Y. Two vectors are *perpendicular* when their dot product is zero.

A *nondegenerate triangle* is a set of three noncollinear points. Two triangles are *congruent* if their vertices can be matched so that the three corresponding distances agree. Thus congruent triangles have the same size and shape; reflections are allowed.

A *coloring* of $\mathbb{R}^3$ is a function $\chi : \mathbb{R}^3 \to \mathcal{K}$, where $\mathcal{K}$ is an arbitrary nonempty set, called the *set of colors*. A finite set of points is *monochromatic* if all its points have the same color. A triangle is *rainbow* if its three vertices have pairwise different colors. Hence a triangle that is neither monochromatic nor rainbow has exactly two vertex colors.

Theorem 1.1 (Main theorem)**.** *Let T be a nondegenerate triangle, let $\mathcal{K}$ be a nonempty set of colors, and let $\chi : \mathbb{R}^3 \to \mathcal{K}$ be any coloring. Then $\mathbb{R}^3$ contains a congruent copy of T that is either monochromatic or rainbow.*

The set $\mathcal{K}$ need not be finite or countable, and the coloring χ need not be measurable, continuous, or otherwise regular. All color deductions in the proof are finite. Continuity is used only for elementary functions of lengths and angles.

1.2. **Background and prior work.** We recall the terminology needed to place Theorem 1.1 in context. For a positive integer r, an *r-coloring* is a coloring whose set of colors is $\{1, \ldots, r\}$. A finite configuration $A \subset \mathbb{R}^d$ is called *Ramsey* if, for every r, there exists an integer $n = n(A, r)$ such that every r-coloring of $\mathbb{R}^n$ contains a monochromatic congruent copy of A. Erdős, Graham, Montgomery, Rothschild, Spencer and Straus initiated Euclidean Ramsey theory in a series of papers beginning with [E+73]; for surveys, see Graham [Gra94]. Among their basic results, every two-coloring of $\mathbb{R}^3$ contains a monochromatic congruent copy of every three-point configuration [E+73, Thm. 8]. Thus Theorem 1.1 reduces to a classical theorem when $|\mathcal{K}| = 2$.

The high-dimensional theory developed rapidly. Frankl and Rödl proved that every triangle is Ramsey [FR86]. They later proved the corresponding statement for every nondegenerate simplex [FR90], where a *nondegenerate d-simplex* means a set of $d+1$ points not contained in an affine

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hyperplane of dimension less than d. We call a triangle *acute*, *right*, or *obtuse* when all its angles are less than $\pi/2$, one angle equals $\pi/2$, or one angle is greater than $\pi/2$, respectively. These results allow the ambient dimension to depend on the number r of colors. In fixed dimension three, Bóna proved that every three-coloring contains a monochromatic copy of every prescribed 30°–60°–90° triangle [Bon93], and Bóna and Tóth extended this to every right triangle [BT96]. Bóna also gave a twelve-coloring of $\mathbb{R}^3$ with no monochromatic copy of the prescribed 30°–60°–90° triangle [Bon93]; this illustrates why a fixed-dimensional theorem for arbitrarily many colors needs a conclusion different from monochromaticity alone.

The relevant conclusion is the monochromatic–rainbow alternative. Following Gehér, Sagdeev and Tóth [GST25], a finite configuration A is called *canonically Ramsey* if there exists an integer $n_0 = n_0(A)$ such that, for every positive integer r and every $n \geq n_0$, every r-coloring of $\mathbb{R}^n$ contains a congruent copy of A that is monochromatic or rainbow. Thus the ambient dimension is independent of the number r of colors. Some results, including Theorem 1.1, hold in the stronger form in which the coloring may use an arbitrary set of colors.

Mao, Ozeki and Wang introduced the Euclidean Gallai–Ramsey problem, which studies the monochromatic–rainbow alternative in Euclidean space [MOW22]. Cheng and Xu proved that every right triangle has the required property already in $\mathbb{R}^3$ for every finite number of colors, and asked whether the same holds for all triangles [CX25, Thm. 1.2 and §5]. Gehér, Sagdeev and Tóth proved the stronger arbitrary-color statement in $\mathbb{R}^3$ for every acute triangle [GST25, Thm. 1.2].

For arbitrary triangles, Fang, Ge, Shu, Xu, Xu and Yang proved the monochromatic–rainbow statement in $\mathbb{R}^4$ for every finite set of colors and asked whether the dimension could be reduced to three [F+25, Thm. 1.2 and §5]. Ge, Shu, Xu and Yu subsequently proved that every nondegenerate simplex is canonically Ramsey in some dimension [GSXY26]. Very recently, Sagdeev announced the more general high-dimensional statement that every Ramsey configuration is canonically Ramsey [Sag26]. Theorem 1.1 is a fixed-dimensional result: it gives a single argument in $\mathbb{R}^3$ for every triangle and every set of colors. In particular, it settles the remaining obtuse-triangle case in dimension three.

The corresponding planar questions remain different. Shader proved that every right triangle is Ramsey in $\mathbb{R}^2$ for two colors [Sha76], while the classical conjecture that every non-equilateral triangle is Ramsey in the plane remains open. Rauch-Müller and Sagdeev recently proved this conjecture for a new family of triangles and, in the canonical setting, proved the monochromatic–rainbow conclusion for thirty-six specific triangles [RMS26]. The ambient dimension in Theorem 1.1 cannot be reduced to two for a statement covering all triangles, since a prescribed equilateral triangle is not Ramsey in the plane [E+73, Sha76].

1.3. **Proof outline.** Assume, for a contradiction, that every congruent copy of T uses exactly two colors. Lemma 2.1 produces a small distance $\varepsilon > 0$ and an odd closed sequence of copies of T. This implies a strong color restriction for points that are at the same prescribed distance from two differently colored points separated by ε. Lemma 3.1 then shows that any two points on a sphere can be joined by finitely many steps of length ε. The color restriction therefore propagates one color across an entire sphere. Finally, equal-length steps propagate that color throughout $\mathbb{R}^3$, giving a contradiction.

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2. An odd closed sequence of triangles

A sequence of points $X_0, X_1, \ldots, X_N$ has N *steps*, one from X_i to X_{i+1} for each $i = 0, \ldots, N-1$. It is *closed* when $X_N = X_0$. Points in the sequence may be repeated. In the next lemma, consecutive points are distinct because they belong to a nondegenerate triangle.

Lemma 2.1. *Let T be a nondegenerate triangle with side lengths $a, b, c > 0$. There exists a real number*

$$0 < \varepsilon < \min\{a, b, c\}$$

with the following property. Let $A, B \in \mathbb{R}^3$ satisfy $|AB| = \varepsilon$, let s be any one of a, b, c, and let $X \in \mathbb{R}^3$ satisfy $|XA| = |XB| = s$. Then there are a positive odd integer N and points

$$X_0 = X,\ X_1, \ldots, X_N = X$$

such that, for every $i = 0, \ldots, N-1$, both triangles AX_iX_{i+1} and BX_iX_{i+1} are congruent to T. The number ε depends only on the side lengths of T.

Proof. The construction uses three circles with a common centre, lying in the plane halfway between A and B and perpendicular to the line AB. We first choose their radii and the angle of each step so that an odd number of steps returns to the starting point.

Choosing the radii and angles. All angles in this proof are measured in radians, so a full turn is 2π. For $-1 < u < 1$, the notation $\arccos u$ denotes the unique angle in $(0, \pi)$ with cosine u.

For a real parameter z with $0 \le z < \min\{a^2, b^2, c^2\}$, set

$$r_a(z) = \sqrt{a^2 - z}, \qquad r_b(z) = \sqrt{b^2 - z}, \qquad r_c(z) = \sqrt{c^2 - z}.$$

These will be the three circle radii. At $z = 0$ they equal a, b, c. The strict triangle inequalities for a, b, c, together with continuity of the square root, allow us to choose a real number

$$0 < \eta < \tfrac{1}{4} \min\{a^2, b^2, c^2\}$$

such that, for every z in the interval $[0, \eta]$,

$$\begin{aligned} |r_a(z) - r_b(z)| &< c < r_a(z) + r_b(z), \\ |r_b(z) - r_c(z)| &< a < r_b(z) + r_c(z), \\ |r_c(z) - r_a(z)| &< b < r_c(z) + r_a(z). \end{aligned} \tag{2.1}$$

To justify this choice, each of these finitely many strict inequalities holds at zero, and therefore continues to hold in a sufficiently small interval starting at zero.

For $0 \le z \le \eta$, define

$$\begin{aligned} \theta_{ab}(z) &= \arccos\left(\frac{r_a(z)^2 + r_b(z)^2 - c^2}{2r_a(z)r_b(z)}\right), \\ \theta_{bc}(z) &= \arccos\left(\frac{r_b(z)^2 + r_c(z)^2 - a^2}{2r_b(z)r_c(z)}\right), \\ \theta_{ca}(z) &= \arccos\left(\frac{r_c(z)^2 + r_a(z)^2 - b^2}{2r_c(z)r_a(z)}\right). \end{aligned}$$

The cosine rule says that two vectors of lengths r, u meeting at angle θ have endpoints at squared distance $r^2 + u^2 - 2ru\cos\theta$. Thus $\theta_{ab}(z)$ is the angle needed for a step of length c between the circles of radii $r_a(z)$ and $r_b(z)$. The other two definitions have the same interpretation. Inequalities (2.1) ensure that each fraction lies strictly between -1 and 1. The three angles are consequently well defined, lie in $(0, \pi)$, and depend continuously on z.

Let

$$S(z) = \theta_{ab}(z) + \theta_{bc}(z) + \theta_{ca}(z).$$

At $z = 0$, the three terms are the three angles of T, so $S(0) = \pi$. We next show that

$$S(z) > \pi \qquad \text{for every } 0 < z \le \eta. \tag{2.2}$$

Fix such a z. The numbers $r_a(z), r_b(z), r_c(z)$ are themselves the side lengths of a nondegenerate triangle: for example, $r_a(z) + r_b(z) > c > r_c(z)$, and the other two sums satisfy the corresponding inequalities. Denote the angles opposite these three sides by ϕ_a, ϕ_b, ϕ_c, respectively. Their sum is π. Since $c^2 = r_c(z)^2 + z$, the cosine rule gives

$$\cos\theta_{ab}(z) = \cos\phi_c - \frac{z}{2r_a(z)r_b(z)} < \cos\phi_c.$$

Cosine is strictly decreasing on $(0, \pi)$, so $\theta_{ab}(z) > \phi_c$. The corresponding comparisons give $\theta_{bc}(z) > \phi_a$ and $\theta_{ca}(z) > \phi_b$. Adding proves (2.2).

Making an odd number of steps close up. Choose an odd integer $q \ge 3$ so large that

$$\frac{\pi}{q} < S(\eta) - \pi.$$

The function S is continuous, starts at π, and at η exceeds $\pi + \pi/q$. The intermediate value theorem, which says that a continuous function takes every value between two of its values, therefore gives a number $z_0 \in (0, \eta)$ with

$$S(z_0) = \pi + \frac{\pi}{q}. \tag{2.3}$$

Set

$$t = \sqrt{z_0}, \qquad \varepsilon = 2t.$$

The choice of η gives $0 < \varepsilon < \min\{a, b, c\}$. This fixes ε using only a, b, c.

Now let A, B be any points with $|AB| = \varepsilon$. By translating and rotating coordinates, we may write

$$A = (0, 0, t), \qquad B = (0, 0, -t).$$

For $s \in \{a, b, c\}$, the points at distance s from both A and B form the circle

$$\mathcal{C}_s = \big\{(r_s(z_0)\cos\psi, r_s(z_0)\sin\psi, 0) : \psi \in \mathbb{R}\big\}.$$

Indeed, subtracting the two squared distance equations shows that the third coordinate is zero; either equation then gives squared radius $s^2 - t^2$. The parameter ψ specifies the angle around the circle. If two side lengths of T are equal, the corresponding circles coincide; the construction still uses their indicated places in the sequence.

First take a starting point X on $\mathcal{C}_a$. Beginning at X, move successively through the circles

$$\mathcal{C}_a,\ \mathcal{C}_b,\ \mathcal{C}_c,\ \mathcal{C}_a, \ldots$$

and repeat this order. At each step increase ψ as follows:

step	increase in ψ	distance between the points
$\mathcal{C}_a$ to $\mathcal{C}_b$	$\theta_{ab}(z_0)$	c
$\mathcal{C}_b$ to $\mathcal{C}_c$	$\theta_{bc}(z_0)$	a
$\mathcal{C}_c$ to $\mathcal{C}_a$	$\theta_{ca}(z_0)$	b

The last column follows directly from the definitions of the angles. For example, if $U \in \mathcal{C}_a$ and $V \in \mathcal{C}_b$ are consecutive points, then

$$|UV|^2 = r_a(z_0)^2 + r_b(z_0)^2 - 2r_a(z_0)r_b(z_0)\cos\theta_{ab}(z_0) = c^2.$$

Also $|AU| = |BU| = a$ and $|AV| = |BV| = b$. Hence both AUV and BUV have side lengths a, b, c and are congruent to T. The other two types of step give the same conclusion.

Every three steps increase ψ by $S(z_0)$. After $3q$ steps the circle label returns to a, and the total increase is

$$qS(z_0) = (q+1)\pi = 2\pi\left(\frac{q+1}{2}\right).$$

Since q is odd, $(q+1)/2$ is an integer. Thus the final point is exactly X, and the number $N = 3q$ of steps is odd. Starting instead with the order $\mathcal{C}_b, \mathcal{C}_c, \mathcal{C}_a$ or $\mathcal{C}_c, \mathcal{C}_a, \mathcal{C}_b$ proves the assertion for the other starting circles. Consecutive points are distinct because their distance is one of the positive numbers a, b, c. Repetition of nonconsecutive points is permitted, as required by the statement. □

3. Equal-length steps on a sphere

For a point $P \in \mathbb{R}^3$ and a real number $s > 0$, write

$$\mathcal{S}_s(P) = \{X \in \mathbb{R}^3 : |XP| = s\}$$

for the *sphere* with centre P and radius s. This denotes the surface, not the solid ball. A plane through P cuts the sphere in a circle of radius s, called a *great circle*.

Lemma 3.1. *Let $P \in \mathbb{R}^3$ and let s, h be real numbers with $0 < h < s$. For any two points $U, V \in \mathcal{S}_s(P)$, there are a nonnegative integer m and points $Y_0, \ldots, Y_m \in \mathcal{S}_s(P)$ such that*

$$Y_0 = U, \qquad Y_m = V, \qquad |Y_{i+1}Y_i| = h \quad (0 \le i < m).$$

When $U = V$, one may take $m = 0$.

Proof. Translate coordinates so that $P = 0$. We first show that any two distinct points U, V on the sphere with $|UV| < h$ can be joined by *two* steps of length h, staying on the sphere.

Put

$$d = |UV|, \qquad M = \frac{U+V}{2}, \qquad \rho = \|M\| = \sqrt{s^2 - d^2/4}.$$

Here $0 < d < h < s$, so $\rho > 0$. Expanding the dot products, using $\|U\| = \|V\| = s$, gives

$$M \cdot (U - V) = 0, \qquad M \cdot U = \rho^2.$$

Consider the plane

$$H = \{W \in \mathbb{R}^3 : W \cdot (U - V) = 0\}.$$

It passes through 0 and M and is perpendicular to $U - V$. Every $W \in H$ is equidistant from U and V, since

$$|WU|^2 - |WV|^2 = -2W \cdot (U - V) = 0.$$

The intersection of H with the sphere is a great circle. It contains the two opposite points

$$W_+ = \frac{s}{\rho} M, \qquad W_- = -\frac{s}{\rho} M.$$

The first is closer than h to U, because

$$|W_+U|^2 = 2s(s - \rho) = \frac{s\, d^2}{2(s + \rho)} < d^2 < h^2.$$

The second is farther than h from U, because

$$|W_-U|^2 = 2s(s + \rho) > 2s^2 > h^2.$$

Move along either semicircle in H from W_+ to W_-. Distance to U varies continuously, so the intermediate value theorem supplies a point W on this semicircle with $|WU| = h$. Since $W \in H$, it also satisfies $|WV| = h$. Thus U, W, V is the required two-step sequence, with all three points on the sphere.

To join arbitrary distinct U, V on the sphere, choose a great circle containing them. Such a circle comes from a plane through $0, U, V$; if U and V are opposite, choose any plane through their line. Take an arc joining them of length at most πs, choosing either semicircle for opposite points. Choose a positive integer $n > \pi s/h$ and divide this arc into n equal pieces. The straight distance between the endpoints of each piece is at most its arc length, and is therefore less than h. Replace each piece by the two-step sequence just constructed. This gives $2n$ steps of length exactly h. When $U = V$, no steps are needed. □

4. Proof of the main theorem

Proof of Theorem 1.1. Fix a nondegenerate triangle T, a nonempty set of colors $\mathcal{K}$, and a coloring $\chi : \mathbb{R}^3 \to \mathcal{K}$. Suppose that no copy of T is monochromatic or rainbow. Consequently every copy of T has exactly two vertex colors. Write a, b, c for the side lengths of T, and fix the number ε supplied by Lemma 2.1. In particular, $0 < \varepsilon < s$ for each $s \in \{a, b, c\}$.

Step 1: a restriction imposed by two different colors. We will show that whenever points $A, B, X \in \mathbb{R}^3$ and a length $s \in \{a, b, c\}$ satisfy

$$|AB| = \varepsilon, \qquad \chi(A) \neq \chi(B), \qquad |XA| = |XB| = s,$$

then

$$\chi(X) \notin \{\chi(A), \chi(B)\}. \tag{4.1}$$

Put $\alpha = \chi(A)$ and $\beta = \chi(B)$, so $\alpha \neq \beta$. Lemma 2.1 supplies a closed sequence $X_0 = X, X_1, \ldots, X_N = X$ with N odd, such that both AX_iX_{i+1} and BX_iX_{i+1} are copies of T at each step.

If $\chi(X_i) = \alpha$, the triangle AX_iX_{i+1} cannot be monochromatic, so $\chi(X_{i+1}) \neq \alpha$. The triangle BX_iX_{i+1} cannot be rainbow, so $\chi(X_{i+1})$ must be α or β. It must therefore be β. Interchanging A and B shows that a color β must be followed by α. Hence, if X_0 had either of these colors, they would alternate along the whole sequence. An odd number of steps would then give X_N a different color from X_0, contrary to $X_N = X_0$. This proves (4.1).

Step 2: two points of the same color give a sphere of that color. Let $P, Q \in \mathbb{R}^3$ satisfy $|PQ| = s \in \{a, b, c\}$ and $\chi(P) = \chi(Q) = \gamma$, where $\gamma \in \mathcal{K}$ is their common color. We show that every point of $\mathcal{S}_s(P)$ has color γ.

Take $U, V \in \mathcal{S}_s(P)$ with $|UV| = \varepsilon$ and $\chi(U) = \gamma$. If $\chi(V) \neq \gamma$, apply Step 1 with $A = U$, $B = V$, and $X = P$. The required equal distances are $|PU| = |PV| = s$. Conclusion (4.1) would give $\chi(P) \neq \chi(U)$, a contradiction. We have proved that

$$\begin{aligned} U, V \in \mathcal{S}_s(P), \quad |UV| = \varepsilon, \quad \chi(U) = \gamma \\ \Longrightarrow \quad \chi(V) = \gamma. \end{aligned} \tag{4.2}$$

The point Q lies on $\mathcal{S}_s(P)$ and already has color γ. Since $0 < \varepsilon < s$, Lemma 3.1, with $h = \varepsilon$, joins Q to any specified point of this sphere by finitely many steps of length ε. Applying (4.2) at each step proves that the entire sphere has color γ. The argument applies to *every* pair of points of the same color at a distance $s \in \{a, b, c\}$.

Step 3: the color spreads to all of space. Choose any copy of T. It uses exactly two colors, so two of its vertices, say P, Q, have the same color γ. Their distance $s = |PQ|$ is one of a, b, c. Step 2 makes $\mathcal{S}_s(P)$ entirely of color γ.

We first explain why every point $Z \in \mathbb{R}^3$ can be reached from P by finitely many steps of length exactly s. If $Z = P$, no steps are needed. Otherwise divide the segment from P to Z into finitely many positive-length pieces, each of length at most $2s$. For endpoints D, E of one such piece, put $d_0 = |DE|$, so $0 < d_0 \leq 2s$, and choose a vector e of norm one perpendicular to $E - D$. The point

$$F = \frac{D + E}{2} + \sqrt{s^2 - d_0^2/4}\, e$$

satisfies $|DF| = |FE| = s$, by the Pythagorean theorem. Replacing each piece DE by the two steps D, F, E gives, for some nonnegative integer m, a finite sequence

$$Z_0 = P, \ Z_1, \ldots, Z_m = Z, \qquad |Z_{i+1}Z_i| = s \quad (0 \leq i < m).$$

If $Z = P$, its color is already γ. Otherwise $m \geq 1$, and we follow the sequence. The point Z_1 belongs to the already monochromatic sphere $\mathcal{S}_s(P)$, so it has color γ. Suppose that $1 \leq i < m$ and that Z_{i-1} and Z_i both have color γ. Applying Step 2 to the pair Z_i, Z_{i-1} shows that $\mathcal{S}_s(Z_i)$

has color γ. Since Z_{i+1} lies on this sphere, it too has color γ. Induction gives $\chi(Z) = \gamma$. The point Z was arbitrary, so every point of $\mathbb{R}^3$ has color γ.

The chosen copy of T is therefore monochromatic, contradicting the assumption at the beginning of the proof. This proves the theorem. □

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