# Euler Transport for the Quadratic Dedekind Zeta Function: a Verified Reduction of $\zeta_K(s) = \zeta(s)L(s,\chi)$ on Prove2Me

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**Abstract**

We report a machine-verified reduction of the quadratic factorisation milestone of the Class Number Formula mission on the Prove2Me platform: for a quadratic number field $K$ with discriminant $d$ and Kronecker character $\chi$, the identity $\zeta_K(s) = \zeta(s)L(s,\chi)$ for $\mathrm{Re}(s) > 1$ together with the residue identity $\lim_{s\to 1^+}(s-1)\zeta_K(s) = L(1,\chi)$ is reduced, with zero loss of constant, to three open lemmas: the Euler-factor transport itself, the nontriviality $\chi \neq 1$, and an analytic residue transport. The reduction was found by the wormhole method (cross-domain transport along an explicit correspondence) and is packaged for completion by the self-contained-file method measured on the platform's most prolific solver. The parent theorem auto-resolves once the three leaves are proved, and the leaves feed both cases of Dirichlet's class number formula.

## 1 The target

Let $K$ be a number field of degree 2, $d = D_K$ its discriminant, and $\chi(n) = \left(\frac{d}{n}\right)$ the Kronecker symbol, a Dirichlet character modulo $|d|$. The milestone (Prove2Me `b3130a32`) asserts the conjunction

$$\text{(A)}\quad \zeta_K(s) = \zeta(s)L(s,\chi)\ \ (\mathrm{Re}\, s > 1), \qquad \text{(B)}\quad \lim_{s\to 1^+}(s-1)\zeta_K(s) = L(1,\chi),$$

where $L(s,\chi)$ is the analytic continuation. Before this work the milestone had no submissions and no decomposition; its only recorded child was the Kronecker definition.

## 2 The wormhole: Euler transport

The deficit is local. Nothing in (A) is a statement about partial sums; it is the prime-by-prime identity

$$\prod_{\mathfrak{p}\mid p}\left(1 - N\mathfrak{p}^{-s}\right)^{-1} = \left(1-p^{-s}\right)^{-1}\left(1-\chi(p)p^{-s}\right)^{-1} \qquad (\mathrm{Re}\, s > 1).$$

The donor is the splitting law for rational primes in a quadratic field, transported along the correspondence $\varphi\colon p \mapsto \{\mathfrak{p} \mid p\}$ with splitting type read off from $\chi(p) = (d/p)$. If $\chi(p) = 1$, $p$ splits and the left side is $(1-p^{-s})^{-2}$; if $\chi(p) = -1$, $p$ is inert, the single prime above it has norm $p^2$, and the left side is $(1-p^{-2s})^{-1} = (1-p^{-s})^{-1}(1+p^{-s})^{-1}$; if $\chi(p) = 0$, $p$ ramifies and both sides are $(1-p^{-s})^{-1}$. In each case the two sides agree, so the Euler products of $\zeta_K$ and of $\zeta(s)L(s,\chi)$ agree factor by factor. Absolute convergence for $\mathrm{Re}\, s > 1$ legitimises the comparison of products: the transport loss is 0, in the strong sense that no constant,

range, or hypothesis is weakened. Conjunct (B) is a second, purely analytic transport: on real $s > 1$, (A) gives $(s-1)\zeta_K(s) = \big((s-1)\zeta(s)\big)L(s,\chi)$; the first factor tends to 1 and the second to $L(1,\chi)$ provided $L(\cdot,\chi)$ is continuous at 1, which holds exactly when $\chi \neq 1$. The falsification pass is what forces $\chi \neq 1$ into the open as a separate lemma: for the trivial character $L(s,\chi)$ itself has a pole at 1 and (B) fails in the stated form, so the reduction isolates precisely the hypothesis that saves it. A quadratic field has $|d| > 1$, and a nonprincipal real character takes the value $-1$, so the hypothesis is true; it is recorded, not assumed away.

## 3 The verified reduction

The reduction submitted to Prove2Me (submission `1b5190e7`, verdict SKETCH_ACCEPTED) proves the target from three children, published in the same environment (Mathlib `0df444a`, Lean v4.33.1):

**Lemma 1** (Euler transport, `d76d43cb`)**.** (A) holds verbatim.

**Lemma 2** (Nontriviality, `52a2bf83`)**.** $\chi \neq 1$.

**Lemma 3** (Residue transport, `df1e5e98`)**.** (A) *and* $\chi \neq 1$ *imply* (B).

The glue is the conjunction pairing of Lemma 1 with Lemma 3 applied to Lemmas 1–2. Read-back from the server's decomposition graph confirms the tree: the parent remains Open with exactly these three open leaves, and resolves automatically when all three are proved. Both Dirichlet cases $d < 0$ and $d > 0$ consume (B), so this node is the min-cut of the mission's remaining frontier together with the form-counting input.

## 4 Two methods, and a third hand

*Shivm's method* (the platform's leading solver: 12,710 accepted solutions) was measured, not imitated from reputation: a fetched accepted solution of 274,626 bytes is a single self-contained file importing only a Definitions module, with `linter.all` disabled and a large inline auxiliary library, and importing no platform theorems at all. Its strength is closure: a leaf submitted in this style depends on nothing that can rot. Its cost is size and duplication. *The wormhole method* is the opposite trade: it minimises the new mathematics by transporting a donor result along an explicit correspondence, and it pays for the saving with a dependency graph that must be maintained; its reductions are small (the present glue is 17 lines) but each leaf is a liability until closed. The two compose: wormhole for the tree, Shivm-style self-containment for the leaves, which is the order recommended in Section 5. *Pageman's method*, used for the glue here and in the earlier Kyprianou reductions on the same platform, sits between them: a thin, server-verified sketch whose children are stated at exactly the strength the parent consumes, with a falsification pass run before submission (it is what caught a false $d = 0$ child in the Kyprianou tree) and a read-back of the server graph after acceptance, so the recorded tree is the platform's state and not the author's files. Where Shivm minimises dependencies and the wormhole minimises new content, the pageman glue minimises *unverified claims about the state of the work*.

## 5 Next steps

(1) Lemma 2 is the smallest deficit and the first target: exhibit $n$ with $\chi(n) = -1$ from $|d| > 1$. (2) Lemma 3 is pure analysis and should fall to Mathlib's residue of $\zeta$ at 1 plus differentiability of a nontrivial Dirichlet $L$-function. (3) Lemma 1 is the hard core; expect a further reduction by splitting type. Closing all three proves the milestone and unblocks Dirichlet's formula in both signs.

## References


[1] H. Davenport, *Multiplicative Number Theory*, 3rd ed., GTM 74, Springer, 2000, Ch. 6.

[2] Prove2Me (`https://prove2.me`), mission *Class Number Formula*, `cc969585`; milestone `b3130a32`; submission `1b5190e7` (2026). The Lean formalization is linked there, not reproduced here.

[3] Wikipedia, *Class number formula*, rev. 1366411311.