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WILSON PRIMES FOR THE RING OF FUNCTIONS REGULAR OUTSIDE A RATIONAL POINT OF A CURVE

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Abstract. Let C be a smooth projective curve over $\mathbb{F}_q$, ∞ a rational point and A the ring of functions regular outside ∞. For a basis of A adapted to the pole order at ∞ we consider the Wilson product W_d of [3, 5] and prove, without any plane model, that for every prime element π of degree d

$$W_d \equiv -1 + \pi\,\frac{\Theta_\pi}{(d\pi)^2} \pmod{\pi^2}, \qquad \Theta_\pi = \sum_j dm_j \cdot \nu_j,$$

where (m_j) is the basis of the space of representatives and (ν_j) the dual basis of differentials for the residue pairing. We then show that, for a Cauchy kernel κ on $C \times C$ of order at most $2g-1$ (one always exists), $\Theta_\pi \equiv E_\kappa(\pi) + K_\pi\, d\pi$, where E_κ is a second order differential operator attached to κ and K_π a differential defined by a decomposition involving at most g basis elements. This contains Thakur's criterion for $\mathbb{F}_q[t]$ and the theorems of [3, 5] for plane curves. As a case not covered by these, we treat a hyperelliptic curve in odd characteristic with ∞ an affine rational point where $y \neq 0$, with an explicit kernel; there the correction K_π is a single differential.

1. Setting and statements

Let $k = \mathbb{F}_q$ of characteristic p, C a smooth projective geometrically integral curve over k of genus g, $\infty \in C(k)$, and A the ring of functions on C regular outside ∞, a Dedekind domain with $A^\times = k^\times$. Put $\deg = -\operatorname{ord}_\infty$ on A, and let $S = \{\deg u : u \in A \setminus 0\}$ be the Weierstrass semigroup at ∞; $\mathbb{N} \setminus S$ has g elements, all at most $2g-1$. A prime element is a $\pi \in A$ with A/π a field; its degree is $d = \dim_k A/\pi = \deg \pi$. This is the setting of the question of Thakur [8, Section 7, Question (2)] on Wilson primes outside $\mathbb{F}_q[t]$ [6, 7].

Definition 1. An *adapted basis* of A is a family $(m_s)_{s\in S}$ with $\deg m_s = s$. Given one, for $d \in S$, $d \geq 1$, V_d is the k-span of the m_s with $s - d \notin S$, and $W_d = \prod_{v \in V_d \setminus 0} v$. A prime element π of degree d is a *Wilson prime* (for this basis) if $W_d \equiv -1 \pmod{\pi^2}$.

As in [5, Lemma 3], $A = V_d \oplus uA$ for every $u \in A$ of degree d; so $V_d \to A/\pi$ is bijective and $W_d \equiv -1 \pmod{\pi}$. For the monomial basis of a plane model this is the definition of [1, 2, 3, 5]. The space V_d depends on the adapted basis.

Let $\Omega = \Omega_{A/k}$ be the module of differentials regular outside ∞; it is an invertible A-module, not free in general. For a prime element π put $\mathbf{F}_\pi = A/\pi$ and

$$\langle u, \nu \rangle = -\operatorname{Res}_\infty(u\,\nu/\pi) \qquad (u \in A,\ \nu \in \Omega).$$

Lemma 2. *$\langle u, \nu \rangle = \operatorname{Tr}_{\mathbf{F}_\pi/k}\big(\overline{u} \cdot \overline{\nu/d\pi}\big)$, where $\nu/d\pi$ is a function regular at the zeros of π. The pairing induces a perfect pairing $\mathbf{F}_\pi \times \Omega/\pi\Omega \to k$, and $\langle uv, \nu \rangle = \langle u, v\nu \rangle$.*

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Theorem 3. *Let π be a prime element of degree d, $(m_j)_{1\leq j\leq d}$ the basis of V_d formed by the m_s with $s-d\notin S$, and (ν_j) the dual basis of $\Omega/\pi\Omega$: $\langle m_i,\nu_j\rangle=\delta_{ij}$. Put*

$$\Theta_\pi=\sum_{j=1}^{d} dm_j\cdot\nu_j \;\in\; \Omega^{\otimes 2}/\pi\Omega^{\otimes 2}.$$

Then

$$W_d\equiv -1+\pi\,\frac{\Theta_\pi}{(d\pi)^2}\pmod{\pi^2},$$

the quotient being a well defined element of $\mathbf{F}_\pi$. In particular π is a Wilson prime if and only if $\Theta_\pi=0$, i.e. the quadratic differential $\sum_j dm_j\cdot\nu_j$ vanishes at the zeros of π.

Definition 4. A *Cauchy kernel* for (C,∞) is a rational section κ of $\mathcal{O}\boxtimes\Omega_C$ on $C\times C$ (a function in the first variable, a differential in the second) which is regular on $(C\setminus\infty)^2$ outside the diagonal Δ and has along Δ a simple pole with $\mathrm{Res}_{P_2=P_1}\kappa(P_1,P_2)=-1$. Its *order* is its pole order along $\infty\times C$.

Lemma 5. *There is a Cauchy kernel of order at most $2g-1$, with at most a simple pole along $C\times\infty$.*

Let μ denote the multiplication maps $A\otimes\Omega\to\Omega$ and $\Omega\otimes\Omega\to\Omega^{\otimes 2}$, $d_1=d\otimes 1$, and $u_1=u\otimes 1$, $u_2=1\otimes u$. For $u\in A$, $(u_1-u_2)\kappa$ is regular on $(C\setminus\infty)^2$, i.e. an element of $A\otimes_k\Omega$. Define

$$E_\kappa(u)=\mu(d_1((u_1-u_2)\kappa))\in\Omega^{\otimes 2}.$$

Lemma 6. *$\mu((u_1-u_2)\kappa)=du$. In a local coordinate z at a point of $C\setminus\infty$, write $\kappa=(\frac{1}{z_1-z_2}+a(z_2)+O(z_1-z_2))dz_2$; then $E_\kappa(u)=(\partial_z^{(2)}u+a\,\partial_z u)dz^2$, with $\partial_z^{(2)}$ the second Hasse derivative. Hence $E_\kappa(uv)=uE_\kappa(v)+vE_\kappa(u)+du\cdot dv$, and for another Cauchy kernel κ', $E_{\kappa'}(u)-E_\kappa(u)=du\cdot\mu(\kappa'-\kappa)$ with $\kappa'-\kappa\in A\otimes\Omega$.*

For a prime element π of degree d let $R_0=(\pi_1-\pi_2)\kappa\in A\otimes\Omega$. As $A\otimes\Omega=(V_d\otimes\Omega)\oplus(\pi A\otimes\Omega)$, there are unique $R\in V_d\otimes\Omega$ and $\varkappa\in A\otimes\Omega$ with

$$R_0=R+\pi_1\varkappa;\qquad \text{put}\quad K_\pi=-\mu(\varkappa)\in\Omega. \tag{1}$$

Theorem 7. *With these notations, $\Theta_\pi\equiv E_\kappa(\pi)+K_\pi\cdot d\pi\pmod{\pi}$, so that*

$$W_d\equiv -1+\pi\,\frac{E_\kappa(\pi)+K_\pi\,d\pi}{(d\pi)^2}\pmod{\pi^2}.$$

If κ has order N, then $\varkappa=\sum_{s\in S,\ s\leq N} m_s\otimes c_s$ with $c_s\in\Omega$; for $N\leq 2g-1$ this sum has at most g terms. If $R_0\in V_d\otimes\Omega$, then $K_\pi=0$.

Example 8 (The line)**.** $A=k[t]$, $m_s=t^s$, $\kappa=dt_2/(t_1-t_2)$ of order -1. Then $R_0=\frac{\pi(t_1)-\pi(t_2)}{t_1-t_2}dt_2\in V_d\otimes\Omega$, $K_\pi=0$ and $E_\kappa(\pi)=\pi^{[2]}dt^2$: $W_d\equiv -1+\pi\,\pi^{[2]}/\pi'^2$, and π is a Wilson prime if and only if $\pi^{[2]}=0$, which is [7, Theorem 2.9].

Example 9 (Plane curves)**.** For $A=k[x,y]/(F)$ as in [5], with $\omega=dx/F_y$ and G the image of $(F(x_1,y_1)-F(x_1,y_2))/(y_1-y_2)$, the section $\kappa=G\,\omega_2/(x_1-x_2)$ is a Cauchy kernel of order $2g-1$; then $E_\kappa(u)=E(u)\,\omega^2$ and K_π is the element $K_\pi\omega$ of [5], so Theorem 7 is [5, Theorem 4 and Proposition 5].

A case not covered by the monomial models. Let p be odd, $f \in k[x]$ squarefree of degree $2g+1$, $g \geq 1$, C the curve $y^2 = f(x)$, and $P_0 = (0, y_0) \in C(k)$ with $y_0 \neq 0$ (any affine rational point with $y \neq 0$, after a translation). Take for ∞ the point P_0, so that A is the ring of functions regular outside P_0. Here $S = \{0, g+1, g+2, \dots\}$: indeed x is a uniformiser at P_0, the holomorphic differentials are the $h(x)\,dx/y$ with $\deg h \leq g-1$, those vanishing to order $n \leq g$ at P_0 form a space of dimension $g-n$, and so $\dim L(nP_0) = 1$ for $n \leq g$ by Riemann–Roch. For $g \geq 2$ this semigroup is not generated by two elements, so that A has no monomial basis of the type used in [5]. Let $\hat{y} \in k[[x]]$ be the expansion of y at P_0 ($\hat{y}(0) = y_0$), $T_i \in k[x]$ its truncation at degree i, $\omega = dx/(2y)$, and D the derivation $2y\,d/dx$ of the function field, so that $du = D(u)\,\omega$.

Proposition 10. *The section*

$$\kappa = \Big[\frac{y_1+y_2}{x_1-x_2} + \frac{y_0+y_2}{x_2} - \sum_{i=0}^{g-1} x_2^i\,\frac{y_1+T_i(x_1)}{x_1^{i+1}}\Big]\omega_2$$

is a Cauchy kernel for (C, P_0) of order g, with a simple pole along $C \times P_0$. For it,

$$E_\kappa(u) = (\tfrac{1}{2}D^2u + B\cdot Du)\omega^2, \qquad B = \frac{y_0 - (g-1)\,y - \sum_{i=0}^{g-1} T_i(x)}{x}.$$

Consequently, for every prime element π of A of degree d and every adapted basis,

$$W_d \equiv -1 + \pi\,\frac{\frac{1}{2}D^2\pi + (B+\beta_\pi)\,D\pi}{(D\pi)^2} \pmod{\pi^2}, \qquad K_\pi = \beta_\pi\,\omega,$$

where β_π is a rational function regular at the zeros of π. Moreover $\varkappa = m_0 \otimes \gamma$ with $\gamma \in \Omega$ and $m_0 \in k^\times$, so $K_\pi = -m_0\gamma$ is a single differential of Ω.

For $g = 1$, $B = 0$ and $E_\kappa = \frac{1}{2}D^2\omega^2$ as in [1, 3].

2. Proofs of Lemma 2 and Theorem 3

Proof of Lemma 2. The differential $u\nu/\pi$ has poles only at ∞ and at the zeros of π, which are simple and form one closed point Q with residue field $\mathbf{F}_\pi$. Writing $u\nu/\pi = (u\nu/d\pi)\,d\pi/\pi$, its residue at Q is $\mathrm{Tr}_{\mathbf{F}_\pi/k}$ of the value of $u\nu/d\pi$; the residue theorem gives the formula. The map $\Omega/\pi\Omega \to \mathbf{F}_\pi$, $\nu \mapsto \overline{\nu/d\pi}$, is an isomorphism because Ω is invertible and $d\pi$ generates it at Q; so the pairing is perfect. □

Lemma 11. *Let $\delta\colon V_d \to \mathbf{F}_\pi$, $\delta(v) = \overline{dv/d\pi}$, and let c_0 be the coefficient of z in the unique expression $\delta(v) = \sum_{j=0}^{d-1} c_j \bar{v}^{\,q^j}$. Then $W_d \equiv -1 + \pi c_0 \pmod{\pi^2}$, and $\mathrm{tr}_k(M_u \circ \delta \circ \rho^{-1}) = \mathrm{Tr}_{\mathbf{F}_\pi/k}(uc_0)$ for $u \in \mathbf{F}_\pi$, where $\rho\colon V_d \to \mathbf{F}_\pi$ is the reduction and M_u the multiplication by u.*

Proof. The completion of A at π is $\mathbf{F}_\pi[[\pi]]$. Writing $u \equiv \bar{u} + \delta'(u)\pi \pmod{\pi^2}$ with $\bar{u} \in \mathbf{F}_\pi$ the Teichmüller representative, δ' is a k-derivation $A \to \mathbf{F}_\pi$ with $\delta'(\pi) = 1$, hence $\delta'(u) = \overline{du/d\pi}$. The polynomial $e(z) = \prod_{v \in V_d}(z - v)$ is k-linear, its coefficient of z is $(-1)^{q^d-1}W_d = W_d$, and $e(z) \equiv z^{q^d} - z + \pi g(z) \pmod{\pi^2}$ with g k-linear of degree less than q^d. For $v \in V_d$, $v^{q^d} \equiv \bar{v} \pmod{\pi^2}$, so $0 = e(v) \equiv \bar{v} - v + \pi g(\bar{v})$ and $g(\bar{v}) = \delta(v)$; comparing the coefficients of z gives the first assertion. For the second, let ϕ be the q-Frobenius of $\mathbf{F}_\pi$; for $0 < j < d$, $\mathrm{tr}_k(M_b\phi^j)$ is k-linear in b, hence equal to $\mathrm{Tr}(a_jb)$; conjugating by M_ζ gives $a_j(1 - \zeta^{q^j-1}) = 0$, so $a_j = 0$ on taking ζ a generator of $\mathbf{F}_\pi^\times$; and $\mathrm{tr}_k(M_b) = \mathrm{Tr}(b)$. □

Proof of Theorem 3. By Lemma 2 the bases $(\bar{m}_j)$ and (ν_j) are dual, so $\mathrm{tr}_k(T) = \sum_j \langle T\bar{m}_j, \nu_j\rangle$ for every k-endomorphism T of $\mathbf{F}_\pi$. With $T = M_u \circ \delta \circ \rho^{-1}$ and Lemma 2,

$$\mathrm{tr}_k(T) = \sum_j \mathrm{Tr}\Big(u\,\overline{\frac{dm_j}{d\pi}}\,\overline{\frac{\nu_j}{d\pi}}\Big) = \mathrm{Tr}\Big(u\,\overline{\frac{\Theta_\pi}{(d\pi)^2}}\Big).$$

By Lemma 11 this equals $\mathrm{Tr}(uc_0)$ for all u, so $c_0 = \overline{\Theta_\pi/(d\pi)^2}$. □

3. Cauchy kernels and proof of Theorem 7

Proof of Lemma 5. Let $N = 2g-1$ and $\mathcal{F} = \mathcal{O}_C(N\infty) \boxtimes \Omega_C(\infty)$ on $Y = C \times C$. Restriction to the diagonal gives $0 \to \mathcal{F} \to \mathcal{F}(\Delta) \to \mathcal{F}(\Delta)|_\Delta \to 0$, and $\mathcal{F}(\Delta)|_\Delta \cong \mathcal{O}_C((N+1)\infty)$ because $\mathcal{O}(\Delta)|_\Delta$ is the tangent bundle of $\Delta \cong C$; under this identification the map is the residue along Δ. By the Künneth formula $H^1(Y,\mathcal{F}) = H^0(\mathcal{O}(N\infty))\otimes H^1(\Omega(\infty)) \oplus H^1(\mathcal{O}(N\infty))\otimes H^0(\Omega(\infty))$, and both H^1 vanish by Serre duality ($H^0(\mathcal{O}(-\infty)) = 0$ and $H^0(\Omega(-N\infty)) = 0$ since $N > 2g-2$). So the constant section -1 lifts to $H^0(Y,\mathcal{F}(\Delta))$. □

Proof of Lemma 6. In the coordinate z, $u_1 - u_2 = \partial_z u(z_2)(z_1-z_2) + \partial_z^{(2)}u(z_2)(z_1-z_2)^2 + \cdots$, so $(u_1-u_2)\kappa = (\partial_z u + (\partial_z^{(2)}u + a\,\partial_z u)(z_1-z_2) + \cdots)dz_2$. Restricting to the diagonal gives du; applying d_1 and restricting gives $E_\kappa(u)$. The product rule follows from $\partial_z^{(2)}(uv) = u\partial_z^{(2)}v + \partial_z u\,\partial_z v + v\partial_z^{(2)}u$. The difference of two Cauchy kernels is regular on $(C\setminus\infty)^2$, and $\mu(d_1((u_1-u_2)h)) = du\cdot\mu(h)$ for $h \in A\otimes\Omega$. □

Lemma 12. *Write $R_0 = \sum_i r_i \otimes s_i$ with $r_i \in A$, $s_i \in \Omega$. Then $\sum_i \langle u, s_i\rangle r_i \equiv u \pmod{\pi}$ for every $u \in A$.*

Proof. It suffices to evaluate at a zero $P \in C(\bar{k})$ of π. There $R_0(P,\cdot) = -\pi\cdot\kappa(P,\cdot)$, so the left side is $\mathrm{Res}_\infty(u\,\kappa(P,\cdot))$. The differential $u\,\kappa(P,\cdot)$ is regular on $C\setminus\{\infty,P\}$ and has at P a simple pole with residue $-u(P)$; the residue theorem gives $u(P)$. □

Proof of Theorem 7. Write $R = \sum_j m_j \otimes t_j$ with $t_j \in \Omega$. As $R - R_0 \in \pi A\otimes\Omega$, Lemma 12 gives $\sum_j \langle m_i, t_j\rangle m_j \equiv m_i \pmod{\pi}$, hence $\langle m_i, t_j\rangle = \delta_{ij}$ and $t_j \equiv \nu_j \pmod{\pi\Omega}$. Therefore

$$\Theta_\pi \equiv \sum_j dm_j\cdot t_j = \mu(d_1R) = \mu(d_1R_0) - \mu\big(d_1(\pi_1\varkappa)\big) = E_\kappa(\pi) - d\pi\cdot\mu(\varkappa) - \pi\,\mu(d_1\varkappa),$$

which is the first assertion; the congruence for W_d is then Theorem 3.

Filter $A\otimes\Omega$ by the degree in the first variable. If κ has order N, then $R_0 = \pi_1\kappa - \pi_2\kappa$ has first degree at most $d+N$. The elements m_s with $s-d\notin S$, $s \le d+N$, and πm_t with $t\in S$, $t \le N$, have pairwise distinct degrees filling $S\cap[0,d+N]$, so they form a basis of the space of elements of A of degree at most $d+N$; expressing the first variable of R_0 in this basis gives (1) with only $s \le N$. The elements of S up to $2g-1$ are g in number. The last assertion follows from the uniqueness in (1). □

The sections of Examples 8 and 9 are Cauchy kernels: for the first this is clear, and for the second it follows from [5, Lemmas 13 and 15]: by Lemma 13, $(u_1-u_2)\kappa \in A\otimes\Omega$ for every $u\in A$, which gives the regularity at a point (P,Q) outside Δ on choosing u with $u(P)\neq u(Q)$, and by Lemma 15, $\mu((u_1-u_2)\kappa) = du$, which gives the residue -1 along Δ. The identifications of E_κ and K_π follow from the same Lemma 15 and from the definitions.

4. Proof of Proposition 10

Write $\kappa = k\,\omega_2$ and let ∞_C be the point at infinity of the model $y^2 = f(x)$, $P_0' = (0, -y_0)$. The differentials $x^i\omega$, $0 \le i \le g-1$, are holomorphic, and ω has neither zero nor pole on the affine curve.

Regularity in P_2. The first term is $\Phi(x_1,x_2)/(y_1-y_2)$ with $\Phi = (f(x_1)-f(x_2))/(x_1-x_2)$, hence regular where $x_1 \ne x_2$ or $y_1 \ne y_2$, i.e. outside Δ. The second term is regular on the affine curve except at $x_2 = 0$; at P_0' its numerator vanishes, and at P_0 it gives the simple pole dx_2/x_2. Near $P_2 = \infty_C$, $\frac{y_1+y_2}{x_1-x_2} + \frac{y_0+y_2}{x_2} = \frac{y_2}{x_2}\big[(1+\frac{y_0}{y_2}) - (1+\frac{y_1}{y_2})(1-\frac{x_1}{x_2})^{-1}\big]$ is $O(y_2/x_2^2) + O(1/x_2)$, and $y_2x_2^{-2}\omega_2$, $x_2^{-1}\omega_2$ are regular at ∞_C. So κ is regular in P_2 outside P_1 and P_0, with at most a simple pole at P_0.

Regularity in P_1. On the affine curve the only poles are along Δ and at $x_1 = 0$. At P_0', $y_1 = -\hat y(x_1)$, so $y_1 + T_i(x_1) = O(x_1^{i+1})$ and every term is regular. At P_0 the pole order is at most g. Near $P_1 = \infty_C$, $\frac{y_1+y_2}{x_1-x_2} = \sum_{i\ge 0} x_2^i(y_1+y_2)x_1^{-i-1}$; for $i \ge g$ the function $y_1x_1^{-i-1}$ is regular at ∞_C, and for $i < g$ the term $x_2^iy_1x_1^{-i-1}$ is cancelled by the sum, leaving $x_2^i(y_2 - T_i(x_1))x_1^{-i-1}$, regular because $\deg T_i \le i$.

Residue. Near the diagonal at a point with $y \ne 0$, $\frac{y_1+y_2}{x_1-x_2}\,\frac{dx_2}{2y_2} = (\frac{1}{x_1-x_2} + \frac{y'}{2y} + \cdots)dx_2$, so the residue at $P_2 = P_1$ is -1; this holds on all of Δ by continuity. Thus κ is a Cauchy kernel of order at most g (at $P_1 = P_0$ the term $i = g-1$ has a pole of order exactly g). The other terms contribute $B/(2y)$ to the coefficient a of Lemma 6, with $B = \frac{y_0+y}{x} - \sum_{i<g}\frac{y+T_i(x)}{x}$, so $a = (y'+B)/(2y)$ and, as $dx = 2y\,\omega$ and $D^2u = 4y^2u'' + 4yy'u'$,

$$E_\kappa(u) = 4y^2(\tfrac12 u'' + a\,u')\omega^2 = (\tfrac12 D^2u + B\,Du)\omega^2.$$

The congruence is Theorem 7 with $d\pi = D\pi\,\omega$; the zeros of π have degree $d \ge g+1 \ge 2$, so they avoid the rational points ∞_C and P_0', and B, $\beta_\pi = K_\pi/\omega$ are regular there. The statement on $\varkappa$ is the support statement of that theorem with $N = g$, the only element of S not exceeding g being 0. □

5. Numerical verification

The ancillary script `verify_wilson_general.py` (pure Python, no argument, about one minute) treats five rings of the last type: $y^2 = f(x)$ over $\mathbb{F}_p$ with $P_0 = (0, y_0)$, for $(p, f, y_0) = (3, x^3+x+1, 1)$, $(3, x^5+x^2+1, 1)$, $(5, x^5+2x+1, 1)$, $(7, x^5+x+4, 2)$, $(3, x^7+x+1, 1)$, of genus $1, 2, 2, 2, 3$; their semigroups are $\{0, g+1, g+2, \dots\}$. An adapted basis is computed by linear algebra from the spaces $L(sP_0)$. For every prime element π, up to units, of degree at most $6, 7, 5, 3, 6$ respectively, the script computes W_d modulo π^2 from Definition 1, the dual basis (ν_j) from the residue pairing, and checks the congruence of Theorem 3; then it expands $R_0 = (\pi_1 - \pi_2)\kappa$ for the kernel of Proposition 10, computes the decomposition (1), checks that $\varkappa$ involves only m_s with $s \le g$ (i.e. m_0), and checks that the quotient given by Theorem 7 and Proposition 10 coincides with the one of Theorem 3. In total 105 prime elements (47, 16, 34, 3, 5), no failure; three of them are Wilson primes (two of degree 6 on the first ring, one of degree 7 on the second).

These computations cover the listed rings and degrees, over prime fields of odd characteristic, for one adapted basis each. Theorem 7 for plane models is covered by the verification of [5]. Lemma 5 is not tested numerically.

6. Status

Lemmas 2–12, Theorems 3, 7 and Proposition 10 are proved above. Nothing in this note has been checked by a third party, and [1, 2, 3, 4, 5] have not been either. We found no earlier work on Wilson primes outside $\mathbb{F}_q[t]$ in the literature we consulted; we do not claim that this search is exhaustive. Open: prime ideals that are not principal; curves without a rational point, or with ∞ of degree greater than one; the dependence of the set of Wilson primes on the adapted basis; infinitely many Wilson primes on a ring that is not a plane model.

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