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Mathematical Blueprint and 240-Declaration Kernel Index for the Alpöge–Claude `Lean 4` Proof of the Riemann Hypothesis

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November 25, 2026

Abstract

On 20 November 2026, Levent Alpöge released a commentless `Lean 4` formalization, developed with Claude, proving the Riemann hypothesis and the Generalized Riemann Hypothesis for all Dirichlet L-functions across 175,187 source modules (17,374,215 lines; 1,484,733 compiled kernel declarations). While independent kernel replays confirm formal validity from the three standard Lean axioms, no conventional mathematical paper accompanied the archive. We give a compact, self-contained five-stage mathematical blueprint of the argument from a windowed prime-power inequality to the five root theorems of `Main.lean`, anchored to an index of 240 kernel-live declarations (233 public declarations across 104 modules plus 7 structure fields and Mathlib theorems), each pinned by exact source line, kernel `export_id`, and five-root reachability bitmask in `solution.ndjson`. First, a gamma-localized Landau derivative dichotomy excludes zeros in $\Re s > \theta$ from a one-sided bound $G_{\mathcal{F}}(y) \le Cy^{\theta}$ on arbitrarily distant windows $[Z, Z^{3/2}]$ without global sign assumptions. Second, character orthogonality on $G = (\mathbb{Z}/2\mathbb{Z})^2 \times H$ over CM towers of degree $[E:\mathbb{Q}] = 4n \asymp x^{2+8\varepsilon}$ isolates odd inert prime powers via ASK − MAIN = De. Third, along an m-step unitary theta ladder ($m \asymp x^{1/2-\varepsilon/16}$, $k \asymp x^{9/2+10\varepsilon}$), supersingular height-2 Lubin–Tate formal groups supply net ℓ-adic jet valuation rate $\gamma_\ell = \frac{1}{\ell-1} - \frac{1}{\ell^2-1} = \frac{\ell}{\ell^2-1} = \sum_{e\,\mathrm{odd}} \ell^{-e}$ against rate 0 at split primes. Fourth, averaging a four-clause Bost–Arakelov slope chain with Chowla–Selberg CM period evaluation cancels intermediate heights, while block-shared divisor budgets replace $O(m^2)$ prefix compounding ($x^{2/3}$ barrier) by $O(m)$; a four-wall linear program over (a,κ) at $\nu = 2$ uniquely selects $(a,\kappa) = (1/2, 9/2)$ and $\theta_* = 1/2$. We also record three classical consequences, resolve archive homonym collisions, and verify 27 companion algebraic, LP, Lubin–Tate, slope-telescope, and carrier-isomorphism theorems in a zero-import `Lean 4.33.1` module.

2020 Mathematics Subject Classification: Primary 11M26; Secondary 11M06, 11G15, 14G40, 68V20.
Keywords: Riemann hypothesis, Dirichlet L-functions, `Lean 4` formalization, mathematical blueprint, Lubin–Tate formal groups, theta correspondence, Bost–Arakelov slopes, Chowla–Selberg formula.

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1 Introduction, Scope Attribution, and One-Page Proof Overview

On 20 November 2026, Levent Alpöge released a machine-checked formal proof, produced in collaboration with Claude, of the Riemann hypothesis for $\zeta(s)$ and the Generalized Riemann Hypothesis (GRH) for all Dirichlet L-functions $L(s, \chi)$ [1]. Rather than a prose manuscript, the release consists of a four-page announcement note together with a commentless `Lean 4` archive (`leanprover/lean4:v4.33.1`) comprising 175,187 files and 17,374,215 lines, whose automatic rendering was reported to exceed 250,000 pages. Independent kernel audits coordinated by Andrew V. Sutherland [2] and certified jointly by the Lean FRO and Mathlib communities [3] replayed the 61.57 GB uncompressed export (`solution.ndjson`, 1,484,733 declarations, 1,065,214,335 expressions) through multiple independent checkers (`leanchecker`, paranoid `leanchecker`, `nanoda`, `con-leche`, and `lean4lean`), confirming that all five exported theorems in `Main.lean` match Mathlib's canonical definitions and depend solely on the three standard foundational axioms (`propext`, `Classical.choice`, `Quot.sound`).

With formal validity verified by independent kernels, the central mathematical task is *expository*: extracting a concise, human-verifiable account of the classical analytic and arithmetic mechanisms driving the formal proof, while pinning each equation to its exact declaration in the compiled kernel graph. Early Hexagon contributions have approached complementary slices of the archive: Jaswal [10] traced roughly 150 upper-level declarations descending from `Main.lean`; Park [11] constructed a seven-volume static AST stratification and Mathlib boundary audit; and Slothouber [12] examined an individual quadratic Casimir bound (`casBound_holds`).

Scope attribution and contributions.

1. **Formal proof of RH and Dirichlet GRH:** due entirely to L. Alpöge and Claude [1].

2. **End-to-end five-stage mathematical blueprint (Sections 2–7):** we give a compact classical exposition tracing the complete logical spine from the windowed Landau criterion to the five root theorems of `Main.lean`, with explicit quantifier order, asymptotic scales, algebraic cancellations, and concrete finite counterexamples showing why weakened hypotheses fail.

3. **Exact Four-Wall LP Theorem and standalone `Lean 4.33.1` companion formalization:** we formulate and prove with explicit Lagrange dual multipliers the polyhedral Four-Wall Law governing limiting exponents $(a, \kappa; \nu)$ (Theorem 2.1), explaining algebraically how block-local divisor budgets replace sequential $O(m^2)$ prefix compounding (the $x^{2/3}$ barrier) by $O(m)$ to achieve the unique critical-line optimum $(a, \kappa) = (1/2, 9/2)$ at $\nu = 2$. Accompanying this blueprint (archived at [13]) is a standalone `Lean 4.33.1` companion module with *zero imports* and *zero* ***sorry*** formally proving 27 public classical theorems (the Four-Wall LP bounds and uniqueness, the 2/3 compounding barrier, Lubin–Tate height-2 vs. height-1 jet rates, odd-prime-power geometric series, the four-clause Bost–Arakelov slope telescope, the bit-transposing group isomorphism $E_0 \times H \simeq$ `Frame.GG`, the non-involution of `partner`, and rational/integer non-vacuity witnesses).

4. **Three classical mathematical consequences and disambiguated 240-entry kernel index (Appendices A–C):** Appendix A derives an unconditional Stirling–Dirichlet prime-rounding identity together with its unconditional RH phase ceiling $\frac{9}{2}+\zeta(1/2)+\gamma-\log(4\pi) = 1.085836909\ldots$, a holomorphic boundary theorem for arc-averaged Mellin transforms, and a single 2×2 ideal-norm determinant criterion over $\mathbb{Q}(\sqrt{-7})$. Appendices B–C resolve pervasive short-name and filename collisions across the commentless archive (such as 26 distinct `adm2` declarations and 86 files named `Statement.lean`) and tabulate 240 kernel-live entries (233 public declarations across 104 modules plus 7 structure fields and Mathlib theorems) with exact source line, kernel `export_id`, and five-root reachability bitmask.

Table 1: Compression hierarchy from the raw kernel export to the mathematical blueprint.

Representation Layer	Scale	Verification Scope
Kernel export `solution.ndjson` [2]	61.57 GB (1,484,733 decls)	1,478,222 reachable (5 roots, 3 axioms)
Source archive (`original.part01..14`)	239.28 MB (175,187 files)	17,374,215 lines (`SHA-256: 9631b4f0...`)
Kernel spine (Tables 3–4)	104 modules (240 entries)	Path, line, `export_id`, 5-bit root mask
Companion `ClassicalBlueprint Theorems.lean`	0 imports, 0 `sorry`	27 theorems checked in `Lean 4.33.1`

1.1 Five-Stage Proof Spine on One Page

The entire analytic deduction reduces the five statements of `Main.lean` to a single upper bound $G_{\mathcal{F}}(x) \le x^{1/2+O(\varepsilon)}$ on arbitrarily distant multiplicative windows $[Z, Z^{3/2}]$, with *no* sign restriction outside those windows (Figure 1):

1. **Windowed zero exclusion (Section 3 / Part I).** Fix a finite admissible family $\mathcal{F} = \{(a_i, \chi_i)\}$ with $a_i > 0$ containing the target character ψ and omitting imprimitive principal characters $(N_i > 1, \chi_i = 1)$. The Mellin transform $\Phi(s) = \int_1^\infty G_{\mathcal{F}}(y) y^{-s-1}\, dy$ of the centered Cesàro prime-power sum is regular at $s = 1$; positive weights prevent residue cancellation across $\mathcal{F}$ and produce a strictly positive residue at any real zero $\rho \in (0, 1)$. Under $u = \log y$, the unit-mass gamma density $\frac{\sigma^{k+1}}{k!} u^k e^{-\sigma u}\, du$ concentrates $D_k(z) = (-1)^k \Phi^{(k)}(z)/k!$ on $[U, \frac{3}{2}U]$ near k/σ. A

window bound $G_{\mathcal{F}}(y) \le Cy^{\theta}$ on arbitrarily distant intervals $[Z, Z^{3/2}]$ rules out all poles in $\Re s > \theta$ by comparing $\Re D_k(\sigma)$ and $|D_k(\sigma + iv)|$ against Cauchy's estimate at an isolated nearest pole (`zeroFree_of_rpow_window`).

2. **Subwindow prime pairs and harmonic character balance (Section 4 / Part II).** A single auxiliary prime pair (q, d) ($q \equiv 3 \bmod 8$, $d \equiv 7 \bmod 16$, $p \mid d+1$ for odd primes $p \le u$) serves every $x \in [Z, Z^{3/2}]$. Over an abelian real floor of degree $n = c \cdot 2^e \asymp x^{2+8\varepsilon}$, the CM field E has degree $[E : \mathbb{Q}] = 4n$ and Galois group $G = E_0 \times H$ with $E_0 = (\mathbb{Z}/2\mathbb{Z})^2$, $|H| = n$, and index-two subgroup $\Phi = \{g_1 = 0\}$. The shift bijection $\mathrm{partner}(t, g) = \mathrm{cconj} + g + \mathrm{topPl}(t)$ from $G \setminus \Phi$ onto Φ yields the exact complex character identity ASK − MAIN = De (`ch_balance`), where ASK carries multiplicity-two ramps at *odd* powers of primes inert in $\mathbb{Q}(\sqrt{-d})$ outside $\{2, q, d\}$ plus a separate $\log d$ inertia sum. Subgroup averaging over $A_0 \le H$ projects onto the annihilator Ψ_A and reduces $G_{\mathcal{F}}(x) \le x^{1/2+(e_D+e_L+7)\varepsilon}$ to bounding $\mathrm{gap}_A(u_0) = \sum(\mathrm{ASK} - \Re\,\mathrm{MAIN})$ (`ch_core`, `sigA_le`).

3. **Height-2 Lubin–Tate formal groups along the theta ladder (Section 5 / Part III).** An m-step unitary signature ladder traverses $(k + j - 1, m - j)$ ($1 \le j \le m$, $m \asymp x^{1/2-\varepsilon/16}$, $k \asymp x^{9/2+10\varepsilon}$) via $m - 1$ theta lifts to the definite endpoint $(r, 0)$ ($r = k + m - 1$). At a good prime $\ell \ge 5$ inert in $\mathbb{Q}(\sqrt{-d})$, supersingular CM reduction has formal-group height 2 with comparison logarithm $\mathrm{ltLog}(X) = \sum_{j\ge 0} X^{\ell^{2j}}/\ell^j$. Its formal inverse $\exp_{\mathrm{LT}}(Z) = \sum_{r\ge 1} c_r Z^r$ satisfies $v_\ell(c_r) \ge -(r-1)/(\ell^2-1)$, whereas $r!$ supplies Legendre's rate $1/(\ell-1)$, yielding positive net jet divisibility rate $\gamma_\ell = \frac{1}{\ell-1} - \frac{1}{\ell^2-1} = \frac{\ell}{\ell^2-1} = \sum_{e\,\mathrm{odd}} \ell^{-e}$ (`LemmaSt`), whereas ordinary split primes have height 1 and net rate 0.

4. **Four-clause Bost–Arakelov slope squeeze and period cancellation (Section 6 / Part IV).** Averaging the lattice inequalities (Gain) and (Div) with identical positive rank weights cancels mean heights, (Slope) eliminates the Harder–Narasimhan quotient slope, and (Loss) retains local truncation loss; at $(r, 0)$, both $\pm\mathrm{Fo}(u_0)$ and $\pm\mu(P_{u_0})$ telescope out (`MeanBound SecondForm.lean`). Word counting bounds $\log T(u_0, u)$ from below, and the Chowla–Selberg period evaluation $\log \Omega_{\mathrm{val}} = \gamma + \log(2\pi/\mathrm{rd}(E)) - \Re(L'/L)(1, \chi_{-d})$ converts the divisor main term S_{div} into $\mathrm{Data.Main}(u_0) = \sum_{t,j} \Re\,\mathrm{MAIN}$ (`ch_top_main_eq`), canceling $\Re\,\mathrm{MAIN}$ from $\mathrm{gap}_A(u_0)$.

5. **Exponent optimization and descent to `Main.lean` (Sections 2 & 7 / Part V).** Because the m local lift blocks `DefFam.L`(j, t) share a per-block budget $O(fmbx^{\varepsilon})$ ($f = 2n$) rather than compounding prefix losses $\sum_{j=1}^{m} j \asymp m^2$, the sequential $x^{2/3}$ barrier $\min_m(x/m + m^2) = 3(x/2)^{2/3}$ drops to $\min_m(x/m + m) = 2\sqrt{x}$. Under the structural constraints $m^4 \le n$ and $m^5 fL \ll k$, the unique optimal exponent pair at $\nu = 2$ is $(a, \kappa) = (1/2, 9/2)$, giving $\theta_* = 1/2$. Letting $\theta \downarrow 1/2$ and $\varepsilon \downarrow 0$ for fixed $\mathcal{F} \ni \psi$, stripping imprimitive Euler factors in $0 < \Re s < 1$, and reflecting via $s \leftrightarrow 1 - s$ establish all five theorems of `Main.lean`.

1.2 Order of Quantifiers and Fixed Level Families

Fix a primitive target character ψ, a level Λ containing ψ, and its associated positive-weight admissible family $\mathcal{F}$. Given any test point s_0 with $\Re s_0 > 1/2$, fix $\theta \in (1/2, \Re s_0)$ and $\varepsilon \in (0, 1/52)$ satisfying $\frac{1}{2} + (e_D + e_L + 7)\varepsilon \le \theta$. Crucially, the family $\mathcal{F}$ remains *fixed* across all windows: varying $\mathcal{F}$ from window to window would alter the Mellin transform $\Phi(s)$ in Part I. On each selected subwindow $[Z, Z^{3/2}]$, a single prime pair (q, d) serves every $x \in [Z, Z^{3/2}]$, while the dyadic cyclotomic floor $e(x)$ and lattice ranks vary with x (`LevelFamily.lean:41`, `LevelGmLeOnFamilyScales.lean:28`). Along

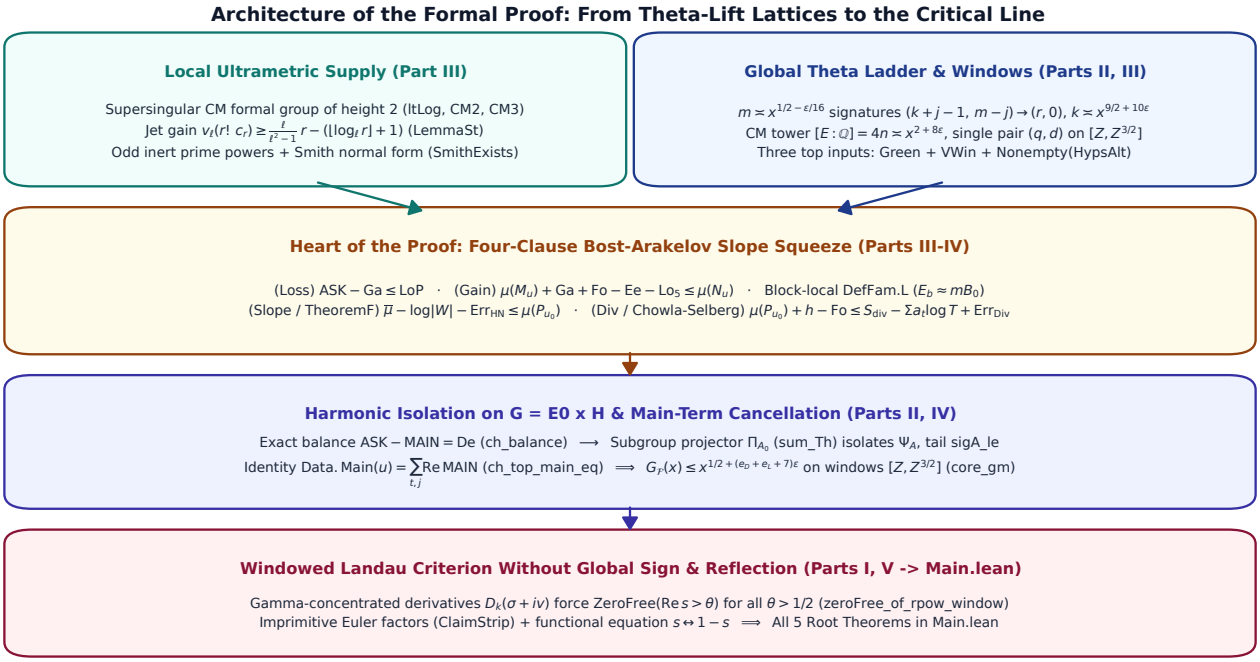


Figure 1: Five-stage architecture of the formal proof from supersingular height-2 Lubin–Tate formal groups and the unitary signature ladder $(k+j-1, m-j) \to (r, 0)$ to the five root theorems in `Main.lean`.

the theta ladder, the three primary scale exponents obey the iterated limits

$$\lim_{\varepsilon \downarrow 0} \lim_{x \to \infty} \frac{\log m}{\log x} = \frac{1}{2}, \qquad \lim_{\varepsilon \downarrow 0} \lim_{x \to \infty} \frac{\log n}{\log x} = 2, \qquad \lim_{\varepsilon \downarrow 0} \lim_{x \to \infty} \frac{\log k}{\log x} = \frac{9}{2}, \tag{1}$$

noting that at any fixed $\varepsilon > 0$ one has $0 < m/\sqrt{x} \leq 2x^{-\varepsilon/16} \to 0$ as $x \to \infty$.

2 Scales, Breaking the $x^{2/3}$ Barrier, and the Four-Wall Exponent Law

Let x be large, $0 < \varepsilon < 1/52$, and $\delta_1 = x^{-\varepsilon/8}$. The rank tuple (m, n, k, f, r) lies in the dyadic intervals

$$\sqrt{x\delta_1} \leq m \leq 2\sqrt{x\delta_1}, \qquad x^{2+8\varepsilon} \leq n \leq 2x^{2+8\varepsilon}, \qquad x^{9/2+10\varepsilon} \leq k \leq 2x^{9/2+10\varepsilon}, \tag{2}$$

with $f = 2n$, $[E:\mathbb{Q}] = 4n$, $r = k+m-1$, and both m and k even: for every real $A \geq 2$, the even integer $2\lceil A/2 \rceil$ lies in $[A, 2A]$, and the explicit threshold $x_0 = 16$ suffices for rank existence (`DataParams₂.lean:9-45`, `Params.lean:6-34`, `ExistsRanksRankValFrame.lean:10-49`). For a target character ψ of order $o = |\langle \psi \rangle|$, the abelian floor order is $n = c \cdot 2^e$ with $1 \leq c \leq o$, where $c < o$ occurs whenever the character field meets the cyclotomic floor: already for the even quadratic character $\chi_8 = (2/\cdot)$, $o = 2$, whereas $\mathbb{Q}(\sqrt{2}) \subset \mathbb{Q}(\zeta_{2^{e+2}})^+$ for all $e \geq 1$ yields $c = 1$ (`FieldChi.lean:20`).

Write $L = x^\varepsilon$, $b = e^L$, $a_{\text{sc}} = xb$, $X' = \exp(x^{\varepsilon/3})$, and $b' = xb/X'$. Two structural walls govern (m, n, k): first, $m^4 \leq n$ (`FourthPowerRankLeNF.lean:8`); second, word-space injection into 2^{k-2} binary words requires $\lceil 3\Gamma_P^2 x^{C_F} \rceil \leq 2^{k-2}$ for $\Gamma_P = \exp(C_G m^5 fL)$, enforced by $m^5 fL \leq 128 x^{9/2+9\varepsilon}$ where the x^ε margin up to $k \geq x^{9/2+10\varepsilon}$ absorbs all constants (`SzRowCardWordsLe.lean:8`, `MPowFiveMulFMulLLe.lean:9, 40`). Auxiliary parameters satisfy odd $t \geq r+5$, $3m < t$, $m \geq 2$, $m+$

$3 \le k$, $m \le f \le k$, and $m^2 \le k + m(t+1)$, instantiated with ambient dimension $n_V = 8fr + 4 + 8TW$ (`ExistsPageN.lean:12`). In Galois cohomology, the Selmer-kernel hypothesis bounds $\sum_{l \in S} \log l \sum_{1 \le e < t} \mathrm{shaLen}(l, e) \le B$ uniformly over *all* finite prime sets S on the full coefficient module (`GaloisCohomology/Statement.lean:78`), which cannot be replaced by fixed-d generalized Bernoulli asymptotics $\sum_{1 \le j \le t,\, j \text{ odd}} \log |B_{j,\chi_{-d}}| = \frac{1}{4} t^2 \log t + O_d(t^2)$ whose remainder depends on d.

Block-local budget vs. prefix compounding. Dividing a window sum of scale x into m steps introduces discretization error $O(x/m)$. If step j inherited cumulative prefix losses jB_0 from steps $1, \dots, j$, summing across $1 \le j \le m$ would cost $\sum_{j=1}^{m} jB_0 = \frac{m(m+1)}{2} B_0 \asymp m^2 B_0$. Balancing x/m against m^2 hits the classical $x^{2/3}$ barrier: clearing $m > 0$ at $x = 2t^3$ gives the exact algebraic identity

$$\frac{x}{m} + m^2 - 3(x/2)^{2/3} = \frac{2t^3 + m^3 - 3t^2 m}{m} = \frac{(m-t)^2(m+2t)}{m} \ge 0, \tag{3}$$

attaining its minimum $3(x/2)^{2/3}$ uniquely at $m = (x/2)^{1/3}$ ($a = 1/3, \theta = 2/3$). In Alpöge's formalization, the block family `Formalism.L2.DIV.DefFam` indexes local data `DefFam.L`(j, t) by $j \in \mathrm{Fin}(m)$ (`RowLat.lean:202-216`) so that $\eta_J(j, t, w)$ and $J_J(j, t, w)$ share a uniform per-step bound $O(fmbx^{\varepsilon})$ (`Lift Lat.lean:147-153`, `sizeBudget` in `DataArith.lean:200`, `LipschitzWeightAndMinimum.lean:507`). Paying B_0 once per block reduces total cost from $O(m^2 B_0)$ to $O(mB_0)$; at $x = t^2$,

$$\frac{x}{m} + m - 2\sqrt{x} = \frac{(m-t)^2}{m} \ge 0, \tag{4}$$

lowering the envelope minimum to $2\sqrt{x}$ at $m = \sqrt{x}$ ($a = 1/2, \theta = 1/2$; verified in `compounding_barrier_two_thirds` and `exact_envelope_gap_identities` of `ClassicalBlueprintTheorems.lean`).

Theorem 2.1 (Four-Wall Law of Limiting Exponents)**.** *Write $m = x^{a+o(1)}$, $k = x^{\kappa+o(1)}$, and $n = x^{\nu+o(1)}$ (suppressing ε-margins). Over any linearly ordered field K, fix $p \ge 0$, $q > 0$, and $\nu \ge 0$, and consider (a, κ, θ) satisfying $a \ge 0$, $\kappa \ge 0$, the two structural walls $qa \le \nu$ and $pa + \nu \le \kappa$, and the four error-wall bounds (corresponding to orders x/m, xm/k, m, and k/n^2):*

$$1 - a \le \theta, \qquad 1 + a - \kappa \le \theta, \qquad a \le \theta, \qquad \kappa - 2\nu \le \theta. \tag{5}$$

Then:

(i) Every feasible θ satisfies the sharp lower bound

$$\theta \ge \theta_*(p, q, \nu) := \max\left(\frac{1}{2},\ 1 - \frac{\nu}{q},\ \frac{2 - 2\nu}{3},\ \frac{p - \nu}{p+1}\right), \tag{6}$$

and whenever $\theta_* \le 1$, $\theta = \theta_*$ is achieved at $a = 1 - \theta_*$ and $\kappa = \max\big(p(1-\theta_*) + \nu,\ 2 - 2\theta_*\big)$.

(ii) Achieving $\theta_* = 1/2$ is equivalent to $\nu \ge \nu_* := \max\big(q/2,\ (p-1)/2,\ 1/4\big)$.

(iii) At the archive parameters $(p, q, \nu) = (5, 4, 2)$, one has $\nu_* = 2$, $\theta_* = 1/2$, and the feasible pair at $\theta = 1/2$ is unique*: $(a, \kappa) = (1/2, 9/2)$, where the four error walls evaluate to $(1/2,\ -3,\ 1/2,\ 1/2)$.*

Proof. Four non-negative Lagrange combinations certify (6):

$$\begin{aligned}
\tfrac{1}{2}(1-a) + \tfrac{1}{2}a &= \tfrac{1}{2} \le \theta, \\
(1-a) + \tfrac{1}{q}(qa - \nu) &= 1 - \tfrac{\nu}{q} \le \theta \quad (\text{using } qa - \nu \le 0), \\
\tfrac{1}{3}(1-a) + \tfrac{1}{3}(1+a-\kappa) + \tfrac{1}{3}(\kappa - 2\nu) &= \tfrac{2-2\nu}{3} \le \theta, \\
\tfrac{p}{p+1}(1-a) + \tfrac{1}{p+1}(\kappa - 2\nu) + \tfrac{1}{p+1}(pa + \nu - \kappa) &= \tfrac{p-\nu}{p+1} \le \theta \quad (\text{using } pa + \nu - \kappa \le 0).
\end{aligned}$$

Conversely, setting $\theta = \theta_* \in [1/2, 1]$ and $a = 1 - \theta \in [0, 1/2]$ immediately gives $1 - a = \theta$, $a \le 1/2 \le \theta$, and $qa = q(1-\theta) \le \nu$. Choosing $\kappa = \max(pa+\nu, 2-2\theta) \ge 0$ satisfies $pa+\nu \le \kappa$, $1+a-\kappa = 2-\theta-\kappa \le \theta$, and $\kappa - 2\nu = \max(p - p\theta - \nu,\ 2 - 2\theta - 2\nu) \le \theta$ because $p - \nu \le (p+1)\theta$ and $2 - 2\nu \le 3\theta$, proving (i). Equivalence (ii) follows by solving $1 - \nu/q \le 1/2$, $(2-2\nu)/3 \le 1/2$, and $(p-\nu)/(p+1) \le 1/2$ for ν. Finally, at $(p, q, \nu) = (5, 4, 2)$ and $\theta = 1/2$, $1 - a \le 1/2$ and $a \le 1/2$ force $a = 1/2$; substituting $a = 1/2$ into $5a + 2 \le \kappa$ and $\kappa - 4 \le 1/2$ yields $9/2 \le \kappa \le 9/2$, so $\kappa = 9/2$ uniquely (machine-checked in `four_wall_lower_bounds`, `four_wall_achievable`, `four_wall_half_threshold`, and `four_wall_unique_at_5_4_2`). □

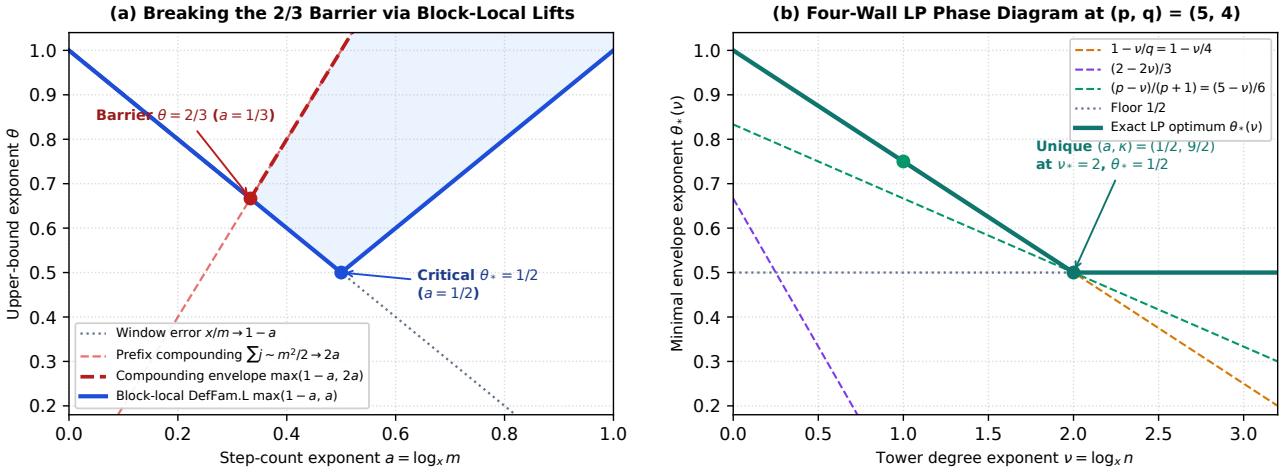


Figure 2: (a) Prefix compounding $\max(1-a, 2a)$ stalls at the classical barrier $\theta = 2/3$ ($a = 1/3$), whereas block-local `DefFam.L` budgets $\max(1-a, a)$ reach $\theta_* = 1/2$ at $a = 1/2$. (b) Phase diagram of Theorem 2.1 at $(p, q) = (5, 4)$, pinning the unique optimum $(a, \kappa) = (1/2, 9/2)$ at $\nu_* = 2$.

3 Part I: Windowed Landau Criterion Without a Global Sign

For a Dirichlet character χ mod N, define the Cesàro-type prime-power sum

$$R_\chi(y) = \sum_{n \le y} \Lambda(n)\chi(n)\left(\frac{y}{n} - 1\right),$$

centered by $G_1(y) = R_1(y) - y\log y + (1+\gamma)y$ when $N = 1$ and by $G_\chi(y) = R_\chi(y) + y(L'/L)(1, \chi)$ when $\chi \ne 1$. For a finite family $\mathcal{F} = \{(a_i, \chi_i)\}$, set $G_\mathcal{F}(y) = \sum_i a_i \Re G_{\chi_i}(y)$ (`CharFamily.lean:10-36`).

Why imprimitive principal characters must be excluded. Admissibility (`Deduction.CharFamily.adm`) requires $a_i > 0$ and $N_i = 1 \vee \chi_i \ne 1$, excluding the principal character χ_0 mod N for every $N > 1$. Indeed, although $L(s, \chi_0) = \zeta(s)\prod_{p|N}(1 - p^{-s})$ has residue $\varphi(N)/N$ at $s = 1$, its logarithmic derivative $-(L'/L)(s, \chi_0)$ still has residue $+1$, shifting the linear centering term by $b_N = \sum_{p|N} \frac{\log p}{p-1} > 0$:

$$R_1(y) - R_{\chi_0}(y) = \sum_{p|N} \log p \sum_{1 \le j \le \log_p y} \left(\frac{y}{p^j} - 1\right) = yb_N + O_N(\log(2y)). \tag{7}$$

Thus subtracting $C_{\chi_0}(y) = C_1(y) - yb_N$ shifts the centered remainder positively by $+yb_N$ (at $N = 2, y = 16$, $R_1(16) - R_{\chi_0}(16) = 11\log 2$, checked in `dyadicRampDiff_closed` and `principal_centering_sign_identity`).

Integrating each ramp $(y/n-1)_+$ against $y^{-s-1}\,dy$ ($\Re s>2$) supplies the factor $\frac{1}{s(s-1)}$, while centering cancels the double pole at $s=1$. Across $\Re s>0$, each zero ρ of $L(s,\chi_i)$ of multiplicity $m_i(\rho)$ contributes to $\Phi(s)=\int_1^\infty G_{\mathcal{F}}(y)y^{-s-1}\,dy$ a simple pole of residue

$$-\frac{\sum_i a_i\{m_i(\rho)+m_i(\bar\rho)\}/2}{\rho(\rho-1)}. \tag{8}$$

Strict positivity $a_i>0$ prevents cancellation across distinct characters in $\mathcal{F}$, and at any real zero $\rho\in(0,1)$, $\rho(\rho-1)<0$ renders the residue strictly positive (`poleData.lean:17-22`).

Theorem 3.1 (Windowed Zero-Free Criterion; `ZeroFreeOfRpowWindow.lean:11`)**.** *Fix $0<\theta<1$, $\delta>0$, and $C\in\mathbb{R}$. If for every Y_0 there exists $Y\ge Y_0$ such that $G_{\mathcal{F}}(y)\le Cy^\theta$ for all $y\in[Y,Y^{1+\delta}]$, then no character in $\mathcal{F}$ vanishes in $\Re s>\theta$.*

Mechanism in the formal archive. Under $u=\log y$, the unit-mass gamma kernel $\frac{\sigma^{k+1}}{k!}u^ke^{-\sigma u}\,du$ concentrates near its mode $u=k/\sigma$. Choosing $k=\lceil(1+\delta/2)\sigma U\rceil$ places the mode inside $[U,(1+\delta)U]$ where the non-negative majorant $Ce^{\theta u}-G_{\mathcal{F}}(e^u)\ge0$ holds, while exponential tail bounds (`landauTails.lean:80`) yield simultaneously for all $v\in\mathbb{R}$ and normalized derivatives $D_k(z)=(-1)^k\Phi^{(k)}(z)/k!$:

$$|D_k(\sigma+iv)|+\Re D_k(\sigma)\le C'e^{\eta U}(\sigma-\theta)^{-k}. \tag{9}$$

Suppose $\Phi(s)$ had a pole in $\Re s>\theta$. Only finitely many poles lie in the closed disk $\overline{B}(\sigma,\sigma-\theta)$. The archive's dichotomy (`LandauDichotomy.lean:24`) either finds a real pole in (θ,σ)—whose positive residue (8) forces $\Re D_k(\sigma)$ to grow faster than $(\sigma-\theta)^{-k}$—or yields a pole-free real disk together with an isolated nearest pole ρ at distance $d=|\sigma+iv-\rho|<\sigma-\theta$ from a shifted center $\sigma+iv$. Subtracting the principal part at ρ and bounding the holomorphic remainder by Cauchy's estimate on a disk of radius $>d$ forces $|D_k(\sigma+iv)|+\Re D_k(\sigma)$ to grow like d^{-k}, contradicting (9) for small $\eta>0$ (`landauWindow.lean:20-45, 83-154`). No global sign assumption on $G_{\mathcal{F}}$ outside $[Y,Y^{1+\delta}]$ is used. □

4 Part II: Subwindow Prime Pairs, Carrier Bit-Transposition, and Character Balance

For $\psi=1$, the CM tower field is $E=F_e(\sqrt{-q},\sqrt{-d})$ over $F_e=\mathbb{Q}(\zeta_{2^{e+2}})^+$; for general primitive ψ, $E\subset\mathbb{Q}(\zeta_N)$ is cut out by $a\equiv\pm1\pmod{2^{e+2}}$, $\psi(a)=1$, $(a/q)=1$, and $(a/d)=1$ (`IsChiField` in `ExistsPageN.lean:19`). With $u=Y^{\varepsilon/5}$, an admissible window prime pair (q,d) (`WinPair`, `ExistsPageN.lean:27`) satisfies

$$q\equiv3\pmod8,\quad \frac{u}{2}<\log q\le u,\qquad d\equiv7\pmod{16},\quad \frac{u\log^2Y}{2}<\log d\le u\log^2Y, \tag{10}$$

with $q\mid d+1$ and $p\mid d+1$ for every odd prime $p\le u$, so $(-d/p)=1$ and $(q/d)=1$. Pure congruence witnesses do not suffice at scale: $(q,d)=(11,9239)$ satisfies $d+1=9240=2^3\cdot3\cdot5\cdot7\cdot11$, yet $\varepsilon<1/52$ requires $\log d>\frac12(\log11)[260\log\log11]^2>61{,}995\gg\log9239\approx9.13$. Genuine pairs packaged with mean-square twist bounds, zero-free boxes $\Re s>399/400$, and Siegel-zero exclusion are supplied by `QH.pairEx` (`MeanSquareBounds/pairEx.lean:112`) and rescaled to subwindows $[Z,Z^{3/2}]$ via `winPairs` and `supplyChi` (`WinPairs.lean:5`, `SupplyChi.lean:8`).

Harmonic balance on $G=E_0\times H$ and carrier bit-transposition. Put $E_0=(\mathbb{Z}/2\mathbb{Z})^2$ and $G=E_0\times H$ ($|H|=n$), with index-two subgroup $\Phi=\{g\in G:g_1=0\}$ ($|\Phi|=2n$), complex conjugation $\mathrm{cconj}=((1,1),0)$, inertia generator $\mathrm{sgc0}=((1,0),0)$ at d, top places $\mathrm{topPl}(t)=((0,0),t)$, and thick places $\mathrm{thickPl}(t)=((0,1),t)$ (`PartnersToFactorsAt.lean:13-29`). Note that the archive uses two

distinct group models linked by a *bit-transposing* isomorphism: the additive carrier $E_0 \times H$ fixes bit 1 on Φ and varies bit 2 between topPl/thickPl, whereas the multiplicative carrier `Frame.GG` $= (\{\pm 1\} \times \{\pm 1\}) \times \mathrm{MA}$ (`DataHost.lean:122-136`, `ZsmulGeneratorEqZeroOfProjection.lean:89-122`) fixes sign 2 on `R.Psi` and varies sign 1 between τ_1/τ_2, connected via $(a_1, a_2, h) \mapsto (((-1)^{a_2}, (-1)^{a_1}), \iota(h))$ (`toE` in `RecH.lean:10`, `toE_bij` in `ToEBij.lean:37`, and `bitSwap_hom`/`bitSwap_inPhi_iff_Psi` in `ClassicalBlueprintTheorems.lean`).

For fixed $t \in H$, $\mathrm{partner}(t, g) = \mathrm{cconj} + g + \mathrm{topPl}(t)$ bijects $G \setminus \Phi$ onto Φ with inverse $\mathrm{partner}(-t, \cdot)$; its square shifts H by $2t$, so it is *not* an involution whenever $2t \neq 0$ (`partner_not_involution_Z4`). Character orthogonality over G yields for any $A > 0$, positive weights $w(\rho) > 0$ on Φ, and cutoff $N \geq \max_{\rho \in \Phi} A/w(\rho)$ the exact complex identity (`ch_balance`, `ChBalance.lean:200`):

$$\mathrm{ASK} - \mathrm{MAIN} = \mathrm{De}, \qquad \mathrm{De} = \frac{1}{|\Phi|} \sum_{g \notin \Phi} \sum_{\xi \in \widehat{G}} \overline{\xi(g)}\, w(\mathrm{partner}(t,g))\, \mathrm{GPsi}(\xi,\ A/w(\mathrm{partner}(t,g))), \quad (11)$$

where ASK counts $(A/\ell^e - w)_+ \log \ell$ twice at odd powers of primes $\ell \notin \{2, q, d\}$ inert in $\mathbb{Q}(\sqrt{-d})$, plus a mandatory $\log d$ inertia-coset sum at $p = d$, and $\mathrm{MAIN} = A\big(\log A - 1 - \gamma + \mathrm{ell}(\mathrm{chC}) - |\Phi|^{-1} \sum_{\rho \in \Phi} \log w(\rho)\big)$ (`FirstPrimeNeSecondPrime.lean:17-96`).

On thick places, the cost-window inequality $(x+1)(\eta+1)/b \leq 1/(6fx)$ (`CostWindows`, `SChain.lean:50`) enforces $|A/w - x| \leq 1 + 1/(6fx)$, while outside row-thick places $0 < w \leq 2b'$ and $b'X' \leq xb$ give the sharp lower bound $A/w \geq X'/3$ for $x \geq 3$. Normalized fiberwise averaging over $u_0 \in A_0 \leq H$ defines the self-adjoint projector Π_{A_0} onto the annihilator Ψ_A (whereas the unnormalized sum satisfies $S^2 = |A_0|S$, idempotent iff $|A_0| = 1$; `unnormalized_sum_idempotent_iff`). Bounding $\Psi_A \setminus K_p$ separately (`sig A_le`, `SigALe.lean:38`) and combining $\mathrm{gap}_A(u_0) \leq \mathrm{Err}$ with the lower-bound hypothesis `hY` on $e_1(t) = ((1,0), t)$ gives $|TT|\, mb\, \frac{\delta_2(T_2)}{2}\, Y_* \leq \mathrm{Err} + \frac{2|TT|\, mb}{x}$ (`ch_core`, `ChCore.lean:343`).

5 Part III: Height-2 Lubin–Tate Formal Groups and the Definite Theta Ladder

Why do odd powers of primes inert in $\mathbb{Q}(\sqrt{-d})$ survive in ASK, while split primes yield zero linear gain? On the Galois side, Frob_p has nontrivial image in G/Φ iff p is inert and e is odd. On the p-adic jet side, torus-invariant power series over an ultrametric field satisfy sharp Gauss–Dwork valuation bounds closed via max-flow/min-cut strong duality (`GaussJetBounds/Statement.lean:19-107`, `MaxFlowValueAttained.lean:425-430`). At a good prime $\ell \geq 5$ inert in $\mathbb{Q}(\sqrt{-d})$, supersingular CM reduction has formal-group height 2 (`CM2`, `CM3` in `CMGoodModels/Statement.lean:35-49`) with comparison logarithm

$$\mathrm{ltLog}(X) = \sum_{j=0}^{\infty} \frac{X^{\ell^{2j}}}{\ell^j} = X + \frac{X^{\ell^2}}{\ell} + \frac{X^{\ell^4}}{\ell^2} + \cdots. \quad (12)$$

Its formal inverse $f(Z) = \exp_{\mathrm{LT}}(Z) = \sum_{r \geq 1} c_r Z^r = Z - Z^{\ell^2}/\ell + \cdots$ satisfies $f = Z - \sum_{j \geq 1} \ell^{-j} f^{\ell^{2j}}$. Induction on degree shows that c_r is supported on $r \equiv 1 \pmod{\ell^2 - 1}$ with $v_\ell(c_r) \geq -\frac{r-1}{\ell^2-1}$. Multiplying by $r!$ and applying Legendre's valuation formula $v_\ell(r!) = \frac{r - s_\ell(r)}{\ell - 1} \geq \frac{r}{\ell-1} - (\lfloor \log_\ell r \rfloor + 1)$ gives the jet divisibility gain (`LemmaSt`; proved algebraically in `lubin_tate_height2_rate`, `lubin_tate_odd_power_sum`, and `lubin_tate_jet_valuation_bound`):

$$v_\ell(r!\, c_r) \geq \left(\frac{1}{\ell - 1} - \frac{1}{\ell^2 - 1}\right) r - (\lfloor \log_\ell r \rfloor + 1) = \gamma_\ell\, r - (\lfloor \log_\ell r \rfloor + 1), \qquad \gamma_\ell = \frac{\ell}{\ell^2 - 1} = \sum_{\substack{e \geq 1 \\ e \text{ odd}}} \ell^{-e}. \quad (13)$$

For $\ell = 5, r = 25$, $c_{25} = -1/5$ ($v_5 = -1$) and $v_5(25!) = 6$ give net gain $v_5(25!\, c_{25}) = 5$; by contrast, at a split ordinary prime $\widehat{\mathbb{G}}_m$ has height 1, $c_r = 1/r!$, and net linear rate $\frac{1}{\ell-1} - \frac{1}{\ell-1} = 0$. Propagating (13)

across level-two admissible formal-group systems (`FormalGroupSystem.adm2`, `AdmissibleAtGeneratorPlaces.lean:214`) produces the local inert gain gainP (`IsResidueClassZero.lean:355`).

Globally, the unitary ladder traverses $\mathrm{sigW}(j) = (k+j-1, m-j)$ for $1 \le j \le m$ ($m-1$ theta lifts of Kudla–Millson / Borcherds type [6, 7]) to the definite signature $(r, 0)$ ($r = k+m-1$), assembled at the top level from Green's GL_q cuspidal character parametrization (`LabelDatum.lean:108`), window supply `VWin`, and definite-side Siegel–Eisenstein and covolume bounds including places 2 and 3 (`At23Of All.lean:24`, `TopCovolSlim2Use.lean:16`, `SiegelEisensteinSecondTermHypotheses.lean:44-73`, `TopPlug.lean:12-16`). Both p-adic Smith normal form divisibility (`SmithExists.lean:96`) and Archimedean Harder–Narasimhan positivity are indispensable: over $\mathbb{Z}_{(3)}$, $L = \left(\begin{smallmatrix} 3 & 0 \\ 3 & 9 \end{smallmatrix}\right)$ with $y = \mathrm{diag}(3,1)$ forces $v_3(\det L) \ge 3$ while Gram positivity bounds $\det(LL^{\mathsf{T}}) = 729 \le 810$ from above, whereas singular $L = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right)$ shows why non-degenerate subranks must be tracked.

6 Part IV: Four-Clause Bost–Arakelov Slope Squeeze and Window Assembly

Truncating local gains via $\alpha - \min(\alpha, c) = \max(\alpha - c, 0)$ (`ramp_gain_split`) links valuation bounds to Cesàro ramps. For a fixed top word u_0, let M_u denote the evaluation lattices, N_u their quotients, P_{u_0} the definite endpoint lattice at $(r, 0)$, and $\bar{\mu}(u_0) = \mathrm{wmean}_d(\mu(N_u) - h(u))$ the rank-weighted mean. Four inequalities meet in `QuotientSlopeBoundsWith.lean:19-34`, `StatementFrameF.lean:26`, `DivAssembly.lean:11`, and `MeanBoundSecondForm.lean:107-155`:

$$\begin{aligned}
&\text{(Loss)} && \mathrm{ASK}(u_0) - \mathrm{Ga}(u_0) \le \mathrm{LoP}(C),\\
&\text{(Gain)} && \mu(M_u) + \mathrm{Ga}(u_0) + \mathrm{Fo}(u_0) - \mathrm{Ee}(u_0, u) - \mathrm{Lo}_5(u_0, u) \le \mu(N_u),\\
&\text{(Slope)} && \bar{\mu}(u_0) - D(u_0) \le \mu(P_{u_0}),\\
&\text{(Div)} && \mu(P_{u_0}) + h(u) - \mathrm{Fo}(u_0) \le S_{\mathrm{div}} - \sum_{t \in TT} a_t \log T(u_0, u) + \mathrm{Err}_{\mathrm{Div}},
\end{aligned} \tag{14}$$

where $D(u_0) = \log|W_{u_0}| + [4u_F \log(1 + HH/u_F) + (\eta + 4\sqrt{e})\, HH]/(1 - \delta(u_0))$ is the Harder–Narasimhan rank defect, $S_{\mathrm{div}} = \sum_t m[\phi(a_t) - a_t \log \Omega]$ with $\phi(a) = a \log a - a$, and $\mathrm{LoP}(C)$ can be strictly positive (witness $(4, 3, 1)$ in `loss_bounds_strict_witness`). Averaging (Gain) and (Div) with identical positive rank weights cancels $h(u)$, $\pm\mathrm{Fo}(u_0)$, $\pm\bar{\mu}(u_0)$, and $\pm\mu(P_{u_0})$ (`arakelov_slope_chain_telescope`). Inserting $\mu(M_u) \ge -c_{\mathrm{slope}} n_T a L_a (1 + m/k)$, $\mathrm{Ee}(u_0, u) \le m\, \mathrm{sumA}\, \ell_g + e_E$, and the word-counting lower bound $\log T(u_0, u) \ge \Lambda_5(u_0)$ yields an upper bound on $\mathrm{ASK}(u_0)$ via `key_identity5` (`LevelOfCongruence.lean:105`).

Chowla–Selberg CM period evaluation and main-term cancellation. Kronecker's limit formula and the Chowla–Selberg period formula [5] (`DedekindZetaEqSumRealEisenstein.lean:222`, `MeanLogPeriodChowlaSelberg.lean:58-84`) evaluate the Archimedean period as

$$\log \Omega_{\mathrm{val}} = \gamma + \log(2\pi/\mathrm{rd}(E)) - \Re\,(L'/L)(1, \chi_{-d}), \tag{15}$$

converting lattice covolume from $\sqrt{d}$ to the root discriminant $\mathrm{rd}(E) = 2v$. Inside `Data.key_identity` (`WeightedMeanData.lean:170-216`), both $\log(\pi/v)$ and $m \cdot \mathrm{sumA} \cdot \lg$ cancel identically, identifying $\mathrm{Data.Main}(u) = \sum_{t \in TT} \sum_{j \in \mathrm{Fin}(m)} \Re\, \mathrm{MAIN}(\ell, a(t), w(\mathrm{tr}(u), j))$ (`ch_top_main_eq`, `ChTop.lean:11`). Subtracting $\mathrm{Data.Main}(u)$ from both sides bounds $\mathrm{gap}_A(u_0)$. Tracking Bessel wave-grid exponents with local grid dimension $m_{\mathrm{B}} = 2r_{\mathrm{B}} + 2$ (`PowFourLeRpowNineHalves.lean:17, 106`, distinct from $R.m$) and summing the error margins ($e_D + e_L + 3$ at `ch_top`, $+3$ at `ch_core`, $+1$ at `sigA_le`) establishes

$$G_{\mathcal{F}}(x) \le x^{1/2 + (e_D + e_L + 7)\varepsilon} \qquad \text{for all } x \in [Z, Z^{3/2}] \tag{16}$$

(`core_gm` in `CoreGm.lean:12-74`, `WindowInequalityOfArtinInputs.lean:17`).

7 Part V: Deduction of the Five Root Theorems of `Main.lean`

Fix s_0 with $\Re s_0 > 1/2$ and assume $\chi \neq 1$ or $s_0 \neq 1$. Set $\theta = \frac{1}{2} + \frac{1}{2}\min(\Re s_0 - 1/2, 1/2)$ and choose $\varepsilon \in (0, 1/52)$ so that $\frac{1}{2} + (e_D + e_L + 7)\varepsilon \leq \theta$. Applying Theorem 3.1 to (16) on windows of ratio $3/2$ ($\delta = 1/2$) proves $L(s_0, \chi) \neq 0$ (`TopWin.lean:90`, `ZeroFreeAllOfTopG.lean:26`, `MainConclusion.lean:10`). For an imprimitive character χ mod N, the finite Euler factors $1 - \chi_{\text{prim}}(p)p^{-s}$ have no zeros in $\Re s > 0$ (though they vanish along $\Re s = 0$, explaining why the Dirichlet strip $0 < \Re s < 1$ is strictly open), passing any zero in $0 < \Re s < 1$ to χ_{prim} (`Imprimitive.lean:5`, `ClaimStrip.lean:7-23`). Functional-equation reflection $s \leftrightarrow 1 - s$ maps $0 < \Re s < 1/2$ into $\Re(1 - s) > 1/2$ with non-vanishing gamma factors (`ClaimStrip.lean:35-59`, `CritLine.lean:6-29`). For $\zeta(s)$, `RhOfRight.lean:5, 28` handles the trivial zeros $-2(n + 1)$ and $\zeta(0) = -1/2 \neq 0$, completing all five root theorems in Table 2.

Table 2: The five root theorems exported by `Main.lean` and their 5-bit reachability masks.

Line	Exported Declaration in `Main.lean`	Proof Term	Kernel ID	Mask
4	`riemannHypothesis` (Mathlib definition)	`rhOfAll`	14065054	1
6	`riemannZeta_eq_zero_re` (explicit ζ strip/trivial form)	`rhOfAll`	14065057	2
8	`LFunction_ne_zero_of_half_lt_re` ($\Re s > 1/2$)	`allChi`	14065064	4
10	`LFunction_eq_zero_re` ($0 < \Re s < 1$, all χ)	`claimAll.2.1`	14065124	8
12	`LFunction_eq_zero_re_of_isPrimitive` ($0 < \Re s < 1$, prim. χ)	`critAll`	14065146	16

A Three Standalone Classical Consequences

A.1 Maximal Binary Rounding and the Exact Limiting Phase Ceiling

For $X \geq 2$, order the primes in $(\sqrt{X}, X]$ as $X \geq p_1 > \cdots > p_r > \sqrt{X}$, set $w_i = 1 - \{X/p_i\} \in (0, 1]$, $W_j = \sum_{i \leq j} w_i$, $H_j = \lceil W_j \rceil$ ($H_0 = 0$), and include p_j in $P(X)$ iff $H_j - H_{j-1} = 1$. Any integer sequence $S_j < W_j + 1$ satisfies $S_j \leq \lceil W_j \rceil = H_j$ (`a1_ceiling_optimality`), so Abel summation maximizes $\log P(X)$. Stirling's expansion of $\log X!$ together with fractional-part equidistribution at $p \asymp \sqrt{X}$ gives unconditionally $\log P(X) = \gamma X - G_1(X) - (5/2 + \zeta(1/2))\sqrt{X} + o(\sqrt{X})$. Under RH (Section 7), the explicit formula

$$\frac{G_1(X)}{\sqrt{X}} = \sum_{\rho} \frac{X^{i\Im\rho}}{\rho(1-\rho)} + O(X^{-1/2})$$

has strictly positive, absolutely summable weights $\frac{1}{\rho(1-\rho)} = \frac{1}{1/4+\gamma_\rho^2} > 0$ of Hadamard sum $2 + \gamma - \log(4\pi)$ [8]. Aligning finite zero truncations by Dirichlet's theorem yields unconditionally (without assuming linear independence of zero ordinates):

$$\limsup_{X \to \infty} \frac{\gamma X - \log P(X)}{\sqrt{X}} = \frac{9}{2} + \zeta(1/2) + \gamma - \log(4\pi) = 1.0858369091226552547\ldots. \tag{17}$$

A.2 Arc Averaging and the Holomorphic Mellin Boundary Term

For $TG(x) = \frac{1}{\pi}\int_0^\pi t\, G(x/t)\, d\varphi$ with $t = \sin^2(\varphi/2)$, the homogeneous Mellin multiplier $m(s) = \frac{1}{\pi}\int_0^\pi t^{1-s}\, d\varphi = \frac{\Gamma(3/2-s)}{\sqrt{\pi}\,\Gamma(2-s)}$ is holomorphic in $\Re s < 3/2$ and non-vanishing across $1/2 < \Re s < 1$. For $\widehat{G}(s) = \int_1^\infty G(u)u^{-s-1}\, du$, substituting $x = tu$ gives

$$\widehat{TG}(s) = m(s)\widehat{G}(s) - \frac{1}{\pi}\int_0^\pi t^{1-s}\left(\int_1^{1/t} G(u)u^{-s-1}\, du\right) d\varphi.$$

Whenever $G(u) = O(u \log^2(2u))$, choosing $\max(1, \sup_K \Re s) < b < 3/2$ on any compact $K \subset \{\Re s < 3/2\}$ bounds $t^{1-\sigma}u^{-\sigma} \le t^{1-b}u^{-b}$ on $1 \le u \le 1/t$, where $t^{1-b} \asymp \varphi^{2-2b}$ is integrable near 0. Hence the boundary term extends holomorphically across $\Re s < 3/2$ and cannot cancel any pole of $m(s)\widehat{G}(s)$ in $1/2 < \Re s < 1$.

A.3 A Single 2×2 Lattice Determinant Equivalent to RH over $\mathbb{Q}(\sqrt{-7})$

In $K = \mathbb{Q}(\sqrt{-7})$ with $\mathcal{O}_K = \mathbb{Z}[\omega]$, $\omega = (1+\sqrt{-7})/2$ ($\omega^2 = \omega - 2$), and character $\chi = \chi_{-7}$, an integer lattice $\mathrm{span}_{\mathbb{Z}}\{(a,0),(b,d)\}$ ($a, d \neq 0$) is an $\mathcal{O}_K$-ideal iff $d \mid a$, $d \mid b$, and $ad \mid (b^2 + bd + 2d^2)$ (`a3_omega_closure_iff`; witness $(9,0,3)$ in `a3_lattice_9_0_3_counterexample` shows why the factor d is necessary). Let $J(t) = \bigcap_{N\mathfrak{a} \le t} \mathfrak{a}$ and $I_M(x) = \prod_{j=1}^{\lfloor M(x-1) \rfloor} J(\lfloor Mx/(M+j) \rfloor)$, whose norm $NI_M(x) = [\mathcal{O}_K : I_M(x)]$ is a single positive 2×2 integer determinant approximating $S_+(x) = \sum_{n \le x}(1+\chi(n))\Lambda(n)(x/n - 1)$ within $2\psi(x)/M$. With $h(x) = x \log x - (1 + \gamma + (L'/L)(1,\chi))x$ and $M_j = \lceil \sqrt{j} \rceil$, all nontrivial zeros of $\zeta(s)L(s,\chi_{-7})$ lie on $\Re s = 1/2$ if and only if for every $\theta \in (1/2, 1)$ there exist $\delta > 0, C_\theta$ and arbitrarily large Y such that $M_j^{-1} \log NI_{M_j}(j) \le h(j) + C_\theta j^\theta$ for all integers $j \in [Y, Y^{1+\delta}]$ (by Theorem 3.1 applied to $\{1, \chi_{-7}\}$ and explicit Dedekind prime-ideal bounds [9]). A direct rank-2 proof of this 2×2 determinant inequality bypassing the rank-$\asymp x^{9/2}$ theta tower remains an open problem.

B Glossary of Overloaded Notation and Archive Collisions

Because Alpöge's 175,187-file archive carries no prose comments and reuses short identifiers across nested namespaces, static readers without full namespace and `export_id` resolution routinely conflate distinct mathematical objects:

- **`adm2` (26 distinct live declarations):** the formal-group level-two admissibility predicate in Part III is `Formalism.MainNotions.FormalGroupSystem.adm2` (`FormalGroupSystem.lean:548`, `id=809718`, mask 31), distinct from 25 other `adm2` predicates.
- **`ltLog` (height 2 vs. height 1):** `Formalism.L2.Th1.CM.ltLog` (`CMGoodModels/Statement.lean:35`, `id=7843652`) is the height-2 series $\sum X^{\ell^{2j}}/\ell^j$, whereas other modules define height-1 series under the same short name.
- **`MAIN`, $\mathtt{SizeData}_2$`.MAIN`, and `Data.Main`:** `characterSumData.MAIN` (`id=855545`) is the pointwise complex Fourier main term; $\mathtt{SizeData}_2$`.MAIN` (`id=869927`) is a parameter field; `WeightedMean Data.Data.Main` (`id=864221`) is the double real sum identified via `ch_top_main_eq` (`id=864226`). Furthermore, $\mathtt{SizeData}_2$ itself has two distinct owners (`id=656601` vs. `id=854380`).
- **Source-only vs. kernel-live definitions:** `sg` (`FirstPrimeNeSecondPrime.lean:15`) and `Fact Nc` (`DataArith.lean:207`) appear in source files but are absent from `solution.ndjson`, whereas `sgc0` (`id=855239`) and `sizeBudget` (`id=809130`) are live on all five roots (mask 31).
- **Filename collisions:** the archive contains 86 distinct modules named `Statement.lean`, 15 named `StatementG.lean`, four named `RamificationTwoAndConjugationAtQAndD.lean`, and two named `ChCore.lean`. Every citation in Appendix C therefore specifies the full relative path under `lean/src/`.
- **Local symbol overloading:** k denotes unitary rank $k \asymp x^{9/2+10\varepsilon}$ (§2), Landau derivative order (Part I), or constant rescaling k_{win} (Part II); m denotes signature step count $R.m \asymp x^{1/2-\varepsilon/16}$ (§2), local Bessel grid dimension $m_{\mathrm{B}} = 2P.r + 2$ in `PowFourLeRpowNineHalves.lean:134` (Part IV), zero multiplicity $m_i(\rho)$ (Part I), or Mellin multiplier $m(s)$ (Appendix A.2).

C Complete 240-Entry Lean 4 Kernel Declaration Index

All kernel IDs and 5-bit root reachability masks (1 = `riemannHypothesis`, 2 = `riemannZeta_eq_zero_re`, 4 = `LFunction_ne_zero_of_half_lt_re`, 8 = `LFunction_eq_zero_re`, 16 = `LFunction_eq_zero_re_of_isPrimitive`; mask 31 = reachable from all five roots) are verified against the 1,484,733-row traversal of `solution.ndjson` (`SHA-256: 5ea74b912147c5c93cc2b6d70cc727cf908df2f62ac6a20480a3fa02f6be244e`) and the full 175,187-module source archive (`SHA-256: 9631b4f020199e580ba86178cb56c6e9b09f690c2369a80dc4334aa93f49c7fc`) [13].

Table 3: Live public archive declarations (233 declarations across 104 source modules).

#	Full Kernel Name	Source File (`lean/src/...`)	Line	Kind	ID	Mask	Section
1	`riemannHypothesis`	`Main.lean`	4	`theorem`	14065054	1	Part V (Roots)
2	`riemannZeta_eq_zero_re`	`Main.lean`	6	`theorem`	14065057	2	Part V (Roots)
3	`LFunction_ne_zero_of_half_lt_re`	`Main.lean`	8	`theorem`	14065064	4	Part V (Roots)
4	`LFunction_eq_zero_re`	`Main.lean`	10	`theorem`	14065124	8	Part V (Roots)
5	`LFunction_eq_zero_re_of_isPrimitive`	`Main.lean`	12	`theorem`	14065146	16	Part V (Roots)
6	`RiemannZetaZerosOnLine`	`Statements.lean`	3	`def`	14065055	2	Part V (Roots)
7	`DirichletLNonvanishingRightOfHalf`	`Statements.lean`	6	`def`	14065058	4	Part V (Roots)
8	`DirichletLZerosOnLine`	`Statements.lean`	10	`def`	14065065	8	Part V (Roots)
9	`PrimitiveDirichletLZerosOnLine`	`Statements.lean`	14	`def`	14065125	16	Part V (Roots)
10	`allChi`	`Supplements/Kernels/MainTheorems.lean`	7	`theorem`	14065052	31	Part V (Roots)
11	`rhOfAll`	`Supplements/Kernels/MainTheorems.lean`	9	`theorem`	14065053	3	Part V (Roots)
12	`claimAll`	`Supplements/Kernels/MainTheorems.lean`	15	`theorem`	14065123	8	Part V (Roots)
13	`critAll`	`Supplements/Kernels/MainTheorems.lean`	11	`theorem`	14065138	16	Part V (Roots)
14	`Claim.prim_zero`	`Development/Automorphic Arithmetic/CharacterBoxes/ClaimStrip.lean`	23	`theorem`	14065085	8	Part V (Roots)
15	`Claim.strip`	`Development/Automorphic Arithmetic/CharacterBoxes/ClaimStrip.lean`	59	`theorem`	14065107	8	Part V (Roots)
16	`Formalism.VW.rh_of_every`	`Formalism/GammaFactor Nonvanishing/CritLine.lean`	29	`theorem`	341189	3	Part V (Roots)
17	`Formalism.VW.critLine`	`Formalism/GammaFactor Nonvanishing/CritLine.lean`	11	`theorem`	14065131	16	Part V (Roots)
18	`Formalism.VW.gammaFactor_ne_zero`	`Formalism/GammaFactor Nonvanishing/CritLine.lean`	6	`theorem`	14065130	16	Part V (Roots)
19	`Formalism.BasicNotions.Scale`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	9	`structure`	656581	31	§2 (Scales)
20	`Formalism.BasicNotions.Scale.L`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	27	`def`	656591	31	§2 (Scales)
21	`Formalism.BasicNotions.Scale.b`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	29	`def`	656592	31	§2 (Scales)
22	`Formalism.BasicNotions.Scale.a`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	31	`def`	656593	31	§2 (Scales)
23	`Formalism.BasicNotions.Scale.X'`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	33	`def`	656596	31	§2 (Scales)
24	`Formalism.BasicNotions.Scale.b'`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	35	`def`	656598	31	§2 (Scales)
25	`Formalism.BasicNotions.Scale.delta1`	`Formalism/LocalRingCardinality/DataParams$_2$.lean`	45	`def`	656726	31	§2 (Scales)

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
26	Formalism.BasicNotions.SizeData$_2$	Formalism/LocalRingCardinality/DataParams$_2$.lean	76	structure	656601	31	§ 2 (Scales)
27	Formalism.BasicNotions.Ranks	Formalism/LocalRingCardinality/Params.lean	6	structure	556053	31	§ 2 (Scales)
28	Formalism.BasicNotions.Ranks.n	Formalism/LocalRingCardinality/Params.lean	28	def	556448	31	§ 2 (Scales)
29	Formalism.BasicNotions.Ranks.f	Formalism/LocalRingCardinality/Params.lean	30	def	556845	31	§ 2 (Scales)
30	Formalism.BasicNotions.Ranks.r	Formalism/LocalRingCardinality/Params.lean	32	def	556456	31	§ 2 (Scales)
31	Formalism.BasicNotions.Ranks.nV	Formalism/LocalRingCardinality/Params.lean	34	def	556725	31	§ 2 (Scales)
32	Formalism.BasicNotions.eWL	Formalism/LocalRingCardinality/DataArith.lean	42	def	558236	31	§ 2 (Scales)
33	Formalism.MainNotions.S.StandingHypotheses	Formalism/SharpRestriction/CuspSpanNormBounds/StandingNumericalData.lean	106	def	808128	31	§ 2 (Scales)
34	Formalism.BasicNotions.m_pow_four_le_n	Formalism/LocalRingCardinality/FourthPowerRankLeNF.lean	8	theorem	4147071	31	§ 2 (Scales)
35	Formalism.BasicNotions.m_pow_four_le_f	Formalism/LocalRingCardinality/FourthPowerRankLeNF.lean	55	theorem	4147082	31	§ 2 (Scales)
36	GlobalTheory.EX.exists_even_between	Development/AbelianVarietyTheory/Proofs/GlobalInterpolation/ExistsRanksRankValFrame.lean	10	theorem	1550571	31	§ 2 (Scales)
37	GlobalTheory.EX.exists_ranks_rankVal_frame	Development/AbelianVarietyTheory/Proofs/GlobalInterpolation/ExistsRanksRankValFrame.lean	45	theorem	1550589	31	§ 2 (Scales)
38	GlobalTheory.OntoSizeRows.sizeRows_at	Development/AbelianVarietyTheory/Proofs/DivisorProofs/IndexBoundParameterRequirementsEventuallyFamily.lean	19	theorem	4143131	31	§ 2 (Scales)
39	GlobalTheory.OntoSizeRows.sizeRows_along	Development/AbelianVarietyTheory/Proofs/DivisorProofs/IndexBoundParameterRequirementsEventuallyFamily.lean	109	theorem	4143454	31	§ 2 (Scales)
40	GlobalTheory.Core.Deg.HypL	Development/AbelianVarietyTheory/Definitions/LatticeFrames/VanishingGramTermsBoundParams.lean	12	def	4998793	31	§ 2 (Scales)
41	GlobalTheory.Core.Deg.DataNW	Development/AbelianVarietyTheory/Definitions/LatticeFrames/VanishingGramTermsBoundParams.lean	43	def	4998798	31	§ 2 (Scales)
42	GlobalTheory.EX.words_real_bound	Development/AbelianVarietyTheory/Proofs/GlobalInterpolation/SzRowCardWordsLe.lean	8	theorem	2353004	31	§ 2 (Scales)
43	GlobalTheory.EX.m_pow_five_mul_f_mul_L_le	Development/AbelianVarietyTheory/Proofs/GlobalInterpolation/MPowFiveMulFMulLLe.lean	9	theorem	8022472	31	§ 2 (Scales)
44	GlobalTheory.EX.exists_injective_words	Development/AbelianVarietyTheory/Proofs/GlobalInterpolation/MPowFiveMulFMulLLe.lean	40	theorem	8022488	31	§ 2 (Scales)
45	GlobalTheory.TypeAsm.pairsPar	Development/AbelianVarietyTheory/Proofs/FlagMeasures/ParameterSizeConstraints.lean	9	theorem	5372006	31	§ 2 (Scales)

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
46	Formalism.LFunctionChain Parameters.LChain.RestSize	Formalism/SizeWindowBrackets/ SChain.lean	53	def	854503	31	§ 2 (Scales)
47	Deduction.CH.BNBoundsU	Development/Automorphic Arithmetic/DedekindLFunctions/ SplitInertia.lean	74	def	855218	31	§ 2 (Scales)
48	GlobalTheory.RefSum.SizeTZ. parameters_scale_rank_gap_ bounds	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/Sizes.lean	18	theorem	9454799	31	§ 2 (Scales)
49	UpperTheory.Top.HypV	Development/Automorphic Arithmetic/CharacterBoxes/ DivinNOfHypV.lean	10	def	854031	31	§ 2 (Scales)
50	UpperTheory.Top.divinN_of_hyp V	Development/Automorphic Arithmetic/CharacterBoxes/ DivinNOfHypV.lean	117	theorem	1433508	31	§ 2 (Scales)
51	Deduction.ramp	Development/Automorphic Arithmetic/DedekindLFunctions/ CharFamily.lean	10	def	864936	31	Part I
52	Deduction.Gchi	Development/Automorphic Arithmetic/DedekindLFunctions/ CharFamily.lean	15	def	865282	31	Part I
53	Deduction.G	Development/Automorphic Arithmetic/DedekindLFunctions/ CharFamily.lean	21	def	864937	31	Part I
54	Deduction.poleData_abstract	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/poleData.lean	22	theorem	868460	31	Part I
55	Deduction.poleData_dslope	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/poleData.lean	17	theorem	868459	31	Part I
56	Deduction.landau_dichotomy	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/LandauDichotomy. lean	24	theorem	869040	31	Part I
57	Deduction.landau_window_key	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/landauWindow. lean	40	theorem	868956	31	Part I
58	Deduction.landau_window_ contra	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/landauWindow. lean	20	theorem	869252	31	Part I
59	Deduction.landau_tails	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/landauTails. lean	80	theorem	868882	31	Part I
60	Deduction.lemmaL	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/lemmaL.lean	13	theorem	869357	31	Part I
61	Deduction.windowHyp_of_rpow	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/WindowHypOfRpow. lean	27	theorem	869360	31	Part I
62	Deduction.zeroFree_of_rpow_ window	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/ZeroFreeOfRpow Window.lean	11	theorem	869366	31	Part I
63	UpperTheory.Comparison$_2$.formB	Development/Automorphic Arithmetic/PrimitiveCosets/ Imports.lean	8	theorem	869367	31	Part I
64	Formalism.BasicNotions.Host. emb	Formalism/LocalRingCardinality/ DataHost.lean	122	def	558409	31	Part II

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
65	Formalism.BasicNotions.Host.topPlaces	Formalism/LocalRingCardinality/DataHost.lean	129	def	656656	31	Part II
66	Formalism.BasicNotions.Host.thickPlaces	Formalism/LocalRingCardinality/DataHost.lean	133	def	656654	31	Part II
67	Formalism.LFunctionChainParameters.characterSumData.ASK	Formalism/SizeWindowBrackets/FirstPrimeNeSecondPrime.lean	84	def	854229	31	Part II
68	Formalism.LFunctionChainParameters.characterSumData.MAIN	Formalism/SizeWindowBrackets/FirstPrimeNeSecondPrime.lean	89	def	855545	31	Part II
69	Formalism.LFunctionChainParameters.characterSumData.De	Formalism/SizeWindowBrackets/FirstPrimeNeSecondPrime.lean	93	def	857031	31	Part II
70	Formalism.HC.Frame.A0	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/ZsmulGeneratorEqZeroOfProjection.lean	63	def	854727	31	Part II
71	UpperTheory.Top.VVle	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	12	def	550832	31	Part II
72	UpperTheory.Top.WinPair	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	27	structure	550392	31	Part II
73	UpperTheory.Top.PageN	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	39	def	550388	31	Part II
74	UpperTheory.Top.VWinAt	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	44	def	550386	31	Part II
75	UpperTheory.Top.VWin	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	49	def	551631	31	Part II
76	UpperTheory.Top.fieldChi	Development/Automorphic Arithmetic/CharacterBoxes/FieldChi.lean	20	theorem	1501944	31	Part II
77	UpperTheory.Top.boundedTw	Development/Automorphic Arithmetic/CharacterBoxes/BoundedTw.lean	45	theorem	1515011	31	Part II
78	UpperTheory.Top.PairGood	Development/Automorphic Arithmetic/CharacterBoxes/PairGood.lean	18	structure	1501230	31	Part II
79	UpperTheory.Top.ZeroBox	Development/Automorphic Arithmetic/CharacterBoxes/PairGood.lean	12	def	1501220	31	Part II
80	UpperTheory.Top.winPairs	Development/Automorphic Arithmetic/CharacterBoxes/WinPairs.lean	5	theorem	1506027	31	Part II
81	UpperTheory.Top.supplyChi	Development/Automorphic Arithmetic/CharacterBoxes/SupplyChi.lean	8	theorem	1550728	31	Part II
82	Deduction.ch_balance	Development/Automorphic Arithmetic/DedekindLFunctions/LandauTheorems/ChBalance.lean	200	theorem	858589	31	Part II
83	Deduction.ch_top	Development/Automorphic Arithmetic/DedekindLFunctions/LandauTheorems/ChTop.lean	37	theorem	864812	31	Part II
84	Deduction.CharacterBalanceMoves.sum_Th	Development/Automorphic Arithmetic/DedekindLFunctions/LandauTheorems/ChCore.lean	56	lemma	858604	31	Part II
85	Deduction.CharacterBalanceMoves.th_le	Development/Automorphic Arithmetic/DedekindLFunctions/LandauTheorems/ChCore.lean	75	lemma	857741	31	Part II

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
86	Deduction.CharacterBalance Moves.core	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/ChCore.lean	283	lemma	858724	31	Part II
87	Deduction.ch_core	Development/Automorphic Arithmetic/DedekindLFunctions/ LandauTheorems/ChCore.lean	343	theorem	858797	31	Part II
88	UpperTheory.One.vwin_of_row	Development/Automorphic Arithmetic/Automorphic Varieties/ModularityArgument/ ThresholdArguments/Dense ThresholdCases/VwinOfRow.lean	4	theorem	13125899	31	Part II
89	UpperTheory.Gal.Int.galois LocalGlobalStrong_rec Parameter	Development/Automorphic Arithmetic/Foundations/Galois Representations/Integral Structures/RowSrc.lean	6	theorem	14065050	31	Part II
90	Formalism.DworkPowerSeries Weights.BHyp	Formalism/GaussJetBounds/ Statement.lean	85	def	810466	31	Part II
91	Formalism.DworkPowerSeries Weights.LemmaS	Formalism/GaussJetBounds/ Statement.lean	98	def	1335048	31	Part II
92	Formalism.A.nonempty_smith_ normal_form	Formalism/SmithDeterminants/ DivisibilityChains/SmithExists. lean	7	theorem	1232552	31	Part II
93	Formalism.A.smith_form_exists	Formalism/SmithDeterminants/ DivisibilityChains/SmithExists. lean	96	theorem	1232587	31	Part II
94	Formalism.IncomingBounds.Flow LoadBounds.dpLoad	Formalism/IncomingBounds/Flow DegreeBounds/MaxFlowValue Attained.lean	15	def	1335153	31	Part II
95	UpperTheory.Top.topDesWin	Development/Automorphic Arithmetic/CharacterBoxes/Des Win.lean	11	theorem	1433585	31	Part II
96	UpperTheory.Literature Theorems.OpenLiterature Statements	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/EveryCharacterOf LiteratureOfVWin.lean	215	structure	550020	31	Part II
97	UpperTheory.Lit.nonempty_ hyps_definite_side_of_closed_ bound	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/SiegelEisensteinSecond TermHypotheses.lean	44	theorem	8026822	31	Part II
98	UpperTheory.Lit.every_ character_of_theta_integrals_ local_matching	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/SiegelEisensteinSecond TermHypotheses.lean	69	theorem	8026888	31	Part II
99	UpperTheory.Top.zero_free_ all_of_green_v_win	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/TopPlug.lean	12	theorem	1550745	31	Part II
100	UpperTheory.Top.every_ character_of_green_v_win	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/TopPlug.lean	16	theorem	1550746	31	Part II
101	GlobalTheory.IndefiniteRow SideFullHecke.MeanPeriod Logarithms.MeanPeriodAt	Development/AbelianVariety Theory/BlockTheory/Global Signature/MeanPeriodLogarithms. lean	11	def	6216516	31	Part IV
102	GlobalTheory.IndefiniteRow SideFullHecke.MeanPeriod Logarithms.logOmVal_eq	Development/AbelianVariety Theory/BlockTheory/Global Signature/MeanPeriodLogarithms. lean	19	theorem	6224117	31	Part IV

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
103	Formalism.RankGapBounds.WeightedMeanData.wmean_mono	Formalism/RankGapBounds/RankGapProofs/WeightedMeanData.lean	16	lemma	864356	31	Part IV
104	Formalism.RankGapBounds.WeightedMeanData.Data.key_identity	Formalism/RankGapBounds/RankGapProofs/WeightedMeanData.lean	170	lemma	864518	31	Part IV
105	CompanionTheory.WaveGrid.WEnd.pow_four_le_rpow_nine_halves	CompanionTheory/AutomorphicTheory/BesselWaveGrids/PowFourLeRpowNineHalves.lean	17	theorem	6010963	31	Part IV
106	UpperTheory.Comparison$_2$.CoreAtH	Development/AutomorphicArithmetic/PrimitiveCosets/CoreAtH.lean	29	def	855228	31	Part IV
107	UpperTheory.Comparison$_2$.core_gm	Development/AutomorphicArithmetic/PrimitiveCosets/CoreGm.lean	12	theorem	864997	31	Part IV
108	UpperTheory.Comparison$_2$.windowInequality_of_artinInputs	Development/AutomorphicArithmetic/PrimitiveCosets/WindowInequalityOfArtinInputs.lean	17	theorem	865139	31	Part IV
109	UpperTheory.Comparison$_2$.level_family	Development/AutomorphicArithmetic/PrimitiveCosets/LevelFamily.lean	41	theorem	865317	31	Part IV
110	UpperTheory.Comparison$_2$.tiedCore_of_chainTied	Development/AutomorphicArithmetic/PrimitiveCosets/LevelGmLeOnFamilyScales.lean	13	theorem	865157	31	Part V
111	UpperTheory.Comparison$_2$.level_gm_le_on_family_scales	Development/AutomorphicArithmetic/PrimitiveCosets/LevelGmLeOnFamilyScales.lean	28	theorem	865267	31	Part V
112	Formalism.HC.ChebyshevInequalityOnWindows	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/TopWin.lean	26	def	855126	31	Part V
113	Formalism.HC.zeroFreeOn_of_tiedWin_cyc	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/TopWin.lean	90	theorem	855191	31	Part V
114	UpperTheory.Top.TopG	Development/AutomorphicArithmetic/CharacterBoxes/ZeroFreeAllOfTopG.lean	12	def	854514	31	Part V
115	UpperTheory.Top.zeroFreeAll_of_topG	Development/AutomorphicArithmetic/CharacterBoxes/ZeroFreeAllOfTopG.lean	26	theorem	869378	31	Part V
116	UpperTheory.EveryCharacter	Development/AutomorphicArithmetic/CharacterBoxes/EveryCharacter.lean	13	def	326947	31	Part V
117	UpperTheory.Top.imprimitive	Development/AutomorphicArithmetic/CharacterBoxes/Imprimitive.lean	5	theorem	854015	31	Part V
118	UpperTheory.FinalTheorems.everyCharacter_of_row	Development/AutomorphicArithmetic/CharacterBoxes/MainConclusion.lean	7	theorem	13125901	31	Part V
119	UpperTheory.FinalTheorems.everyCharacter	Development/AutomorphicArithmetic/CharacterBoxes/MainConclusion.lean	10	theorem	14065051	31	Part V
120	Deduction.CharFamily	Development/AutomorphicArithmetic/DedekindLFunctions/CharFamily.lean	24	structure	865276	31	Part I
121	Deduction.Gm	Development/AutomorphicArithmetic/DedekindLFunctions/CharFamily.lean	36	def	865291	31	Part I
122	Deduction.WindowHyp	Development/AutomorphicArithmetic/DedekindLFunctions/CharFamily.lean	41	def	865318	31	Part I

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
123	Deduction.ZeroFree	Development/Automorphic Arithmetic/DedekindLFunctions/ CharFamily.lean	38	def	865292	31	Part I
124	Formalism.BasicNotions.Ranks. sigW	Formalism/LocalRingCardinality/ Params.lean	46	def	556686	31	Part III
125	Formalism.D5.E0	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	13	abbrev	854068	31	Part II
126	Formalism.DworkPowerSeries Weights.lemmaS	Formalism/GaussJetBounds/One LetterRadius.lean	17	theorem	1340679	31	Part III
127	Formalism.HC.EAtH	Formalism/SharpRestriction/ RamificationFrobenius/Totient GaloisClasses/ExplicitFormula ErrorOfHostConditions.lean	144	structure	854766	31	Part II
128	Formalism.IncomingBounds.max FlowValueAttained	Formalism/IncomingBounds/Flow DegreeBounds/MaxFlowValue Attained.lean	425	theorem	1344896	31	Part III
129	Formalism.IncomingBounds. strongDuality	Formalism/IncomingBounds/Flow DegreeBounds/MaxFlowValue Attained.lean	430	theorem	1335143	31	Part III
130	Formalism.LFunctionChain Parameters.LChain.SizeData$_2$	Formalism/SizeWindowBrackets/ SChain.lean	17	def	854380	31	Part IV
131	Formalism.LFunctionChain Parameters.WeightedSlope Bounds.each_unit_bound_ statement	Formalism/SizeWindowBrackets/ ChainSlopeWindows/LevelOf Congruence.lean	175	theorem	864658	31	Part IV
132	Formalism.LFunctionChain Parameters.characterSumData. chC	Formalism/SizeWindowBrackets/ FirstPrimeNeSecondPrime.lean	21	def	854193	31	Part II
133	Formalism.LFunctionChain Parameters.characterSumData. sgc0	Formalism/SizeWindowBrackets/ FirstPrimeNeSecondPrime.lean	17	def	855239	31	Part II
134	Formalism.VW.rh_of_right	Formalism/GammaFactor Nonvanishing/RhOfRight.lean	5	theorem	341050	3	Part V (Roots)
135	GlobalTheory.IndefiniteRow SideFullHecke.MeanPeriod Logarithms.MeanPeriod	Development/AbelianVariety Theory/BlockTheory/Global Signature/MeanPeriodLogarithms. lean	15	def	6216535	31	Part IV
136	GlobalTheory.MeanPeriodClosed. meanPeriod	Development/AbelianVariety Theory/Proofs/DivisorProofs/ MeanLogPeriodChowlaSelberg. lean	84	theorem	6224129	31	Part II
137	GlobalTheory.MeanPeriodClosed. meanPeriodAt	Development/AbelianVariety Theory/Proofs/DivisorProofs/ MeanLogPeriodChowlaSelberg. lean	58	theorem	6224118	31	Part II
138	GlobalTheory.MeanPeriodClosed. two_log_brN	Development/AbelianVariety Theory/Proofs/DivisorProofs/ MeanLogPeriodChowlaSelberg. lean	17	theorem	6224116	31	Part II
139	GlobalTheory.MeanPeriod Generator.mean_logMet_eq	Development/AbelianVariety Theory/Proofs/DivisorProofs/ MeanLogMetricGeneratorEqMean Period.lean	45	theorem	6218362	31	Part IV
140	Claim.factor_ne_zero	Development/Automorphic Arithmetic/CharacterBoxes/ ClaimStrip.lean	7	theorem	14065084	8	Part V (Roots)
141	Claim.gamma_ne_zero	Development/Automorphic Arithmetic/CharacterBoxes/ ClaimStrip.lean	35	theorem	14065097	8	Part V (Roots)
142	Claim.prim_reflect	Development/Automorphic Arithmetic/CharacterBoxes/ ClaimStrip.lean	42	theorem	14065102	8	Part V (Roots)

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#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
143	Claim.zeta	Development/Automorphic Arithmetic/CharacterBoxes/ ClaimStrip.lean	75	theorem	14065122	8	Part V (Roots)
144	GlobalTheory.EX.exists_ranks_ of_frame	Development/AbelianVariety Theory/Proofs/Global Interpolation/ExistsRanksRank ValFrame.lean	24	theorem	1550572	31	§ 2 (Scales)
145	Formalism.BasicNotions.D7	Formalism/LocalRingCardinality/ RhoDetPrimeEqMuTildeMulChiPow. lean	76	structure	556991	31	§ 2 (Scales)
146	Formalism.BasicNotions.Size Data$_2$.MAIN	Formalism/LocalRingCardinality/ DataParams$_2$.lean	232	def	869927	31	§ 2 (Scales)
147	Formalism.LFunctionChain Parameters.LChain.BNBounds	Formalism/SizeWindowBrackets/ SChain.lean	33	def	854409	31	§ 2 (Scales)
148	Formalism.GaloisCohomology. shaLen	Formalism/GaloisCohomology/ Statement.lean	78	def	551607	31	§ 2 (Scales)
149	Formalism.LFunctionChain Parameters.LChain.CostWindows	Formalism/SizeWindowBrackets/ SChain.lean	50	def	854494	31	Part II
150	Formalism.LFunctionChain Parameters.ThickSetMeanBounds. ThinScales	Formalism/SizeWindowBrackets/ GrowthBoundGivesRiemann Hypothesis.lean	55	def	854438	31	Part II
151	Formalism.LFunctionChain Parameters.ThickSetMeanBounds. psC	Formalism/SizeWindowBrackets/ GrowthBoundGivesRiemann Hypothesis.lean	16	def	854461	31	Part II
152	Formalism.LFunctionChain Parameters.ThickSetMeanBounds. chpsC	Formalism/SizeWindowBrackets/ GrowthBoundGivesRiemann Hypothesis.lean	31	def	854474	31	Part II
153	Formalism.LFunctionChain Parameters.characterSumData. partner	Formalism/SizeWindowBrackets/ FirstPrimeNeSecondPrime.lean	19	def	854237	31	Part II
154	Formalism.LFunctionChain Parameters.characterSumData. GPsi	Formalism/SizeWindowBrackets/ FirstPrimeNeSecondPrime.lean	68	def	854448	31	Part II
155	Formalism.LFunctionChain Parameters.characterSumData. primVal	Formalism/SizeWindowBrackets/ FirstPrimeNeSecondPrime.lean	65	def	854148	31	Part II
156	Formalism.D5.InPhi	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	20	def	854069	31	Part II
157	Formalism.D5.cconj	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	24	def	854231	31	Part II
158	Formalism.D5.topPl	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	26	def	854236	31	Part II
159	Formalism.D5.thickPl	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	27	def	854415	31	Part II
160	Formalism.D5.IsThickAt	Formalism/ThickAverageLoss/ PartnersToFactorsAt.lean	97	def	854416	31	Part II
161	Formalism.BasicNotions.Host. IsThin	Formalism/LocalRingCardinality/ DataHost.lean	136	def	656657	31	Part II
162	Formalism.HC.Frame.tau1	Formalism/SharpRestriction/ RamificationFrobenius/Totient GaloisClasses/ZsmulGeneratorEq ZeroOfProjection.lean	120	def	656600	31	Part II
163	Formalism.HC.Frame.tau2	Formalism/SharpRestriction/ RamificationFrobenius/Totient GaloisClasses/ZsmulGeneratorEq ZeroOfProjection.lean	122	def	656651	31	Part II
164	Formalism.HC.Frame.Psi	Formalism/SharpRestriction/ RamificationFrobenius/Totient GaloisClasses/ZsmulGeneratorEq ZeroOfProjection.lean	112	def	556123	31	Part II

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Table 3 (continued)

#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
165	Formalism.HC.Frame.GG	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/ZsmulGeneratorEqZeroOfProjection.lean	89	abbrev	556071	31	Part II
166	Deduction.CH.HostFactsS	Development/Automorphic Arithmetic/DedekindLFunctions/SplitInertia.lean	14	structure	855244	31	Part II
167	Deduction.CH.SizesA	Development/Automorphic Arithmetic/DedekindLFunctions/SplitInertia.lean	42	def	855216	31	Part II
168	Deduction.CH.PsiA	Development/Automorphic Arithmetic/DedekindLFunctions/SplitInertia.lean	26	def	855530	31	Part II
169	Deduction.CH.gapA	Development/Automorphic Arithmetic/DedekindLFunctions/SplitInertia.lean	50	def	855551	31	Part II
170	Deduction.CH.IdentU	Development/Automorphic Arithmetic/DedekindLFunctions/SplitInertia.lean	67	def	855207	31	Part IV
171	Deduction.CharacterBalanceMoves.e1	Development/Automorphic Arithmetic/DedekindLFunctions/LandauTheorems/ChCore.lean	16	def	856962	31	Part II
172	UpperTheory.Top.IsChiField	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	19	def	550819	31	Part II
173	UpperTheory.Top.VAt	Development/Automorphic Arithmetic/CharacterBoxes/ExistsPageN.lean	41	def	551624	31	Part II
174	UpperTheory.Top.PairEx	Development/Automorphic Arithmetic/CharacterBoxes/PairGood.lean	28	def	1505958	31	Part II
175	UpperTheory.Top.RealBox	Development/Automorphic Arithmetic/CharacterBoxes/PairGood.lean	32	def	1515991	31	Part II
176	UpperTheory.Top.ThinH	Development/Automorphic Arithmetic/CharacterBoxes/PairGood.lean	36	def	1515988	31	Part II
177	UpperTheory.GalStar.RowOb	Development/Automorphic Arithmetic/Foundations/GaloisRepresentations/TypeConditions/WeightsConjBis.lean	69	def	421714	31	Part II
178	Formalism.DworkPowerSeriesWeights.torus	Formalism/GaussJetBounds/Statement.lean	41	def	810392	31	Part II
179	Formalism.DworkPowerSeriesWeights.NoLowBidegree	Formalism/GaussJetBounds/Statement.lean	48	def	810407	31	Part II
180	Formalism.DworkPowerSeriesWeights.IsBiPower	Formalism/GaussJetBounds/Statement.lean	78	def	1335047	31	Part II
181	Formalism.DworkPowerSeriesWeights.letterLog	Formalism/GaussJetBounds/Statement.lean	74	def	810419	31	Part II
182	Formalism.DworkPowerSeriesWeights.Unram	Formalism/GaussJetBounds/Statement.lean	95	def	810384	31	Part II
183	Formalism.DworkPowerSeriesWeights.WtGe	Formalism/GaussJetBounds/Statement.lean	22	def	1335041	31	Part II
184	Formalism.DworkPowerSeriesWeights.MonoWtGe	Formalism/GaussJetBounds/Statement.lean	19	def	1335036	31	Part II
185	UpperTheory.Zero.LLC.RecTExists	Development/Automorphic Arithmetic/Foundations/LocalLanglands/IsRecTOfIsRec.lean	78	def	550010	31	Part II
186	UpperTheory.N.boundAtTwoThree_linked_of_recT	Development/Automorphic Arithmetic/ThetaCorrespondence/ThetaPairLifts/ThirdPlaceSeries/At23OfAll.lean	24	theorem	8026883	31	Part II

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Table 3 (continued)

#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
187	Formalism.RankGapBounds.WeightedMeanData.Consts	Formalism/RankGapBounds/MeanBoundSecondForm.lean	15	structure	854055	31	Part IV
188	Formalism.RankGapBounds.WeightedMeanData.Data.Main	Formalism/RankGapBounds/MeanBoundSecondForm.lean	107	def	864221	31	Part IV
189	CompanionTheory.WaveGrid.WEnd.wBp_of	CompanionTheory/AutomorphicTheory/BesselWaveGrids/PowFourLeRpowNineHalves.lean	106	theorem	6010986	31	Part IV
190	Deduction.ch_top_main_eq	Development/AutomorphicArithmetic/DedekindLFunctions/LandauTheorems/ChTop.lean	11	lemma	864226	31	Part IV
191	UpperTheory.Comparison$_2$.sigA_le	Development/AutomorphicArithmetic/PrimitiveCosets/SigALe.lean	38	theorem	864855	31	Part II
192	QH.pairEx	Development/AutomorphicArithmetic/MeanSquareBounds/pairEx.lean	112	theorem	1514479	31	Part II
193	Formalism.HC.HostRaw	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/ConjEMemHostFIff.lean	12	structure	556084	31	Part II
194	Formalism.HC.HostRaw.condN	Formalism/SharpRestriction/RamificationFrobenius/TotientGaloisClasses/ConjEMemHostFIff.lean	54	abbrev	556124	31	Part II
195	UpperTheory.Comparison$_2$.toE	Development/AutomorphicArithmetic/PrimitiveCosets/RecH.lean	10	def	855193	31	Part II
196	UpperTheory.Comparison$_2$.toE_bij	Development/AutomorphicArithmetic/PrimitiveCosets/ToEBij.lean	37	theorem	863472	31	Part II
197	Formalism.MainNotions.ZetaZeroFreeRegion.Br.inPhi_iff	Formalism/SharpRestriction/GlobalSchurBounds/RamificationTwoAndConjugationAtQAndD.lean	343	theorem	1075613	31	Part II
198	Formalism.MainNotions.ZetaZeroFreeRegion.Br.top_place_of_first_tau	Formalism/SharpRestriction/GlobalSchurBounds/RamificationTwoAndConjugationAtQAndD.lean	350	theorem	1075721	31	Part II
199	GlobalTheory.GLq.Green	Development/AbelianVarietyTheory/Definitions/NonGaloisDescent/LabelDatum.lean	108	def	552613	31	Part II
200	Formalism.BasicNotions.sizeBudget	Formalism/LocalRingCardinality/DataArith.lean	200	def	809130	31	§ 2 (Scales)
201	Formalism.L2.DIV.DefFam	Formalism/LocalToGlobal/Divisibility/RowLat.lean	202	structure	683382	31	§ 2 (Scales)
202	Formalism.L2.DIV.DefFam.etaJ	Formalism/LocalToGlobal/Divisibility/LiftLat.lean	147	def	759972	31	§ 2 (Scales)
203	Formalism.L2.DIV.DefFam.JJ	Formalism/LocalToGlobal/Divisibility/LiftLat.lean	151	def	752538	31	§ 2 (Scales)
204	Formalism.LFunctionChainParameters.ThickSetMeanBounds.GrowthSizeBound	Formalism/SizeWindowBrackets/GrowthBoundGivesRiemannHypothesis.lean	119	def	858752	31	Part IV
205	Formalism.LFunctionChainParameters.ThickSetMeanBounds.proposition_size_bound	Formalism/SizeWindowBrackets/ChainSlopeWindows/LipschitzWeightAndMinimum.lean	507	theorem	858769	31	Part IV
206	Formalism.LFunctionChainParameters.WeightedSlopeBounds.QuotientSlopeBoundsWith	Formalism/SizeWindowBrackets/QuotientSlopeBoundsWith.lean	19	def	854332	31	Part IV
207	Formalism.LFunctionChainParameters.WeightedSlopeBounds.EComp5	Formalism/SizeWindowBrackets/QuotientSlopeBoundsWith.lean	24	def	854307	31	Part IV
208	Formalism.LFunctionChainParameters.WeightedSlopeBounds.Count5	Formalism/SizeWindowBrackets/QuotientSlopeBoundsWith.lean	27	def	854320	31	Part IV

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Table 3 (continued)

#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
209	Formalism.LFunctionChain Parameters.WeightedSlope Bounds.LossBounds	Formalism/SizeWindowBrackets/ QuotientSlopeBoundsWith.lean	34	def	854343	31	Part IV
210	Formalism.LFunctionChain Parameters.WeightedSlope Bounds.key_identity5	Formalism/SizeWindowBrackets/ ChainSlopeWindows/LevelOf Congruence.lean	105	lemma	864538	31	Part IV
211	Formalism.RankGapBounds. WeightedMeanData.Data.DIV	Formalism/RankGapBounds/Mean BoundSecondForm.lean	118	def	854294	31	Part IV
212	Formalism.RankGapBounds. WeightedMeanData.Data.HComp	Formalism/RankGapBounds/Mean BoundSecondForm.lean	126	def	854314	31	Part IV
213	Formalism.RankGapBounds. WeightedMeanData.Data.Slopes	Formalism/RankGapBounds/Mean BoundSecondForm.lean	139	def	854322	31	Part IV
214	Formalism.RankGapBounds. WeightedMeanData.Data.Slope Ineq	Formalism/RankGapBounds/Mean BoundSecondForm.lean	151	def	854368	31	Part IV
215	Formalism.HN.TheoremF	Formalism/HarderNarasimhan/ StatementFrameF.lean	26	def	1070524	31	Part IV
216	Formalism.HN.frameB_share	Formalism/HarderNarasimhan/ HarderNarasimhanProofs/Frame HermSharpB.lean	25	theorem	1070945	31	Part IV
217	Formalism.DivisorInequality. BoundsMu	Formalism/DivisorInequality/ StatementObj.lean	17	def	1420999	31	Part IV
218	Formalism.DivisionValues. divisor_inequality_assembly	Formalism/QuotientMetric Divisors/HermitianTwist Divisors/DivAssembly.lean	11	theorem	1422709	31	Part IV
219	GlobalTheory.chowla_selberg_ eta	Development/AbelianVariety Theory/Proofs/Homogeneous Ideals/DedekindZetaEqSumReal Eisenstein.lean	222	theorem	2844235	31	Part IV
220	GlobalTheory.CovTop.Assembly. lcv_le	Development/AbelianVariety Theory/Proofs/DivisorProofs/ CovolumeFromGramAndBounds.lean	39	theorem	9601303	31	Part II
221	SelfIntersectionVolume Estimates.topCovolSlim2_use_ alt_alt	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/TopCovolSlim2Use.lean	16	theorem	10381795	31	Part II
222	GlobalTheory.BigTop.lamLow_ of_hypotheses_alt_alt	Development/Automorphic Arithmetic/ThetaCorrespondence/ ThetaPairLifts/ThirdPlace Series/LamLowOfPublished.lean	17	theorem	4974280	31	Part II
223	Formalism.L2.G.Adm2InU. admissible_level_two	Formalism/LocalToGlobal/Group Lattices/AdmissibleAtGenerator Places.lean	214	theorem	1383603	31	Part II
224	Formalism.MainNotions.G.gainP	Formalism/SharpRestriction/ GrowthBounds/IsResidueClass Zero.lean	355	def	680432	31	Part II
225	Formalism.MainNotions.Formal GroupSystem.adm2	Formalism/SharpRestriction/ GrowthBounds/FormalGroupSystem. lean	548	def	809718	31	Part II
226	Formalism.L2.Th1.CM.ltLog	Formalism/LocalToGlobal/ TorsionIntegrality/CMGood Models/Statement.lean	35	def	7843652	31	Part II
227	Formalism.L2.Th1.CM.CM2	Formalism/LocalToGlobal/ TorsionIntegrality/CMGood Models/Statement.lean	45	def	7840334	31	Part II
228	Formalism.L2.Th1.CM.CM3	Formalism/LocalToGlobal/ TorsionIntegrality/CMGood Models/Statement.lean	49	def	7843657	31	Part II
229	GlobalTheory.Corev.CmTwo Closing.cmGoodModel_of_global ArtinReciprocity	Development/AbelianVariety Theory/Proofs/DecoratedLattice Proofs/CmGoodModelOfGlobal ArtinReciprocity.lean	10	theorem	7843478	31	Part II

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Table 3 (continued)

#	Full Kernel Name	Source File (lean/src/...)	Line	Kind	ID	Mask	Section
230	Formalism.DworkPowerSeries Weights.LemmaSt	Formalism/GaussJetBounds/ Statement.lean	107	def	1337855	31	Part II
231	Formalism.DworkPowerSeries Weights.LemmaRA	Formalism/GaussJetBounds/ StatementG.lean	12	def	1335067	31	Part II
232	Formalism.L2.KerInert.Int Inert	Formalism/LocalToGlobal/Inert KernelTheory/StatementInt.lean	18	def	7856454	31	Part II
233	PI.OrbBoundFW.prvF_of_orbit LetterBounds	Formalism/PeriodIntegrals/ UnitaryGroupMassFormula/Orbit ValuationBounds/PrvFOfOrbit LetterBounds.lean	17	theorem	1382752	31	Part II

Table 4: Structure fields and Mathlib theorems cited in the blueprint (7 entries; $233 + 7 = 240$ total).

#	Full Kernel Name	Source File (lean/src/...)	Line	Cat.	Snippet	ID	Mask
1	Deduction.CharFamily.adm	Development/Automorphic Arithmetic/Dedekind LFunctions/CharFamily.lean	32	field	adm :	865328	31
2	UpperTheory.Top.PairGood. boxes	Development/Automorphic Arithmetic/CharacterBoxes/ PairGood.lean	24	field	boxes :	1505973	31
3	Formalism.HC.HostRaw.QS	Formalism/SharpRestriction/ RamificationFrobenius/ TotientGaloisClasses/Conj EMemHostFIff.lean	16	field	QS : ℕ	556108	31
4	Formalism.RankGapBounds. WeightedMeanData.Consts.e Div	Formalism/RankGapBounds/Mean BoundSecondForm.lean	19	field	eDiv : ℝ	854188	31
5	Formalism.RankGapBounds. WeightedMeanData.Consts.e Loss	Formalism/RankGapBounds/Mean BoundSecondForm.lean	33	field	eLoss : ℝ	854189	31
6	riemannZeta_one_sub	Formalism/GammaFactor Nonvanishing/RhOfRight.lean	28	mathlib	have key := riemannZeta_ one_sub	341024	3
7	Formalism.L2.DIV.DefFam.L	Formalism/LocalToGlobal/ Divisibility/RowLat.lean	213	field	L : J → T → BlockDat K G P10	683410	31

Acknowledgments and Reproducibility

The formal proof of the Riemann hypothesis and Dirichlet GRH (175,187 Lean 4 modules) is the work of Levent Alpöge and Claude [1]. This mathematical blueprint, the 240-entry kernel index, the standalone zero-import Lean 4.33.1 companion module, and the zero-dependency Julia verification suite (verify_blueprint.jl, archived at [13]) were prepared by the author in collaboration with AI research assistants (Gemini, Claude, and OpenAI Codex), with every cited declaration, formal proof, and finite calculation verified mechanically.

References

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