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WILSON PRIMES FOR THE COORDINATE RINGS OF CLASS NUMBER ONE IN CHARACTERISTIC TWO

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Abstract. Besides the polynomial rings, four coordinate rings A of a curve over a finite field minus one rational point are known to have class number one, and by the classification of function fields of class number one there is no other. For the one of characteristic 3, the Wilson primes were characterised in [1]. We treat the three rings of characteristic 2: $y^2+y=x^3+x+1$ over $\mathbb{F}_2$, $y^2+y=x^3+\alpha$ over $\mathbb{F}_4$, and $y^2+y=x^5+x^3+1$ over $\mathbb{F}_2$ (genus 2). With the Wilson product W_d of [1], a prime π of degree d is a Wilson prime, i.e. $W_d \equiv -1 \pmod{\pi^2}$, if and only if π divides $D^{(2)}\pi+(f'+\kappa)D\pi$, where $D=d/dx$, $D^{(2)}$ is the second Hasse derivative and $\kappa \in \mathbb{F}_q$ is an explicit function of the leading coefficients of π. In genus 1 this is equivalent to the identity $D^{(2)}\pi = t\,D(t\pi)$ with $t=x+a$, $a\in\mathbb{F}_q$ explicit, and there is no Wilson prime of degree $\equiv 0,3 \pmod 4$; in genus 2 it is equivalent to $D^{(2)}\pi+(x^4+x^2+\kappa)D\pi=(x^2+x+1+\kappa)\pi$ for $d\geq 4$, and there is no Wilson prime of degree $\equiv 2,3 \pmod 4$, $d\geq 4$. The criteria are confirmed by exact computation on 15 302 primes. The product W_d and the notion of Wilson prime refer to the given model of the ring.

1. Setting and statements

The function fields of positive genus over a finite field with class number one are classified: there are eight of them [3, 6, 5]. A field of positive genus and class number one has at most one place of degree one, and exactly four of the eight have one; they are the four fields found by MacRae [4], defined by $y^2=x^3-x-1$ over $\mathbb{F}_3$ and

$$\text{(I)}\quad y^2+y=x^3+x+1 \ \text{ over } \mathbb{F}_2,$$
$$\text{(II)}\quad y^2+y=x^3+\alpha \ \text{ over } \mathbb{F}_4 \quad (\alpha^2+\alpha+1=0),$$
$$\text{(III)}\quad y^2+y=x^5+x^3+1 \ \text{ over } \mathbb{F}_2.$$

In each case there is a unique place ∞ of degree one, and the ring A of functions regular outside ∞ is a principal ideal domain with $A^\times=\mathbb{F}_q^\times$. The first field is treated in [1]. Here we fix one of (I), (II), (III), write $k=\mathbb{F}_q$, $y^2+y=f(x)$, $\gamma=\deg f\in\{3,5\}$, so that $A=k[x,y]/(y^2+y-f)$, the genus is $g=(\gamma-1)/2$, and $\deg := -\operatorname{ord}_\infty$ satisfies $\deg x=2$, $\deg y=\gamma$. Let $S=\{0,2,4,\dots\}\cup\{\gamma,\gamma+2,\dots\}$; for $j\in S$ put $m_j=x^{j/2}$ (j even), $m_j=x^{(j-\gamma)/2}y$ (j odd). The m_j form a k-basis of A, $\deg m_j=j$, and we write $a=\sum_{j\in S}a_jm_j$, with $a_j=0$ for $j\notin S$. For a prime π, $\deg\pi=\dim_k A/\pi$.

Definition 1. For $d\in S$, $d\geq 2$, V_d is the k-span of the m_j with $j\in S$, $j-d\notin S$, and $W_d=\prod_{v\in V_d\setminus\{0\}}v$. A prime π of degree d is a *Wilson prime* if $W_d\equiv -1 \pmod{\pi^2}$.

In genus 1, V_d is spanned by the m_j, $j\leq d+1$, $j\neq d$, as in [1]; in genus 2 and $d\geq 4$ by the m_j with $j<d$ together with m_{d+1}, m_{d+3}, and $V_2=k+ky$. The space V_d, hence W_d and the notion of Wilson prime, depend on the chosen coordinates x,y. In $\mathbb{F}_q[t]$ the analogous product is Thakur's F_d [7, 8]; the generalisation to class number one is asked for in [8, §7.5, Question (2)].

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Lemma 2. $\dim_k V_d = d$, *and for every prime* π *of degree* d *the reduction* $V_d \to A/\pi$ *is bijective. Hence* $W_d \equiv -1 \pmod{\pi}$.

Proof. The degree of a non-zero element of V_d lies in $S \setminus (d+S)$, a set with d elements, and the degree of a non-zero multiple of π lies in $d+S$. So $V_d \cap \pi A = 0$ and $\dim V_d = d = \dim A/\pi$. □

Derivatives. A is étale over $k[x]$ (the polynomial Y^2+Y-f is separable at every point), so $\Omega_{A/k} = A\,dx$ and the iterative Hasse–Schmidt derivation $(D^{(n)})_{n\geq 0}$ of $k[x]$, $D^{(n)}x^i = \binom{i}{n}x^{i-n}$, extends uniquely to the function field [2]; it maps A into A and remains iterative, because $Y^2+Y-f(x+\varepsilon)$ has a unique root in $A[[\varepsilon]]$ lifting y (its derivative in Y is 1). We use $D = D^{(1)}$ and $D^{(2)}$. Explicitly $Dy = f'$ and $D^{(2)}y = f^{[2]} + f'^2$, where $f^{[2]} = D^{(2)}f$: indeed $(y+\delta)^2 + (y+\delta) = f(x+\varepsilon)$ modulo ε^3 has the solution $\delta = f'\varepsilon + (f^{[2]} + f'^2)\varepsilon^2$. We use the rules

$$\begin{gathered} D^{(2)}(ab) = D^{(2)}a \cdot b + Da \cdot Db + a \cdot D^{(2)}b, \\ D \circ D = 0, \qquad D \circ D^{(2)} = D^{(2)} \circ D, \qquad D^{(2)} \circ D^{(2)} = 0, \end{gathered} \tag{1}$$

which follow from $D^{(i)} \circ D^{(j)} = \binom{i+j}{i} D^{(i+j)}$ in characteristic 2. If π is prime, its zeros are simple and $x - x(P)$ is a uniformiser at every affine point P, so $D\pi$ is a unit modulo π.

For a prime π of degree d, normalised by $\pi_d = 1$, put

$$\kappa(\pi) = \pi_{d-1} \ \text{ in cases (I), (II)}, \qquad \kappa(\pi) = \pi_{d-3} + \pi_{d-1}\pi_{d-2} \ \text{ in case (III)},$$

an element of $\mathbb{F}_q$.

Theorem 3. *Let* π *be a prime of degree* $d \geq 2$, *normalised by* $\pi_d = 1$. *Then* π *is a Wilson prime if and only if*

$$\pi \ \ \textit{divides} \ \ \Theta(\pi) := D^{(2)}\pi + (f' + \kappa(\pi))D\pi.$$

Theorem 4. *In cases* (I), (II) *write* $f' = x^2 + \varphi$ ($\varphi = 1$, *resp.* 0), *and let* $a \in \mathbb{F}_q$ *be the square root of* $\varphi + \kappa(\pi)$, $t = x + a$. *Then* π *is a Wilson prime if and only if*

$$D^{(2)}\pi = t\, D(t\pi).$$

There is no Wilson prime of degree $d \equiv 0$ *or* $3 \pmod 4$.

Theorem 5. *In case* (III) *let* $d \geq 4$ *and* $\kappa = \kappa(\pi)$. *Then* π *is a Wilson prime if and only if*

$$D^{(2)}\pi + (x^4 + x^2 + \kappa)\, D\pi = (x^2 + x + 1 + \kappa)\, \pi.$$

There is no Wilson prime of degree $d \equiv 2$ *or* $3 \pmod 4$, $d \geq 4$.

In case (III) the two primes of degree 2, x and $x+1$, are Wilson primes, and the identity of Theorem 5 does not hold for them.

2. Reduction to a linear coefficient

Fix a prime π of degree d, $\mathbf{F}_\pi = A/\pi$ its residue field, $\rho\colon V_d \to \mathbf{F}_\pi$ the bijection of Lemma 2, and define the k-linear map $\partial\colon \mathbf{F}_\pi \to \mathbf{F}_\pi$ by $\partial(\rho(v)) = D(v) \bmod \pi$. Every k-linear endomorphism of $\mathbf{F}_\pi$ is uniquely $z \mapsto \sum_{j=0}^{d-1} \lambda_j z^{q^j}$; let λ_0 be the coefficient of z for ∂.

Proposition 6. $W_d \equiv -1 + \pi\lambda_0/D\pi \pmod{\pi^2}$. *So* π *is a Wilson prime if and only if* $\lambda_0 = 0$.

Proof. The completion of A at π is $\mathbf{F}_\pi[[\pi]]$. For $a \in A$ write $a \equiv \bar{a} + \delta(a)\pi \pmod{\pi^2}$, $\bar{a}, \delta(a) \in \mathbf{F}_\pi$. Then δ is a k-derivation with $\delta(\pi) = 1$, so $\delta = \overline{D(\cdot)}/\overline{D\pi}$ because $\Omega_{A/k} = A\,dx$. The polynomial $e(z) = \prod_{v \in V_d}(z - v)$ is k-linear, monic of degree q^d, with coefficient of z equal to $\prod_{v \neq 0}(-v) = W_d$. Modulo π it is $z^{q^d} - z$, so $e(z) \equiv z^{q^d} - z + \pi g(z) \pmod{\pi^2}$ with g a k-linear polynomial over $\mathbf{F}_\pi$ of degree $< q^d$. For $v \in V_d$, $v^{q^d} \equiv \bar{v} \pmod{\pi^2}$, so $0 = e(v) \equiv \bar{v} - v + \pi g(\bar{v})$, i.e. $g(\bar{v}) = \delta(v)$. Hence $g = \partial/\overline{D\pi}$ as functions on $\mathbf{F}_\pi$, hence as polynomials. Compare the coefficients of z. □

Lemma 7. $\operatorname{tr}_k(M_a \circ \partial) = \operatorname{Tr}_{\mathbf{F}_\pi/k}(a\lambda_0)$ *for* $a \in \mathbf{F}_\pi$, *where* M_a *is multiplication by* a. *So* $\lambda_0 = 0$ *if and only if* $\operatorname{tr}_k(M_a \circ \partial) = 0$ *for all* a.

Proof. Let ϕ be the q-Frobenius. For $0 < j < d$ the trace $\operatorname{tr}_k(M_b\phi^j)$ is k-linear in b, hence equal to $\operatorname{Tr}(c_j b)$ for some c_j; conjugation by M_ζ gives $\operatorname{Tr}(c_j b) = \operatorname{Tr}(c_j b \zeta^{q^j-1})$ for all b, so $c_j(1 - \zeta^{q^j-1}) = 0$, and $c_j = 0$ on taking ζ a generator of $\mathbf{F}_\pi^\times$. Finally $\operatorname{tr}_k(M_b) = \operatorname{Tr}_{\mathbf{F}_\pi/k}(b)$. □

3. The pairing and the kernel

For $b, c \in A$ put $\langle b, c\rangle = \operatorname{Res}_\infty(bc\,dx/\pi)$.

Lemma 8. $\langle b, c\rangle = \operatorname{Tr}_{\mathbf{F}_\pi/k}\big(\bar{b}\bar{c}/\overline{D\pi}\big)$. *It induces a non-degenerate symmetric* k*-bilinear form on* $\mathbf{F}_\pi$ *with* $\langle ab, c\rangle = \langle a, bc\rangle$.

Proof. dx has neither zero nor pole at the affine points, so $bc\,dx/\pi$ has poles only at ∞ and at the zeros of π, which are simple and form one Galois orbit; since $d\pi = D\pi\,dx$ the residue at a zero Q is $(bc/D\pi)(Q)$. Apply the residue theorem (signs are immaterial in characteristic 2). □

In $A \otimes_k A$ write $a_1 = a \otimes 1$, $a_2 = 1 \otimes a$, and let $G = y_1 + y_2 + 1$.

Lemma 9. *There is a unique* $R_0 \in A \otimes_k A$ *with* $(x_1 + x_2)R_0 = (\pi_1 + \pi_2)\,G$.

Proof. $A \otimes A$ is a domain, and $A \otimes A/(x_1 + x_2) \cong A[Y]/\big((Y - y)(Y - y - 1)\big) \cong A \times A$, the two factors corresponding to $y_2 = y_1$ and $y_2 = y_1 + 1$. The image of $(\pi_1 + \pi_2)G$ is zero in both: in the first $\pi_1 = \pi_2$, in the second $G = 0$. □

Lemma 10. *Write* $R_0 = \sum_i r_i \otimes s_i$. *Then* $\sum_i \langle b, s_i\rangle r_i \equiv b \pmod{\pi}$ *for every* $b \in A$.

Proof. It suffices to evaluate at a zero P of π over $\bar{k}$, extending the pairing $\bar{k}$-linearly in the second variable. As $\pi(P) = 0$, the left side becomes $\operatorname{Res}_{Q=\infty} b(Q)\,C_P(Q)\,dx$ with $C_P = (y + y(P) + 1)/(x + x(P))$. The function $x + x(P)$ has simple zeros at P and at the conjugate point P', where $y(P') = y(P) + 1$; the numerator of C_P vanishes at P' and equals 1 at P. So $b\,C_P\,dx$ has poles only at P and ∞, with residue $b(P)$ at P. Apply the residue theorem. □

Proposition 11. *Write* $R_0 = \sum_{j \in S} m_j \otimes r_j$, *and assume* $d \geq \gamma - 1$. *Then* $r_j = 0$ *for* $j > d + \gamma - 2$, *and:*

in genus 1: $r_{d+1} = 1$, $r_d = \pi_{d-1}$;
in genus 2: $r_{d+3} = 1$, $r_{d+2} = \pi_{d-1}$, $r_{d+1} = \pi_{d-2} + x$, $r_d = \pi_{d-3} + \pi_{d-1}x$.

Consequently

$$R := R_0 + \pi \otimes \pi_{d-1} \quad (g = 1), \qquad R := R_0 + x\pi \otimes \pi_{d-1} + \pi \otimes \big(\kappa(\pi) + \pi_{d-1}x\big) \quad (g = 2)$$

belongs to $V_d \otimes_k A$.

Proof. We have $x\,m_j = m_{j+2}$; $y\,m_j = m_{j+\gamma}$ for j even, and $y\,m_j = m_{j+\gamma}$ plus a combination of m_i with $i \le j+1$ for j odd ($y \cdot x^i y = x^i y + x^i f$). Recall $\pi_j = 0$ for $j \notin S$. Hence the coefficient N_n of m_n (first variable) in $(\pi_1+\pi_2)G = \pi_1 y_1 + \pi_1(y_2+1) + \pi_2(y_1+y_2+1)$ is $\pi_{n-\gamma}$ for every $n \ge d+2$, because the other terms have degree at most $\max(d+1,\gamma) \le d+1$ in the first variable. Comparing coefficients of m_n in $(x_1+x_2)R_0$ gives $r_{n-2} + x\,r_n = N_n$ for all n. As R_0 has finite support, descending induction gives $r_j = 0$ for $j > d+\gamma-2$, then the listed values from $n = d+\gamma, \dots, d+2$. The elements π and $x\pi$ have coefficients 1 on m_d, resp. m_{d+2}, and $x\pi$ has coefficient π_{d-2} on m_d; so R has no component on m_j with $j - d \in S$. □

Proposition 12. *Let $R = \sum_j m_j \otimes t_j$, the sum over the monomials of V_d. Then $\langle m_i, t_j\rangle = \delta_{ij}$ and*

$$\sum_j D(m_j)\,t_j \equiv D^{(2)}\pi + (f' + \kappa(\pi))D\pi \pmod{\pi}.$$

Proof. $R - R_0 \in \pi A \otimes A$, so Lemma 10 gives $\sum_j \langle m_i, t_j\rangle m_j \equiv m_i \pmod{\pi}$, and $V_d \cap \pi A = 0$ gives the duality. Let $\mu\colon A \otimes A \to A$ be the multiplication, and let D_1, $D_1^{(2)}$ act on the first factor. The left side is $\mu(D_1 R)$. Since $D(\pi)c \equiv D\pi \cdot c$ and $D(x\pi)c \equiv xD\pi \cdot c \pmod{\pi}$, Proposition 11 gives $\mu(D_1 R) \equiv \mu(D_1 R_0) + \kappa(\pi)D\pi$ in both genera (in genus 2 the terms $\pi_{d-1}xD\pi$ cancel). Apply $D_1^{(2)}$ to $(x_1+x_2)R_0 = (\pi_1+\pi_2)G$, using (1), $D_1(x_1+x_2) = 1$, $D_1 G = (f')_1$:

$$(x_1+x_2)D_1^{(2)}R_0 + D_1 R_0 = (D^{(2)}\pi)_1 G + (D\pi)_1 (f')_1 + (\pi_1+\pi_2)D_1^{(2)}G.$$

Applying μ, and $\mu(G) = 1$, gives $\mu(D_1 R_0) = D^{(2)}\pi + f' D\pi$. □

Proof of Theorem 3. By the duality, $\mathrm{tr}_k(T) = \sum_j \langle T\bar{m}_j, \bar{t}_j\rangle$ for every endomorphism T of $\mathbf{F}_\pi$. With $T = M_a \circ \partial$ and Lemma 8, $\mathrm{tr}_k(M_a \circ \partial) = \langle a, \sum_j D(m_j)t_j\rangle = \langle a, \Theta(\pi)\rangle$. By Proposition 6, Lemma 7 and non-degeneracy, π is a Wilson prime iff $\Theta(\pi) \equiv 0 \pmod{\pi}$. The hypothesis $d \ge \gamma - 1$ of Proposition 11 is $d \ge 2$ in genus 1 and $d \ge 4$ in genus 2. In genus 2 and $d = 2$ the primes are x and $x+1$, $\kappa = 0$, $\Theta(\pi) = f' = x^2(x+1)^2$, and both are Wilson primes by direct computation. □

4. Proof of Theorem 4

Here $\gamma = 3$, $f = x^3 + \varphi x + f_0$. Put $h = f' + \kappa(\pi) = x^2 + \varphi + \kappa$ and $L = D^{(2)} + hD$, so that $\Theta(\pi) = L\pi$. From the formulas for D, $D^{(2)}$ on x^i and y,

$$L(x^i) = i\,x^{i+1} + i(\varphi+\kappa)x^{i-1} + \tbinom{i}{2}x^{i-2},$$
$$L(x^i y) = L(x^i)\,y + \kappa x^{i+2} + (i+1)x^{i+1} + \kappa\varphi x^i + i\varphi x^{i-1}.$$

So $L(m_j) = \beta_j m_{j+2} + (\text{terms of degree} \le j+1)$, with $\beta_j = 1$ if $j \equiv 1, 2 \pmod 4$ and $\beta_j = 0$ otherwise. Therefore $\deg L\pi \le d+2$, and if $\pi \mid L\pi$ then $L\pi = (bx+a)\pi$ with $a \in \mathbb{F}_q$ and $b = \beta_d$. Note $Dh = 0$, $D^{(2)}h = 1$.

Case $b = 0$, i.e. $d \equiv 0, 3 \pmod 4$. Apply D to $D^{(2)}\pi + hD\pi = a\pi$: by (1), $DD^{(2)}\pi = aD\pi$. Apply $D^{(2)}$: $D\pi + h\,DD^{(2)}\pi = a\,D^{(2)}\pi$, i.e. $D\pi + ahD\pi = a(a\pi + hD\pi)$, so $D\pi = a^2\pi$. This contradicts the fact that $D\pi$ is a unit modulo π. So there is no Wilson prime of such a degree.

Case $b = 1$. Put $t = x + a$. Applying D to $D^{(2)}\pi + hD\pi = t\pi$ gives $DD^{(2)}\pi = \pi + tD\pi$. Applying $D^{(2)}$ gives $D\pi + h\,DD^{(2)}\pi = D\pi + t\,D^{(2)}\pi$, i.e. $h(\pi + tD\pi) = t(t\pi + hD\pi)$, so $h = t^2$, that is $a^2 = \varphi + \kappa$. Hence $\pi \mid L\pi$ implies $D^{(2)}\pi + t^2 D\pi = t\pi$ with $t = x + \sqrt{\varphi+\kappa}$, and the converse is trivial. Finally $t\,D(t\pi) = t\pi + t^2 D\pi$. □

5. Proof of Theorem 5

Here $f = x^5 + x^3 + 1$, $f' = x^4 + x^2$, $h = x^4 + x^2 + \kappa$, $L = D^{(2)} + hD$, $Dh = 0$, $D^{(2)}h = 1$. One finds $L(x^i) = i\,x^{i+3} + \cdots$ and $L(x^i y) = i\,x^{i+3}y + \cdots$, the omitted terms having degree at most $j+4$, where j is the degree of the monomial. So $\deg L\pi \le d+6$, and the coefficient of m_{d+6} in $L\pi$ is 1 exactly when $d \equiv 2, 3 \pmod 4$. Assume $L\pi = Q\pi$; then Q is a combination of $1, x, x^2, y, x^3$. Applying D, then $D^{(2)}$, to $D^{(2)}\pi + hD\pi = Q\pi$ gives as above

$$DD^{(2)}\pi = DQ\cdot\pi + Q\,D\pi, \qquad (1+DQ)\,D\pi = \big(D^{(2)}Q + Q^2 + h\,DQ\big)\pi.$$

So π divides $1+DQ$, which has the form $c_0 + c_1x^2 + c_2x^4 = (c_0 + c_1x + c_2x^2)^2$ with $c_i \in \mathbb{F}_2$. If it is not zero, its prime divisors in A are among x, $x+1$ (of degree 2) and x^2+x+1 (the prime of degree 4). Hence $DQ = 1$ if $d \ge 5$. Thus $Q = x + q_2x^2 + q_0$, and then $D^{(2)}Q + Q^2 + h = 0$ reads $q_2 + q_0^2 + x^2 + q_2x^4 + x^4 + x^2 + \kappa = 0$, i.e. $q_2 = 1$, $q_0 = 1 + \kappa$. In particular Q has no term in x^3, which excludes $d \equiv 2, 3 \pmod 4$. The converse is trivial. For $d = 4$ the only prime is $x^2 + x + 1$; it is a Wilson prime and satisfies the identity, by direct computation. □

6. Numerical verification

For every prime in the ranges below, W_d modulo π^2 was computed from the definition in exact arithmetic and the Wilson property compared with Theorem 3, and with Theorems 4, 5 (in case (III) for $d \ge 4$): no discrepancy on the 15 302 primes of the table. The numbers of primes agree with the zeta functions.

d	2	3	4	5	6	7	8	9	10	11	12	13	14
(I) primes	2	4	5	8	8	16	25	52	98	192	345	640	
(I) Wilson	1	0	0	4	0	0	0	0	7	0	0	20	
(II) primes	4	16	54	192	652	2304	8100						
(II) Wilson	1	0	0	0	15	0	0						
(III) primes	2	0	1	8	10	16	35	64	98	192	357	640	1162
(III) Wilson	2	0	1	0	0	0	0	4	0	0	1	16	0

In addition, for all primes of degree at most 9 in case (I), 10 in case (III) and 8 in case (II), the element R of Proposition 11 was constructed explicitly and the congruence of Proposition 12 was checked. The ancillary script `verify_wilson_char2.py` reproduces the table up to degree 13 in cases (I), (III) and 7 in case (II), that is 6 040 of these primes, and also computes W_d as a plain product in small degrees.

7. Status and questions

Theorems 3, 4, 5 are proved above. Nothing in this note has been checked by a third party. Open: some degrees allowed by Theorems 4, 5 carry no Wilson prime in the range of the table (6, 9 in case (I), 5 in case (II), 5, 8 in case (III)); in case (I) the Wilson primes of the table occur exactly in the degrees $d \equiv 2, 5 \pmod 8$. We do not know the exact set of degrees with Wilson primes, nor whether there are infinitely many Wilson primes in any of the three rings.

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References

[1] D. Niedbala Giraudin, Wilson primes for an elliptic coordinate ring of class number one, Hexagon 2611.01007 (2026).

[2] H. Hasse, F. K. Schmidt, Noch eine Begründung der Theorie der höheren Differentialquotienten in einem algebraischen Funktionenkörper einer Unbestimmten, J. reine angew. Math. **177** (1937), 215–237.

[3] J. Leitzel, M. Madan, C. Queen, Algebraic function fields with small class number, J. Number Theory **7** (1975), 11–27.
[4] R. E. MacRae, On unique factorization in certain rings of algebraic functions, J. Algebra **17** (1971), 243–261.
[5] P. Mercuri, C. Stirpe, Classification of algebraic function fields with class number one, J. Number Theory **154** (2015), 365–374.
[6] C. Stirpe, A counterexample to 'Algebraic function fields with small class number', J. Number Theory **143** (2014), 402–404.
[7] D. S. Thakur, Differential characterization of Wilson primes for $\mathbb{F}_q[t]$, Algebra Number Theory **7** (2013), no. 8, 1841–1848.
[8] D. S. Thakur, Fermat–Wilson supercongruences, arithmetic derivatives and strange factorizations, J. Théor. Nombres Bordeaux **36** (2024), no. 2, 481–491.

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