A conditional bound $H_2 \le 19\,284$ from the sharp minorant of the 182 argument

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Abstract

Let $H_2 = \liminf_{n\to\infty}(p_{n+2} - p_n)$. Assuming a distribution statement for the sharp prime minorant constructed in the recent argument for $H_1 \le 182$ [1, 2, 3], in the form in which that argument uses it, we prove DHL$[2240, 3]$ and hence $H_2 \le 19\,284$. The best bound announced so far is 19 844 [5]. The proof combines Kapila's support-restricted sieve criterion [5] with the minorant and a certificate in ball arithmetic; the programs and data are in the ancillary files. The result is conditional on Hypothesis H below and has not been checked by a third party.

1 Statement

Following Polymath [7], DHL$[k, j]$ asserts that every admissible k-tuple has infinitely many translates containing at least j primes. If DHL$[k, 3]$ holds and $h_1 < \cdots < h_k$ is admissible, then $H_2 \le h_k - h_1$.

Theorem 1. *Assume Hypothesis H. Then* DHL$[2240, 3]$ *and* DHL$[2241, 3]$ *hold. Consequently* $H_2 \le 19\,284$.

The admissible tuples used have 2240 and 2241 elements and diameters 19 284 and 19 290. Kapila [5] proved DHL$[2305, 3]$ and $H_2 \le 19\,844$ from the distribution theorem of [6] on triply densely divisible moduli; a public discussion of [5] reports 19 842 from the same certificate. The gain here comes from the larger level and the small mass deficit of the minorant of [1]; the sieve criterion and the trial-function family are those of [5].

2 The hypothesis

Put $a = 41361/100000$ and $\lambda = 1 - 2a = 0.17278$. For $x \le n < 2x$ let $\mathcal{S}_5(a; x)$ be the set of products of five distinct primes all of whose pair products are below x^a, and let

$$\varrho_x(n) = \mathbf{1}_{\mathbb{P}}(n) - 24 \cdot \mathbf{1}_{\mathcal{S}_5(a;x)}(n)$$

be the sharp minorant of [1, §4]. Write $\Delta(f; b, q) = \sum_{n\equiv b\,(q)} f(n) - \frac{1}{\varphi(q)} \sum_{(n,q)=1} f(n)$, and let $\mathcal{D}^{(3)}(Y)$ be the strong triply densely divisible integers [1, Def. 3.1], [6, Def. 2.1]. Let

$$\omega = 0.014186, \qquad \delta = 3.5 \cdot 10^{-5}, \qquad \bar\kappa = \frac{46653544162031}{5 \cdot 10^{17}}.$$

Hypothesis H. *(H1)* $\varrho_x \le \mathbf{1}_{\mathbb{P}}$, $|\varrho_x| \le 24$, *and* $\varrho_x(n) \ne 0$ *implies that every prime factor of* $n \in [x, 2x)$ *exceeds* x^λ.

(H2) $\sum_{x\le n<2x} \varrho_x(n) = (1 - \kappa + o(1))\, x/\log x$ *with* $0 \le \kappa \le \bar\kappa$.

(H3) *For every fixed $A, \varepsilon > 0$, every finite set of primes I with product P_I, every primitive class $b \bmod P_I$ and every interval $J \subseteq [x, 2x)$,*

$$\sum_{\substack{q \le x^{1/2+2\omega-\varepsilon},\ q \mid P_I \\ q \in \mathcal{D}^{(3)}(x^\delta)}} |\Delta(\varrho_x \mathbf{1}_J; b, q)| \ll x(\log x)^{-A}.$$

(H4) $\sum_{q \le x^{1/2-\varepsilon}} \max_{(b,q)=1} |\Delta(\varrho_x \mathbf{1}_J; b, q)| \ll x(\log x)^{-A}$.

(H1) and (H2) are proved in [1, §§4.1–4.4]; the value $\kappa = 0.0000933070883240\ldots$ was recomputed independently (Section 5). (H4) is a Bombieri–Vinogradov statement for the primes and for a convolution of five prime variables. (H3) is the distribution property that [1] uses for this minorant on its order-three rows [1, §§3.3–3.6], deduced there from the Type II estimate [2, Thm. 2.1, Cor. 9.3], the Harman construction [2, Thm. 11.1, Prop. 10.1] and the Type III estimate [3, Thm. 1.1]. At the present (ω, δ), with the cutoffs a, $I_0 = 0.34941$, $c = 0.41364$, $\tau = 10^{-10}$ and handoff $h = 9/20$ of [1, 2], all inequalities required in [1, §3.3, §3.6] and [2, (2.3), (10.2)–(10.5)] hold, the smallest margin being $3 \cdot 10^{-5}$ (file `run_final.py`). The order-three rows of [1] use pairs (ω, δ) on the same boundary, up to $\omega = 71/5000$. We do not prove (H3); the papers [1, 2, 3] have not been peer reviewed, and Theorem 1 is conditional on them.

3 The criterion

Put $\rho = \frac{1}{4} + \omega$. For $u_0, \kappa_0 > 0$ let $\mathcal{R}_k(u_0, \kappa_0)$ be the set of $t \in [0, \infty)^k$ with $\frac{3}{2}v + \sum_{t_i \ge v} t_i \le \kappa_0$ for all v with $u_0 < v \le \max_i t_i$ [5, (2.4)]. For F supported in the simplex $\{\sum t_i \le 1\}$ put $Q(F) = \sum_i J_i(F)/I(F)$, with $I(F) = \int F^2$ and $J_i(F) = \int (\int F\, dt_i)^2 \prod_{j \ne i} dt_j$.

Lemma 1. *Assume Hypothesis H. Let $u_0 < \delta/\rho$, $\kappa_0 < (\frac{1}{4} + \frac{\delta}{2})/\rho$, $T > 0$ with $\rho T < \lambda$ and $T < \frac{2}{5}\kappa_0$. Let $F \ne 0$ be square integrable, supported in $\{\sum t_i \le 1\} \cap \mathcal{R}_k(u_0, \kappa_0) \cap [0, T]^k$. If $\rho(1 - \bar\kappa)\, Q(F) > m$, then* $\mathrm{DHL}[k, m+1]$ *holds.*

Proof sketch. This is [5, Thm. 3.1] with enlargement parameter 0, the prime indicator $\mathbf{1}_{\mathbb{P}}(n + h_i)$ being replaced by $\varrho_x(n + h_i)$, as in the minorant method of Baker and Irving [4] and [1, (6.3)]. Take the Maynard–Tao weights $w_n = A_f(n)^2 \ge 0$ with f a smooth approximation of $F(\cdot/\lambda')$, $\lambda' < \rho$. Since the coordinate divisors have prime factors at most $x^{\rho T} < x^{\lambda}$, (H1) forces $d_i = e_i = 1$ on the support of $\varrho_x(n + h_i)$ [1, §4.1]. The remaining moduli $q = W[D, E]$ are either below $x^{1/2+2\omega-c'}$, where (H4) applies, or lie below $x^{1/2+2\omega-c'}$ and in $\mathcal{D}^{(3)}(x^\delta)$ by [5, Lemma 2.5], which uses only the support of F; there (H3) applies. Hence $\sum_n w_n \varrho_x(n + h_i)$ equals $(1 - \kappa + o(1))$ times the main term obtained for primes, with $\kappa \le \bar\kappa$ by (H2); the coprimality of the mean to q follows from the roughness in (H1). Because the enlargement parameter is 0, the sum is evaluated directly and no positivity of ϱ_x is used (this is the only point where the argument of [5, Thm. 3.1], which uses $\mathbf{1}_{\mathbb{P}} \ge 0$ for its enlarged support, is modified). Summing over i and using $\sum_i \mathbf{1}_{\mathbb{P}}(n + h_i) \ge \sum_i \varrho_x(n + h_i)$ with $w_n \ge 0$ gives a translate with at least $m + 1$ primes once $\rho(1 - \bar\kappa) Q(F) > m$, after letting $\lambda' \to \rho$. □

4 Trial functions and certificates

As in [5, §4], $F_0(t) = \psi(t_1 + \cdots + t_k) \prod_i g(t_i)$ with $g(t) = \mathbf{1}_{[0,T]}(t)/(c + (k-1)t)$, and $\psi \ge 0$ a step function on 256 equal cells of $[0, 1]$ with values in $2^{-30}\mathbb{Z}$. We use $F = F_0 \mathbf{1}_{\mathcal{R}_k(u_0, \kappa_0)}$ with

$$u_0 = 0.999\, \delta/\rho, \qquad \kappa_0 = (1 - 10^{-6})(\tfrac{1}{4} + \tfrac{\delta}{2})/\rho.$$

The exact rationals c, T and ψ are in `cert_k2240.json` and `cert_k2241.json`; one checks $T < \frac{2}{5}\kappa_0$ and $\rho T < 0.1 < \lambda$.

Quotient. $Q(F_0) = \frac{k}{m_2}\,\mathbb{E}[\Psi(S_{k-1})^2]/\mathbb{E}[\psi(S_k)^2]$, where $m_2 = \int g^2$, S_n is a sum of n independent variables of density g^2/m_2 and $\Psi(s) = \int \psi(s+t)g(t)\,dt$. The program `certq.py` bounds it from below in ball arithmetic (Arb, through python-flint): the law of S_n is expanded in a Fourier series of period 8, using the characteristic function of g^2/m_2 in closed form (cosine and sine integrals); the truncation of the series, the periodisation and the variation of Ψ^2 on each of about 10^4 (resp. $3 \cdot 10^4$) subintervals are bounded explicitly.

Support restriction. Since $\psi \geq 0$, for each i, $J_i(F_0) - J_i(F) \leq 2C^2 m_1^2 m_2^{k-1}\,\mathbb{P}^*(\text{violation},\ S \leq 1)$, with $C = \max\psi$, $m_1 = \int g$ and $\mathbb{P}^*$ the law in which one coordinate has density g/m_1 and the others g^2/m_2. A violation is witnessed at an actual coordinate. The program `losscert.py` covers the witness by 800 geometric bins and bounds each bin by the joint heavy-count/light-sum inequality of [5, (5.4)] with interval-arithmetic integrals and binomial tails.

k	$Q(F_0) \geq$	loss $\leq$	$2/(\rho(1-\bar\kappa))$	margin $\geq$
2240	7.571154116	$1.99 \cdot 10^{-6}$	7.571130308	$2.18 \cdot 10^{-5}$
2241	7.571511885	$1.75 \cdot 10^{-6}$	7.571130308	$3.79 \cdot 10^{-4}$

With $m = 2$ Lemma 1 gives DHL$[2240, 3]$ and DHL$[2241, 3]$.

Tuples. `tuple_2240.txt` and `tuple_2241.txt` are windows of the admissible set defined by the residue system of [5] for H_2; their diameters are 19 284 and 19 290. The program `verify_tuple.py` checks, by two independent methods, that for every prime $p \leq k$ some class modulo p is missed.

5 Consistency checks

- For the parameters of [5, §4] at $k = 2355$ ($\beta = 2.94$), `certq.py` gives $Q \geq 7.62016$; the first version of [5] (hexagon:2610.00419v1) reports 7.62015 for its own trial of this family at $k = 2355$.
- A separate floating-point computation of the same quotient on grids of 2^{21} to 2^{24} points converges to 7.62025 at $k = 2355$, above the certified bound.
- The mass deficit κ of [1, (4.2)], integrated independently on the three simplices of [1, §4.5], is $9.330708832406 \cdot 10^{-5}$. This agrees with the enclosure $0.000093307088324061 < \kappa < 0.000093307088324062$ of [1]. The three simplex volumes are 125/5184, 625/20736 and 625/6912.
- `verify_cert.py` recomputes both certificates from the stored rational data.

Statement on verification and on the use of AI

The mathematics, the computations and the writing were produced with Claude (Anthropic) under the author's direction. Nothing in this note has been checked by a third party. The result is computer-assisted and conditional: it rests on Hypothesis H, taken from [1, 2, 3], and on the programs in the ancillary files, which anyone can rerun. To the author's knowledge, as of 23 November 2026 no bound for H_2 below 19 842 had been announced.

References

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[3] GPT-6 Astra (OpenAI), prompted by R. Sharifi, *Kloosterman correlations with finitely many exceptional Fourier frequencies*, companion paper in the same bundle, 18 September 2026.

[4] R. C. Baker and A. J. Irving, *Bounded intervals containing many primes*, Math. Z. 286 (2017), 821–841.

[5] A. Kapila, *Finite prime gaps from weighted Selberg sieves and pair penalties*, hexagon:2610.00419v2, 6 November 2026; `https://github.com/a-kapila/maths-proofs`.

[6] OpenAI, *Improved short gaps between primes*, preprint, 30 August 2026.

[7] D. H. J. Polymath, *Variants of the Selberg sieve, and bounded intervals containing many primes*, Res. Math. Sci. 1 (2014), Art. 12.