Vanishing of $K_{4k}(\mathbb{Z})$ through degree 104 from Alpöge's Step 17

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Abstract

Assuming the arithmetic construction in Alpöge's Step 17, we prove that $K_{4k}(\mathbb{Z}) = 0$ for $1 \le k \le 26$. We identify the rational strict Selmer group detected by Step 17 and use the Grothendieck–Montpellier joint representation theorem and Bernoulli criterion to reduce vanishing to integral Hecke tests. In degree 104 the Bernoulli criterion selects four large primes for the second auxiliary field. Two rank-324 determinants exclude all other large primes in the first field. A rank-2,925 determinant is a unit at all four in the second. The accompanying certificates cover every lower multiple of four.

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1 The theorem and the two auxiliary fields

Alpöge asks in Step 17, footnote 6 of his *Remark* [1] about the consequences of the intermediate Selmer estimate for the higher algebraic K-groups of the integers. For the degrees divisible by four the groups are finite, so the problem is to exclude each possible prime divisor of their orders. We use K_n for Quillen's algebraic K-group and $e = 2k$, so that the degree in question is $2e = 4k$.

Theorem. *Assume the arithmetic construction underlying* `corGK_of` *in Alpöge's development at commit* `7cbf7a0`*, with the inputs supplied by* `corGK`*. Then*

$$K_{4k}(\mathbb{Z}) = 0 \qquad (1 \le k \le 26).$$

The Grothendieck–Montpellier archive proves two consequences of this construction that we use and reports finite certificates through degree 80 at the cited checkpoint [2]. First, a nonzero strict Selmer group supplies a single representation whose traces can be tested at several places. Second, three Bernoulli units force that representation into a specified rank-three class. Our calculation uses conjugation to pass to smaller matrices and gives the prime exclusions through degree 104.

To see why two fields help, fix a hypothetical large prime divisor ℓ. The Selmer group that would detect it lives over $\mathbb{Q}$. We can therefore choose either $\mathbb{Q}(\sqrt{-3})$ or $\mathbb{Q}(\sqrt{-7})$ to produce the trace tests. We choose the field in which the Bernoulli criterion holds at ℓ. A witness in that field would make the determinant of a particular matrix divisible by ℓ. Computing a determinant coprime to the candidate modulus excludes every prime divisor of that modulus at once. The proof below establishes this divisibility before using the computed values.

Which primes require Step 17? For $\ell = 2$, the known 2-primary computation applies in every degree under consideration. For odd $\ell < 2^{31}$, the published cyclotomic class-group computation applies. The use of Step 17 begins at $\ell \ge 2^{31}$. Even there, individual pairs of degrees and primes can be checked directly by cyclotomic methods. We give two such checks in degree 104. This separates the dependence of the full theorem on Step 17 from assertions about individual primes that one can verify independently.

2 From a strict Selmer class to one representation

Fix an even integer $2 \le e \le 52$ and a prime $\ell \ge 2^{31}$. Write $G_F = \mathrm{Gal}(\overline{\mathbb{Q}}/F)$, let ω be the mod-ℓ cyclotomic character, and put $D = (\mathbb{Q}_\ell/\mathbb{Z}_\ell)(-e)$. For each finite prime q choose a decomposition group G_q and its inertia group I_q. The rational strict Selmer group used here is

$$S_e = \ker\left(H^1(\mathbb{Q}, D) \longrightarrow \prod_{q \text{ finite}} H^1(I_q, D) \;\times\; H^1(G_2, D) \;\times\; H^1(G_\ell, D) \right).$$

The auxiliary field $E_d = \mathbb{Q}(\sqrt{-d})$, $d = 3, 7$, will enter only when we apply Step 17.

Identifying the length. Specialize Alpöge's construction to level $N = 2$, the trivial Dirichlet character, and the coefficient ring $\mathbb{Z}_\ell$ in $\overline{\mathbb{Q}}_\ell$. The coefficient module is $\mathbb{Z}_\ell \otimes_{\mathbb{Z}_\ell} (\mathbb{Q}_\ell/\mathbb{Z}_\ell)$ with cyclotomic action of exponent $-e$, hence is D. In the pinned development, `vlen_one` identifies Vlen with the length for this twist over $\mathbb{Q}$. The cocycle criterion `len_eq_zero_iff` says that the length is zero precisely when every cocycle that is a coboundary on each I_q and on both G_2, G_ℓ is a global coboundary. Each local group has its own coboundary witness. Thus Vlen $= 0$ if and only if $S_e = 0$, for either E_d.

Suppose $K_{2e}(\mathbb{Z})/\ell \neq 0$. By [9, Thm. 5.1], the ω^{-e}-component of the class group of $L = \mathbb{Q}(\zeta_\ell)$ modulo ℓ is nonzero. An equivariant quotient onto $D[\ell]$ gives an unramified class over L by class field theory. The unique prime above ℓ is principal, so its Frobenius in this quotient is trivial. Since $[L : \mathbb{Q}] = \ell - 1$, inflation–restriction descends the class to $\mathbb{Q}$ and preserves its local triviality at ℓ. At every other finite prime the extension $L/\mathbb{Q}$ is unramified. As $0 < e < \ell - 1$, we have $D^{G_\mathbb{Q}} = 0$, so the exact sequence for multiplication by ℓ injects the descended class into $H^1(\mathbb{Q}, D)$. At 2 the unramified group is $D/(2^{-e} - 1)D = 0$, since the nonzero scalar acts surjectively on the divisible module D. We have proved $K_{2e}(\mathbb{Z})/\ell \neq 0 \Rightarrow S_e \neq 0$.

One representation at all tested places. Both fields satisfy the eligibility conditions of Step 17. The Minkowski bound $2\sqrt{d}/\pi < 2$ gives class number one. The discriminant $-d$ is prime to 2ℓ, and ramification at d implies $E_d \not\subset L$. This gives the kernel condition for the trivial character. Its order, square and parity satisfy the character conditions since e is even. Finally $0 < 2e < \ell - 1$, and the coefficient ramification index is one.

Apply [2, K17-S17-JOINT] with the nine inputs assembled inside `corGK_of` and supplied by `corGK` in [1]. If the deformation model had no admissible points, its upper bound at the identity would have an empty covering set, making its finite length zero. The comparison of lengths would force Vlen $= 0$. Positive length therefore supplies a point f. Its integral representation has diagonal residual characters $\omega^e, 1, \omega^{-e}$ on G_{E_d}. In particular, at the coefficient prime λ determined by this point,

$$\operatorname{tr} \rho_f(g) \equiv 1 + \omega(g)^e + \omega(g)^{-e} \pmod{\lambda} \qquad (g \in G_{E_d}).$$

Thus all tests in one field concern a single system of Hecke eigenvalues. The common eigenvector allows us to add its two Hecke eigenvalues. Separate witnesses at the two places would give no such identity. The construction can supply a different point in the other field. The nine input terms and their source declarations are listed in the Montpellier checkpoint.

3 Choosing the field and identifying the Hecke operators

For $\eta_d = \chi_{-d}$ set $b_{n,d} = |\operatorname{num}(B_{n,\eta_d}/n)|$ in lowest terms and $A_d(e) = b_{e+1,d}b_{2e+1,d}$. The three units in [2, ES.5] have different roles. B_{1,η_d} gives the class-number condition, $B_{1,\eta_d\omega^{2e}}$ excludes the rank-two source type, and $B_{1,\eta_d\omega^e}$ excludes a self-dual rank-three source. Under the first unit, essential self-duality of an actual source would give self-duality. Thus a surviving point in this field gives a cuspidal rank-three representation which is not essentially self-dual. Here the Bernoulli numbers use the Teichmüller lift of ω. The first value is $-1/3$ or -1, and the other two are congruent modulo ℓ to $B_{a+1,\eta_d}/(a+1)$ for $a = 2e, e$, respectively [2, FV.3]. The certificate checks that all these rational denominators are smaller than 2^{31}. Consequently any field with $\ell \nmid A_d(e)$ satisfies all three conditions. We use E_3 when $\ell \nmid A_3(e)$ and E_7 otherwise. The numerical checks below verify the required units in the second case.

The full lattice stabilizer. At level two the allowed inertial characters factor through the action on $\mu_2 \subset \mathbb{Q}$, which is trivial. Applying the level theorem to $\rho_f\chi_\ell^{-1}$ with modulus one gives a regular algebraic conjugate-self-dual cuspidal representation Π of GL_3/E_d, spherical at every finite place. Its infinitesimal parameter is $(e, 0, -e)$. Its central character is unramified at finite places and trivial at infinity, hence trivial by class number one [2, compact transfer].

Use the definite unitary group of the standard ternary Hermitian lattice. It is a pure inner form, quasi-split at finite places. The infinity characters $(z/\bar{z})^e, 1, (z/\bar{z})^{-e}$ have positive conjugate parity, as required for untwisted stable base change in odd rank. The simple generic parameter of Π has trivial reduced component group. The global multiplicity formula therefore allows the compact member at infinity, of highest weight $(e-1, 0, 1-e)$, and the spherical members at split and unramified inert places [3, Ch. 5]. At the ramified prime d, let J_d be the full lattice stabilizer. Conjugate self-duality and determinant one give unramified roots $z, 1, z^{-1}$ for Π_w, unitary by [6, Cor. 1.3]. On the diagonal torus $(E_d)_w^\times \times U(1)$, the unramified character with uniformizer value z on the first factor and trivial on the second has precisely these stable base-change roots. The constituents of its normalized principal series belong to the corresponding tempered packet [3, §4.7]. For the ramified principal series the stable base change and its reducible cases are described in [4, §4.4, Props. 6.1 and 7.1]. The inducing character and Borel modulus are trivial on $B \cap J_d$. The induced function supported on BJ_d with value one at the identity is therefore J_d-fixed. Exactness of compact invariants in characteristic zero gives a fixed vector in a constituent, including when roots coincide. This is the level used by the neighbor matrices.

Two trace normalizations. At geometric Frobenius of norm Q, the twist χ_ℓ^{-1} and the algebraic Satake normalization each multiply by Q. Thus $\operatorname{tr}\rho_f$ is the sum of the unitary Satake roots. Set $u_Q = Q^{2e} + Q^e + 1$. At split p the unweighted neighbor operator H_w has eigenvalue $p\sum_i z_i$, with the inverse roots in the opposite direction. Each scaled operator $p^{e-1}H_w$ and $p^{e-1}H_{\bar{w}}$ has residual eigenvalue u_p. At inert 2 in E_3 the residue norm is 4. Relative to an end of the unitary tree, the 18 distance-two neighbors consist of one toward the end, one at the original height, and sixteen away. The normalized Satake coefficients at the two extreme heights are both $\sqrt{16} = 4$. Hence the unweighted operator H has eigenvalue $4z + 1 + 4z^{-1}$, whereas base change has roots $z, 1, z^{-1}$ [7, Thm. 4.1]. Its trace on the eigenline is $(H + 3I)/4$, so $4^{e-1}(H + 3I)$ has residual eigenvalue u_4.

4 Degree 104

Set $e = 52$ and let $\ell \geq 2^{31}$ be prime. A nonzero strict Selmer group would force a determinant to vanish modulo ℓ in one of the two fields. The Bernoulli numbers determine which field to use. Put

$$p_1 = 43\,279\,339\,111, \qquad p_2 = 490\,440\,141\,803,$$
$$p_3 = 63\,421\,406\,191\,963\,401\,438\,831\,238\,997, \qquad p_4 = b_{105,3}/2162.$$

The certificate proves primality of the four integers. Exact arithmetic gives

$$b_{53,3} = 2p_1p_2p_3, \qquad b_{105,3} = 2 \cdot 23 \cdot 47\, p_4, \qquad \gcd(b_{53,3}b_{105,3}, b_{53,7}b_{105,7}) = 4.$$

Thus the unit criterion applies over E_3 at every large prime outside $M = p_1p_2p_3p_4$, and over E_7 at each prime dividing M.

4.1 Why the negative eigenspace sees the witness

Let v be a compact Hecke eigenvector belonging to the representation Π supplied in the selected field. Conjugation of the field, together with the exchange of the two sets of coefficient variables, induces a linear involution J on the space of forms. It fixes the standard lattice class. Over E_7 it also fixes the other class, since there are exactly two. The choice of identification of a class with its conjugate has no effect on vectors invariant under its stabilizer.

The eigenvector Jv belongs to the conjugate system Π^c. Conjugate self-duality gives $\Pi^c \simeq \Pi^\vee$. Equality with Π would make the attached Galois representation self-dual by Frobenius compatibility and Chebotarev, contrary to the Bernoulli criterion. Strong multiplicity one distinguishes the two systems at a finite unramified place. Thus $v - Jv$ is nonzero in characteristic zero. An operator commuting with J has the same eigenvalue on $v - Jv$ as on v, provided v is its eigenvector. This applies to an inert neighbor operator and to the sum of the two directions at a split prime.

4.2 The two first-field determinants

In the single lattice class over E_3, counting the invariant polynomial basis in bidegree $(51, 51)$ gives dimension 657 and trace 9 for J. The negative eigenspace has dimension $(657 - 9)/2 = 324$. Write $u_Q = 1 + Q^{52} + Q^{104}$. For the inert neighbor operator H at 2 and the two directed operators at 7, the shifted matrices on that space are

$$X = 4^{51}(H + 3I) - u_4 I, \qquad Z = 7^{51}(H_w + H_{\bar{w}}) - 2u_7 I.$$

The shift at 2 uses residue norm 4 and trace $(H + 3I)/4$. At 7 each direction has scaled target u_7 at the same coefficient prime. The eigenvalue of each shifted operator on $v - Jv$ therefore reduces to zero. Each characteristic polynomial is integral at ℓ. Evaluating it at its characteristic-zero eigenvalue and reducing shows that its constant term vanishes modulo ℓ, even when a chosen integral representative of $v - Jv$ reduces to zero.

The exact integer determinants give

$$\gcd(\det X, \det Z) = 3^{1578}5^{33}11^{31}19^{18}23^6 \cdot 29 \cdot 79 \cdot p_1p_2p_3.$$

This excludes every large $\ell \nmid M$. At p_4 the determinant gcd is a unit, but $p_4 \mid b_{105,3}$. The unit criterion that put the hypothetical witness in the negative eigenspace over E_3 therefore fails. All four primes therefore go to the second field.

There is an independent check at the first two primes. For the *ordinary* Bernoulli numbers the residues of B_{p_i-53} modulo p_i are $14\,311\,957\,058$ for $i = 1$ and $449\,277\,795\,842$ for $i = 2$. Each nonzero residue already excludes its prime from $K_{104}(\mathbb{Z})$ by the cyclotomic argument below. The second-field computation tests all four.

4.3 One second-field determinant

Over E_7, the prime 2 splits. Write the scaled sum $2^{51}(H_w+H_{\bar{w}})$ in the standard and nonstandard lattice classes as $\left(\begin{smallmatrix} A & B \\ D & 0 \end{smallmatrix}\right)$. The zero block expresses the fact that all seven neighbors of the nonstandard class in each direction return to the standard class [8, §3]. Each class has its own negative eigenspace. Write $v - Jv = (x, y)$ in these two spaces. Its eigenvalue s satisfies $s \equiv t = 2u_2 \pmod{\lambda}$, where λ is the common coefficient prime above ℓ. Since $\gcd(t, M) = 1$, we have $s \neq 0$. The equations $Ax + By = sx$ and $Dx = sy$ imply $x \neq 0$ and

$$(s^2 I - sA - BD)x = 0.$$

Put $L(T) = T^2 I - TA - BD$ on the standard negative space and $F(T) = \det L(T)$. The dimension of that space is $\binom{27}{3} = 2925$. One way to obtain this rank is to take, for $j = 51, 50$, the cube-rotation invariants of $\bigwedge^2 \mathrm{Sym}^j(\mathbb{Q}^3)$. Subtracting their dimensions imposes the incidence relation $x \cdot y = 0$ on the two polynomial variables. For $j = 2k-1, 2k-2$, twice the exterior-square character difference on the identity, half-turns, quarter-turns and third-turns is, respectively, $2k(4k^2 - 1), -2k, 2k, 0$. Weighting by the class sizes $1, 9, 6, 8$ and dividing by $2 \cdot 24$ gives $\binom{k+1}{3}$; here $k = 26$.

The scaled neighbor maps preserve local coefficient lattices away from 2. The polynomial lattice modulo the incidence relation is torsion-free by division by its monic leading term. Taking stabilizer invariants and applying $(I - J)/2$ gives local lattices for the operators above, so F is integral at each candidate prime. Hence $F(s) = 0$ implies $F(t) \equiv 0 \pmod{\lambda}$.

The calculation evaluates a basis of integral polynomials at integral incidence pairs. If V is their evaluation matrix and W evaluates their images under $L(t)$, then $W = V[L(t)]$ in this basis. Consequently $\det W = \det V \, F(t)$, with $\det V$ integral. The divisibility $\ell \mid \det W$ follows even if V becomes singular after reduction. In the signed polynomial basis, each column is obtained by substituting neighbor representatives in its basis polynomial. The term BD sums two-step paths through the other lattice class. Both same-direction and opposite-direction paths enter because the operator is a sum of directions. Pairing each directed contribution with its conjugate gives an integral field trace; evaluating at the chosen incidence pairs produces the entries of W. The certificate constructs W modulo the 602-bit integer M and computes its determinant over the four prime fields. For example the first two residues are 32 189 186 543 and 445 745 447 561, respectively. The four residues give $\gcd(\det W, M) = 1$, a contradiction in each case. This finishes the large-prime argument in degree 104.

An independent check on the adjoint branch. The weight-53 cusp space of level 3 and character χ_{-3} gives a separate check of the two first-field trace targets for adjoint systems. Write a_q for the eigenvalue of T_q and Δ_q for the trace of the trace-zero adjoint representation at the tested place minus $1 + Q^{52} + Q^{-52}$. The residue norms at $q = 2, 7$ are $Q = 4, 7$, respectively. At the inert prime the Frobenius over E_3 is the square of the rational Frobenius. Thus $\mathrm{tr}\,\mathrm{Ad}^0(g) = (\mathrm{tr}\, g)^2/\det g - 1$ gives

$$2^{104}\Delta_2 = (a_2^2 + 2^{53})^2 - (1 + 2^{104})^2,$$
$$7^{52}\Delta_7 = a_7^2 - (1 + 7^{52})^2.$$

The adjoint verifier constructs both 16×16 matrices from Fourier expansions. The gcd of the determinants of the displayed polynomials in those matrices is $3^{76}5^{12}11^2 19^2 (p_1 p_2 p_3)^2$.

5 The other 25 degrees and the small primes

For each even $2 \le e \le 50$, the odd part of $\gcd(A_3(e), A_7(e))$ equals 1 except at $e = 12, 18, 36, 42$, where it equals $47, 37, 37^2, 43^2$, respectively. All relevant Bernoulli denominators are smaller than 2^{31}. Hence, at every large prime ℓ, the three unit conditions hold over E_3 if $\ell \nmid A_3(e)$, and over E_7 if $\ell \mid A_3(e)$.

Write C_q, U_q for the archived shifted classical and full compact determinants in the first field at the probe $q = 2, 7$, and put $G_e = \gcd(C_2U_2, C_7U_7)$. With $b_j = |\operatorname{num}(B_{j,\eta_3}/j)|$, the recorded integers satisfy

$$v_\ell(G_e) = v_\ell(b_{e+1}) + 3v_\ell(b_{2e+1}) \qquad (\ell > 337).$$

For each of the 25 exponents the certificate expresses the reduced numerator and denominator of $G_e/(b_{e+1}b_{2e+1}^3)$ as products of factors at most 337. This proves the displayed finite identity. At a large $\ell \nmid A_3(e)$, a witness would force $\ell \mid U_2$ and $\ell \mid U_7$, hence $\ell \mid G_e$. The identity rules this out. The following table accounts for the large divisors of $A_3(e)$ that are left.

Degree $2e$	Exclusion of remaining large primes
4, 8, 12, 16, 24, 28	$A_3(e)$ has no prime divisor at or above 2^{31}.
20, 36	One cyclotomic exclusion in each degree.
32, 40	One directed E_7 test in each degree, and one further cyclotomic exclusion in degree 40.
44, 48	Directed E_7 tests at two primes and one prime, respectively.
52, 56, 60, 64, 68	Directed E_7 tests at $4, 2, 4, 4, 3$ primes, respectively.
72	A rank-1,944 directed test modulo a product containing all large divisors.
76, 80, 84, 88, 92, 96, 100	One signed E_7 test per degree, of ranks 1,140, 1,330, 1,540, 1,771, 2,024, 2,300 and 2,600, respectively.

In a cyclotomic row put $j = \ell - 1 - e$. The group in [9, Thm. 5.1] is the even ω^j-component modulo ℓ of the cyclotomic class group. For these primes $2 \le j \le \ell - 3$. The reflection inequality bounds its dimension by that of the ω^{1-j}-component. Herbrand's theorem makes the latter zero if $B_j \not\equiv 0 \pmod{\ell}$. These are ordinary Bernoulli numbers. For instance, in degree 20 the remaining prime is 9 496 386 083 and the residue is 3 753 150 554. The other two nonzero residues appear in the same certificate archive. For the cyclotomic criterion see also [5, §1].

A directed test uses one of the two places above 2 in E_7 and the target $u_2 = 2^{2e} + 2^e + 1$. The block elimination uses this target in place of the doubled target for the sum of directions. The certificates check it is a unit. For the prime-by-prime tests the norm of the determinant, or the product from its two coefficient embeddings, is a unit modulo the tested prime. At degree 72 the calculation verifies right inverses for both the evaluation and numerator matrices modulo the recorded modulus. These identities persist modulo every prime divisor. In the final seven degrees the archived modulus M_e is obtained from $A_3(e)$ by removing specified factors below 2^{31}. The signed calculation gives $\gcd(2u_2, M_e) = \gcd(\det W_e, M_e) = 1$ in every row. Thus every possible large prime is excluded.

For odd $\ell < 2^{31}$ the character ω^{-e} is even. The Kummer–Vandiver computation of Hart, Harvey and Ong [10] gives trivial ℓ-part in the plus class group throughout this range, so [9, Thm. 5.1] gives $K_{2e}(\mathbb{Z})/\ell = 0$. Weibel's table gives trivial 2-primary part in positive degrees divisible by four [11, VI, Cor. 9.8]. Since these groups are finite, the exclusion of every prime proves the theorem, including degree 104.

6 Independent checks and reproducibility

Two further cyclotomic checks in degree 104. In Section 4, the two ordinary Bernoulli residues at p_1, p_2 give separate exclusions by reflection and Herbrand's theorem. Here is the reflection argument for the even index $j = p - 53$. Let A be the class group of $\mathbb{Q}(\zeta_p)$ modulo p. The Kummer radical of its maximal unramified elementary abelian p-extension is dual to A with values in μ_p. Its ω^{1-j}-part has dimension $\dim A_j$ and lies in the classes of elements whose finite valuations are divisible by p. Modulo units these classes inject into the p-torsion of the class group, via $(a) = \mathfrak{a}^p \mapsto [\mathfrak{a}]$. The only odd character in the units modulo p is ω, from roots of unity. For the nontrivial even j here the ω^{1-j}-unit part vanishes. This proves $\dim A_j \leq \dim A_{1-j}$, since the p-torsion and the quotient modulo p have the same dimension in each character.

The release's `check_reflection104.py` computes these residues with FLINT and checks its evaluation against exact rational Bernoulli numbers at two small primes.

Verifying the computation. In the release, `python3 check_step17_coverage.py` recomputes the Bernoulli quantities, checks the 25 support identities and tests the exclusions against the stored records. For degrees through 72 it removes from $A_3(e)$ the prime powers covered by the recorded tests and verifies that only small factors remain. The nested through-40, through-48 and through-72 archives supply the original calculations. `check_through100.py` also checks the products of the recorded determinant blocks in the seven signed rows. For degree 104 the separate reconstruction program `reproduce.py` regenerates the matrices. The archived replay reproduced both rank-324 matrices and all four rank-2,925 residues, and a second evaluator checked three complete rows (8,775 entries). The reproduction instructions give the commands for each type of check.
The versioned release contains the manuscript, TeX source, arithmetic verifiers and certificate bundle.

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