THE DEGREE PAIR $(84, 126)$ IN THE PLANE JACOBIAN PROBLEM: EXCLUSION OF THE SHAPE WITH FINAL CORNER $(19/7, 5)$

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Abstract. In the list of Guccione, Guccione, Horruitiner and Valqui of the possible shapes of a counterexample to the plane Jacobian conjecture with degrees at most 150, two shapes have maximal degree 126. One of them consists of the standard $(2, 3)$-pairs of degrees $(84, 126)$ with starting corner $(7, 35)$ and final corner $(19/7, 5)$, normalised as in Corollary 5.21 of Guccione–Guccione–Valqui. We show that no such pair exists. The two sides of the Newton polygon are described by the results of Guccione–Guccione–Valqui; in suitable polynomial coordinates they confine the pair to an explicit finite system of polynomial equations, which is solved order by order and has no solution. All statements are proved relative to the cited results; the final step is an exact computation, reproduced by an ancillary script (SymPy, and Singular for one ideal membership). The other shape of maximal degree 126 is the subject of a companion note. The argument has not yet been examined by specialists.

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1. Statement

Throughout, K is an algebraically closed field of characteristic zero, $L = K[x, y]$, $[P, Q] = P_x Q_y - P_y Q_x$ and $(m, n) = (2, 3)$. We use the notation of [1]: for a weight (ρ, σ), $v_{\rho,\sigma}$ is the weighted degree, $\ell_{\rho,\sigma}$ the leading form, $\mathrm{st}_{\rho,\sigma}$ and $\mathrm{en}_{\rho,\sigma}$ its end points; $L^{(l)} = K[x^{\pm 1/l}, y]$. *Standard (m, n)-pairs*, their *starting triple* and their *regular corners* are defined in [1, Definitions 4.3, 6.2, 5.5]. Put $t := xy^7$.

Standing data 1.1. (P, Q) is a standard $(2, 3)$-pair in L with $[P, Q] = J \in K^\times$, $\deg P = 84$, $\deg Q = 126$, starting triple $\big((7, 35), (2, 0), (7, -1)\big)$, and $(-1, 1) < \mathrm{Succ}_P(1, 0) < (-1, 0)$ as in [1, Corollary 5.21(4)]. Thus, after rescaling P and Q:

(D1) the support of P lies in $\{(i, j) : 7i - j \leq 28\}$, that of Q in $\{7i - j \leq 42\}$, and

$$\ell_{7,-1}(P) = \lambda_P \big(x^2 h(t)\big)^2, \qquad \ell_{7,-1}(Q) = \lambda_Q \big(x^2 h(t)\big)^3, \qquad \lambda_P, \lambda_Q \in K^\times,$$

with $h \in K[t]$ monic of degree 5, $h(0) \neq 0$ [1, Proposition 6.1, Theorem 7.6(2)].

Moreover h has a single root: $h = (t - c)^5$ with $c \in K^\times$.

Theorem 1.2. *There is no pair (P, Q) as in Standing data 1.1.*

Remark 1.3. A root of multiplicity k of h which satisfies inequality (5.9) of [1, Proposition 5.18], that is $2k \geq 14/3$, generates the regular corner $(2, 0) + k(1/7, 1)$ of a $(2, 3)$-pair in $L^{(7)}$, and such a root exists [1, Propositions 5.16, 5.18]; so $k \geq 3$. The last condition of the standing data is the case $k = 5$, with corner $(19/7, 5)$; it is the only shape with these degrees in the list of [2, Section 6]. If h has no root of multiplicity at least 4, then $k = 3$, which is impossible by the arithmetic of Lemma 3.1 below: the corner $(17/7, 3)$ admits neither Case I nor Case II. The remaining case $h = (t - c)^4(t - c')$, with corner $(18/7, 4)$, is not excluded by that arithmetic alone; for it we rely on [2].

The first arXiv version of [3] (August 2017) listed the shape of the standing data among those discarded, assuming that two expressions I_M and I_m for the intersection number of a Jacobian pair, due to Y. Xu, coincide. The published version [3] establishes the formula I_M but only an inequality for I_m, and no longer contains that list. To our knowledge the shape has not been excluded elsewhere.

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2. The left side

Lemma 2.1. *There is $g = g_2v^2 + g_3v^3 + g_4v^4 \in K[v]$ such that*

$$\hat{P}(z,v) := P\big(zv^5 + g(v),\, v^{-1}\big), \qquad \hat{Q}(z,v) := Q\big(zv^5 + g(v),\, v^{-1}\big)$$

are polynomials in z, v. Give z the weight 2 and v the weight 1. Then $\hat{P}$ has weighted degree 28 with top form $\lambda_P z^4(z - cv^2)^{10}$, $\hat{Q}$ has weighted degree 42 with top form $\lambda_Q z^6(z - cv^2)^{15}$, and

$$\hat{P}_z\hat{Q}_v - \hat{P}_v\hat{Q}_z = -J\,v^3.$$

Proof. Apply the flip ψ_1 of [1, Section 8], so that the weight $(7,-1)$ becomes $(\rho_0,\sigma_0) = (-1,7)$, with $\mathrm{st}_{\rho_0,\sigma_0} = (70,14)$ and $\mathrm{en}_{\rho_0,\sigma_0} = (0,4)$ for the flipped P, and $\frac{1}{m}\,\mathrm{st} = \frac{1}{n}\,\mathrm{st} = (35,7)$. Let F be the element of [1, Theorem 2.6] for (ρ_0,σ_0); then $v_{-1,7}(F) = 6$. As in the proof of [1, Proposition 8.1]: if $\mathrm{st}(F) = (1,1)$ then $\mathrm{en}(F) \neq (1,1)$, since F is not a monomial [1, Remark 2.5], so $\mathrm{en}(F) \sim \mathrm{en}(P) \sim (0,1)$ and $\mathrm{en}(F) = (0,6/7)$, which is impossible; hence $\mathrm{st}(F) \sim (35,7)$, so $\mathrm{st}(F) = (15,3) = \frac{3}{7}(35,7)$ and $q = 7$ in [1, Corollary 7.4]. That corollary applies to the flipped pair and to every pair obtained from it by a translation of the second variable by a Laurent monomial in the first (the form ℓ_{ρ_0,σ_0}, the degrees $v_{1,1}$, $v_{0,1}$ and F are unchanged). It yields: for every weight between $(1,-5)$ and (ρ_0,σ_0) at which the pair has an edge, the edge form of P is a power R^{14}. In the original variables: for a weight $(-s,1)$ with $s < 5$ carrying an edge with end point $(14,70)$, the form is $(xy^5 - \gamma y^{5-s})^{14}$ up to a constant, so $s \in \mathbb{Z}$, and $s \geq 2$ by the condition on $\mathrm{Succ}_P(1,0)$. Here $(14,70) = \mathrm{en}_{7,-1}(P)$ is the starting point of the edge at $\mathrm{Succ}_P(1,0)$, because $\deg P = 84 = v_{1,1}(14,70)$ (compare [1, Section 8]); and the statement applies at every stage of the process below, so that no edge with a non-integer $s < 5$ appears after a translation. The translation $x \mapsto x + \gamma y^{-s}$ removes this edge and leaves the forms of smaller s unchanged [1, Proposition 3.9]. After at most three such translations ($s = 2,3,4$) no edge of weight $(-s,1)$, $s < 5$, has end point $(14,70)$. Since the weight $(-5,1)$ lies within a half-turn of $(7,-1)$, the point $(14,70)$ is then still leading for $(-5,1)$ [1, Remark 2.10, Proposition 3.7]; hence every monomial x^iy^j of the translated P satisfies $j \leq 5i$, that is, $x^iy^j = z^iv^{5i-j}$ is a monomial in $z = xy^5$, $v = y^{-1}$. The same holds for Q. For every weight $(-s,1)$ with $s < 5$ the values of P and Q are positive, since $(14,70)$ and $(21,105)$ remain in the supports of the translated polynomials and $v_{-s,1}(14,70) = 14(5-s)$, $v_{-s,1}(21,105) = 21(5-s)$. At a weight where P has an edge, [1, Proposition 7.3] with $T_0 = Q$ (its hypothesis on the brackets holds, because $[Q,P]$ is a constant) gives $[\ell(Q),\ell(P)] = 0$ and proportional values, so $\ell(Q) \sim R^{21}$ and the same translation applies. At the remaining weights the leading forms commute by [1, Proposition 1.13], so $\ell(Q)$ is proportional to a power of the monomial $\ell(P)$ [1, Proposition 2.1(2)], that is, $\ell(Q)$ is the monomial at $(21,105)$. Hence $j \leq 5i$ on the support of Q.

A translation $x \mapsto x + \gamma y^{-s}$ with $s < 7$ replaces x^iy^j by monomials $x^{i-r}y^{j-sr}$ of $(7,-1)$-weight $7i - j - (7-s)r$, so (D1) holds for the translated pair with the same leading forms. Since $x^iy^j = z^iv^{5i-j}$ has $7i - j = 2i + (5i - j)$, the bound $7i - j \leq 28$ is the weighted degree bound, and for $x = zv^5$, $y = v^{-1}$ one has $x^4(xy^7 - c)^{10} = z^4(z - cv^2)^{10}$, which gives the top form; likewise for Q. The Jacobian follows from $dx \wedge dy = -v^3\,dz \wedge dv$. □

3. The right side

Write $X := x^{1/7}$, so that $L^{(7)} = K[X^{\pm 1}, y]$.

Lemma 3.1. *There are $\lambda \in K^\times$ with $\lambda^7 = c$ and $\lambda_2, \lambda_3 \in K$ such that, with*

$$Y := \eta + \lambda X^{-1} + \lambda_2 X^{-2} + \lambda_3 X^{-3}, \qquad P_1(X,\eta) := P\big(X^7, Y\big), \qquad Q_1(X,\eta) := Q\big(X^7, Y\big),$$

and with the weight $W(X^i\eta^j) := 3i - 11j$, every monomial of P_1 has $W \leq 4$ and every monomial of Q_1 has $W \leq 6$. The monomials of P_1 with $W = 4$ lie between $X^5\eta$ and $X^{38}\eta^{10}$, both with non-zero coefficient; those of Q_1 with $W = 6$ lie between X^2 and $X^{57}\eta^{15}$, both with non-zero coefficient.

Proof. In $\tilde{z} := x^{1/7}y$ one has $\ell_{7,-1}(P) = \lambda_P x^4(\tilde{z}^7 - c)^{10}$, so every root λ of $\tilde{z}^7 = c$ has multiplicity $10 \geq \frac{m}{l}\frac{a\rho + bl\sigma}{\rho + \sigma} = \frac{14}{3}$ (with $l = 1$, $(a,b) = (7,35)$, $(\rho,\sigma) = (7,-1)$). By [1, Proposition

5.18] the automorphism $\varphi(y) = y + \lambda x^{-1/7}$ gives a $(2,3)$-pair $(\varphi P, \varphi Q)$ in $L^{(7)}$ with a regular corner at $A_1 := (2,0) + 5(1/7,1) = (19/7,5)$, of direction $(\rho',\sigma') < (7,-1)$. We follow this corner as in the proof of [1, Proposition 5.19]. Case II is impossible, because [1, Proposition 5.14(4)] requires $\gcd(19,5) > 1$. In Case III, [1, Proposition 5.17] gives $\rho' \mid 7$, hence $\sigma'/\rho' = -j/7$ with $2 \le j \le 6$, and the translation $y \mapsto y + \lambda' x^{-j/7}$ produces a $(2,3)$-pair in $L^{(7)}$ with a regular corner at the same point A_1 and a smaller direction. After finitely many steps Case I occurs, for a pair (P_1,Q_1) and a direction (ρ,σ). There $[\ell(P_1),\ell(Q_1)] \neq 0$, so $v_{\rho,\sigma}(P_1) + v_{\rho,\sigma}(Q_1) = \rho + \sigma$ [1, Proposition 1.13], while $\operatorname{en}(P_1) = 2A_1 = \frac{2}{3}\operatorname{en}(Q_1)$ [1, Corollary 5.7(2)]. Hence $2(\frac{19}{7}\rho + 5\sigma) = \frac{2}{5}(\rho+\sigma)$, that is $(\rho,\sigma) = (21,-11)$, $v(P_1) = 4$, $v(Q_1) = 6$. By [1, Proposition 5.13] either $\frac{1}{2}\operatorname{st}(P_1) = (a'/7, 0)$ with $a' \in \mathbb{Z}$, which would give $6a' = 4$; or $\{\operatorname{st}(P_1), \operatorname{st}(Q_1)\} = \{(k/7,0), (1-k/7,1)\}$ with $k \in \mathbb{N}$. If $\operatorname{st}(P_1) = (k/7,0)$ then $3k = 4$; so $\operatorname{st}(Q_1) = (k/7,0)$ with $3k = 6$, that is $\operatorname{st}(Q_1) = (2/7,0)$ and $\operatorname{st}(P_1) = (5/7,1)$. The directions of the Case III steps lie strictly between $(21,-11)$ and $(7,-1)$, so $j \in \{2,3\}$, each at most once. Finally $v_{21,-11}(x^{i/7}y^j) = 3i - 11j$, and $\operatorname{en}(Q_1) = (57/7, 15)$. □

4. Common coordinates

Let $\mathcal{A}$ be the set of formal sums $\sum c_{ij} X^i \eta^j$ $(i \in \mathbb{Z},\ j \ge 0)$ such that, for every r, only finitely many terms have $W = 3i - 11j \ge r$. It is a ring containing $K[X^{\pm 1}, \eta]$; for $f \neq 0$ let $W(f)$ be the largest weight of its terms and $\ell(f)$ the sum of the terms of that weight. Then $\ell(fg) = \ell(f)\ell(g)$ and $W(fg) = W(f) + W(g)$, and $1 + \varepsilon$ is invertible in $\mathcal{A}$ when $W(\varepsilon) < 0$.

Lemma 4.1. *Let g, $\lambda, \lambda_2, \lambda_3$ be as in Lemmas 2.1 and 3.1, and put*

$$\zeta := z - c\,v^2 - \kappa_1 v - \kappa_2, \qquad \kappa_1 := 7\lambda^5\lambda_2, \quad \kappa_2 := 7\lambda^3(\lambda\lambda_3 + 2\lambda_2^2).$$

Write $\hat P = \sum p_{ab}\zeta^a v^b$ and $\hat Q = \sum q_{ab}\zeta^a v^b$ in $K[z,v] = K[\zeta, v]$. Then $p_{ab} \neq 0$ implies $3b - 2a \le 4$, and $q_{ab} \neq 0$ implies $3b - 2a \le 6$. Moreover $p_{1,2} \neq 0$ and $q_{0,2} \neq 0$.

Proof. In $\mathcal{A}$ the element Y of Lemma 3.1 is $\lambda X^{-1}(1+\varepsilon)$ with $W(\varepsilon) < 0$; it is invertible. The assignment $v \mapsto Y^{-1}$, $z \mapsto X^7Y^5 - g_2Y^3 - g_3Y^2 - g_4Y$ defines a ring morphism $K[z,v] \to \mathcal{A}$ which sends $\hat P$ to P_1 and $\hat Q$ to Q_1, because $P(x,y) = \hat P\big((x - g(y^{-1}))y^5, y^{-1}\big)$ in $K[x, y^{\pm 1}]$. Expanding, the image of v has $W = 3$ and leading term X/λ, and the image of ζ has $W = -2$ and leading term $7\lambda^4\eta X^3$: the terms of weights 6, 3, 0 cancel by the choice of $c = \lambda^7$, κ_1, κ_2, and no monomial has weight -1. Hence the image of $\zeta^a v^b$ has leading term a non-zero multiple of $\eta^a X^{3a+b}$, of weight $3b - 2a$, and these monomials are distinct for distinct (a,b). Therefore, for a non-zero polynomial, $W\big(\sum c_{ab}\zeta^a v^b\big) = \max\{3b - 2a : c_{ab} \neq 0\}$, and its leading part is the image of the corresponding terms. Lemma 3.1 gives $W(\hat P) = 4$, $W(\hat Q) = 6$; the monomials $X^5\eta$ and X^2 correspond to ζv^2 and v^2. □

5. The finite system

By Lemmas 2.1 and 4.1, and since the substitution $z = \zeta + cv^2 + \kappa_1 v + \kappa_2$ preserves the weighted degree (ζ of weight 2) and turns the top form of $\hat P$ into $\lambda_P \zeta^{10}(\zeta + cv^2)^4$, a pair as in the standing data yields a solution of the following system.

- (S1) $\hat P = \sum p_{ab}\zeta^a v^b$ with $a,b \ge 0$, $2a + b \le 28$, $3b - 2a \le 4$; $\hat Q = \sum q_{ab}\zeta^a v^b$ with $a,b \ge 0$, $2a + b \le 42$, $3b - 2a \le 6$; and $p_{1,2}\, q_{0,2} \neq 0$.
- (S2) $\hat P_\zeta \hat Q_v - \hat P_v \hat Q_\zeta = -J\, v^3$ with $J \in K^\times$.
- (S3) The part of $\hat P$ on the line $2a + b = 28$ is $\lambda_P\, \zeta^{10}(\zeta + c\, v^2)^4$, with $\lambda_P, c \in K^\times$.

Call the terms with $3b - 2a = 4 - k$ the *layer* k of $\hat P$ and those with $3b - 2a = 6 - k$ the layer k of $\hat Q$, and write $\hat P = \sum_{k \ge 0} P_k$, $\hat Q = \sum_{k \ge 0} Q_k$. The bracket of a term of layer i of $\hat P$ with a term of layer j of $\hat Q$ is a monomial $\zeta^a v^b$ with $3b - 2a = 9 - (i + j)$, and v^3 has $3b - 2a = 9$. So (S2) is equivalent to the *equations of order k*:

$$\sum_{i+j=k} \{P_i, Q_j\} = \begin{cases} -J\, v^3, & k = 0, \\ 0, & k \ge 1, \end{cases} \qquad \{f,g\} := f_\zeta g_v - f_v g_\zeta. \tag{1}$$

Lemma 5.1. *If* (S1)–(S3) *has a solution, it has one with* $c = 1$, $\lambda_P = \frac{1}{12}$, $J = -2$,

$$P_0 = \zeta v^2\big(1 + \tfrac{5}{4}s + \tfrac{1}{2}s^2 + \tfrac{1}{12}s^3\big), \qquad s := \zeta^3 v^2,$$
$$Q_0 = v^2\big(1 + 5s + \tfrac{16}{3}s^2 + \tfrac{8}{3}s^3 + \tfrac{2}{3}s^4 + \tfrac{2}{27}s^5\big),$$

and $p_{0,0} = q_{0,0} = q_{10,8} = 0$.

Proof. The system is preserved by $(\hat{P}, \hat{Q}) \mapsto (\gamma\hat{P}(\alpha\zeta, \beta v), \delta\hat{Q}(\alpha\zeta, \beta v))$, which multiplies $p_{1,2}$ by $\gamma\alpha\beta^2$, $q_{0,2}$ by $\delta\beta^2$, c by β^2/α and λ_P by $\gamma\alpha^{14}$. So we may assume $p_{1,2} = q_{0,2} = c = 1$ and $\lambda_P = \frac{1}{12}$; the remaining freedom is $\alpha^{12} = 1$, $\beta^2 = \alpha$, $\gamma = \alpha^{-2}$, which multiplies s by α^4, a cube root of unity. By (S1) the layers of order 0 are $P_0 = \zeta v^2(1 + a_4 s + a_7 s^2 + a_{10} s^3)$ and $Q_0 = v^2(1 + h_1 s + \cdots + h_5 s^5)$, and $a_{10} = \lambda_P c^4 = \frac{1}{12}$ by (S3). The equation of order 0 then has exactly three solutions, with $J = -2$, permuted by $s \mapsto \omega s$, $\omega^3 = 1$; one of them is the pair displayed (ancillary script). Finally replace $\hat{Q}$ by $\hat{Q} - 12 q_{10,8}\hat{P} -$ const and $\hat{P}$ by $\hat{P} -$ const: this preserves (S1)–(S3) and the layers of order 0, since $\hat{P}$ has no term in layers 0 and 1 of $\hat{Q}$, and $p_{10,8} = \frac{1}{12}$. □

Proposition 5.2. *The system* (S1)–(S3)*, normalised as in Lemma 5.1, has no solution.*

Proof (exact computation). For $k \geq 1$ the equation of order k in (1) is $\{P_k, Q_0\} + \{P_0, Q_k\} = -\sum_{0<i<k}\{P_i, Q_{k-i}\}$: a linear system $A_k x_k = b_k$ in the vector x_k of the coefficients of layer k not fixed by (S3) or by the normalisation, with a matrix A_k of rational numbers and a right-hand side depending on the earlier layers. Every solution satisfies $A_k^{\mathsf{T}} A_k x_k = A_k^{\mathsf{T}} b_k$, whose general solution expresses x_k through b_k and those coordinates of x_k which remain free; substituting it, the entries of $A_k x_k - b_k$ are *conditions*: polynomials with rational coefficients in the free coordinates of the layers $\leq k$, which vanish on every solution. Carrying this out for $k = 1, 2, \ldots$ (ancillary script):

(A) For $k \leq 23$ exactly five coordinates remain free, namely $q_{14,11}$, $q_{15,11}$, $q_{14,10}$, $q_{15,10}$, $q_{13,8}$, and the script records 28 distinct conditions. The ideal they generate in the polynomial ring over $\mathbb{Q}$ in these five variables contains $q_{14,11}$, $q_{15,11}$ and $q_{15,10}$ (Gröbner basis, Singular [8]).

(B) With $q_{14,11} = q_{15,11} = q_{15,10} = 0$, for $k \leq 36$ the free coordinates are $q_{14,10}$, $q_{13,8}$ and the script records 26 distinct conditions; they generate the unit ideal of $\mathbb{Q}[q_{14,10}, q_{13,8}]$ (Gröbner basis, SymPy).

By (A) the three coordinates vanish on every solution, and by (B) there is none. Independently of this two-step argument, the 73 distinct recorded conditions of orders $k \leq 30$ in the five free coordinates generate the unit ideal (Gröbner basis, Singular; optional step of the script). □

As a check of the procedure, the script repeats step (B) without condition (S3) (with the same three coordinates equal to zero, orders $k \leq 36$): the conditions obtained all vanish at the origin of the free coordinates.

Proof of Theorem 1.2. A pair as in the standing data gives a solution of (S1)–(S3), hence a normalised one by Lemma 5.1, contradicting Proposition 5.2. □

6. Scope

- **Proved here, relative to the cited results:** the reduction of Theorem 1.2 to Proposition 5.2 (Lemmas 2.1–5.1). The inputs are (D1), which rests on [1, Proposition 6.1, Theorem 7.6(2), Remark 3.1], and [1, Theorem 2.6, Remarks 2.5, 2.10, 5.8, Corollaries 5.7, 7.4, Propositions 1.13, 2.1, 3.7, 3.9, 5.13, 5.14, 5.17, 5.18, 5.19, 7.3], as indicated. The condition on $\mathrm{Succ}_P(1, 0)$ in the standing data is the one of [1, Corollary 5.21(4)], which is established there for the standard pairs attached to a minimal counterexample; the theorem is stated under this condition.
- **Verified by exact computation:** Proposition 5.2; the equation of order 0 in Lemma 5.1; the arithmetic of Lemma 3.1 and Remark 1.3; the leading terms in Lemma 4.1 (ancillary

file `verify_84_126.py` in the folder `anc`; SymPy, and Singular for step (A), for which the script also writes the input file). Step (A) was not obtained with SymPy alone.

- **Not claimed:** a statement on the plane Jacobian conjecture itself; on the pairs with the same starting triple whose polynomial h has more than one root (see Remark 1.3). The other shape of maximal degree 126 in [2], with starting corner $(12, 30)$ and degrees $(126, 84)$, is treated in the companion note [7]. The same shape with P and Q exchanged is covered by exchanging them. No new bound on the degree of a counterexample (see [4] for the published one) is claimed here.
- [1, Proposition 7.3 and Corollary 7.4], on which Lemma 2.1 rests, are stated in [1] with the indication that their proofs mimic those of Proposition 7.1 and Corollary 7.2; they are applied here to Laurent translates, as in [1, Section 8] and in [5, 6].
- The argument has been checked only by the author with AI assistance (Claude, Anthropic); it has not been validated by professionals.

References

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