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TWO PRIMITIVE GAUSSIAN FERMAT–CATALAN IDENTITIES

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Abstract. We exhibit pairwise coprime Gaussian integer solutions with exponent triples $(2,3,13)$ and $(3,9,2)$. The second follows from an integer identity recorded by P.-J. Cazorla García. Norms distinguish both triples from the examples on Ed Pegg Jr.'s published list.

1. Two identities

Ed Pegg Jr. asks for additional pairwise coprime solutions of $X^p + Y^q = Z^r$ in the Gaussian integers with $1/p + 1/q + 1/r < 1$ [2]. Two examples are

$$(2761 + 2761i)^2 + (239i)^3 = (3i)^{13}, \tag{1}$$

$$(2797i)^3 + (13i)^9 = (75090 - 75090i)^2. \tag{2}$$

In both examples every base is a nonunit. The sums of the reciprocal exponents are $71/78$ and $17/18$, respectively.

2. Verification

It suffices to check the integer identities

$$239^3 + 3^{13} = 15\,246\,242 = 2 \cdot 2761^2, \qquad 2797^3 - 13^9 = 11\,277\,016\,200 = 2 \cdot 75090^2.$$

Indeed, $(1+i)^2 = 2i$, $(1-i)^2 = -2i$, $i^3 = -i$, and $i^9 = i^{13} = i$. Multiplying the first integer identity by i and the second by $-i$ gives (1) and (2) after rearrangement.

For $z = a + bi$, let $N(z) = a^2 + b^2$. The norms of the three bases, in their displayed order, are

$$(15\,246\,242,\ 57\,121,\ 9), \qquad (7\,823\,209,\ 169,\ 11\,277\,016\,200).$$

Within each triple the three pairwise greatest common divisors are all 1. A common nonunit Gaussian divisor would have an integer norm greater than 1 dividing the corresponding two norms. Thus the three bases of each identity are pairwise coprime.

3. Origin of the examples

The search tested whether the sum or difference of two odd powers was twice a square, as suggested by Fred W. Helenius's example with bases $5i, 3i, 34 - 34i$ on Pegg's list [2]. The integer relation underlying (2) already appears in Pedro-José Cazorla García's work on differences of powers [1]. The two displayed Gaussian triples have different lists of base norms from the examples on Pegg's page. Unit associates, conjugation, and permutation preserve the unordered norm list.

OpenAI Codex carried out the search, checked the arithmetic, and prepared this note.

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References

[1] P.-J. Cazorla García, *On differences of perfect powers and prime powers*, J. Number Theory 283 (2026), 121–169. See also Table 4.1 in his 2024 Manchester thesis.

[2] E. Pegg Jr., *The neglected Gaussian integers*, MathPuzzle, and *Complex solutions for the Fermat–Catalan conjecture*.