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THE EDGE SUMS OF SEVEN DOUBLE-SIX DOMINO FRAMES

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Abstract. Partition the 28 double-six dominoes into seven square frames. When the four edge sums of each frame agree, exactly 10,821 sorted lists of the seven sums are attainable. We enumerate every admissible frame, prove completeness by an exact-cover recurrence, and provide a physical witness for every list.

1. Enumeration of the edge-sum lists

Use each of the 28 double-six dominoes once to tile the borders of seven 3-by-3 squares. Within every border, the three pips along any side must have the same sum [1].

Theorem 1.1. *There are exactly* $\mathbf{10,821}$ *different sorted seven-term edge-sum lists. Grouping by the smallest sum gives*

smallest sum	2	3	4	5	6
attainable lists	3323	2721	2721	1511	545

Proof. Let the four corner pips in cyclic order be $a_0, \ldots, a_3$. For a common edge sum s, the intermediate pips must be $b_i = s - a_i - a_{i+1}$, cyclically. Choose the clockwise orientation of the frame by reflection if necessary. Each domino is then the unordered pair $\{a_i, b_i\}$, retaining doubles. Enumerate $s = 0, \ldots, 18$ and the 7^4 corner choices. Keep $0 \le b_i \le 6$ and four distinct dominoes. This gives exactly 1,070 different pairs (M, s) of four-domino masks and edge sums. Conversely each retained pair has a physical realization.

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For a remaining domino set U, let $G(U)$ be the set of attainable sum histograms. Start with $G(\varnothing) = \{0\}$. Choose any $i \in U$ and take the union of $\{e_s + v : v \in G(U \setminus M)\}$ over all retained (M, s) with $i \in M \subseteq U$. The frame containing i proves both directions of the recurrence. On all 28 dominoes it gives the table. The supplied program independently regenerates frames by closing walks of four oriented dominoes, repeats the covering recursion with a different pivot order, and checks reflection $s \mapsto 18 - s$ on the complete answer set. □

Constructive certificates. For the sum list in the question, the following seven frames give one physical arrangement. Each four-digit word lists its clockwise corner pips.

sum	16	13	12	10	6	6	2
corners	4656	4566	3645	2455	2233	0021	0102

The supplementary file `domino-certificates.json` gives seven frame indices for each sum list. A frame is encoded by its sum and its four clockwise corner pips. Put the three supplementary files in one directory. Running `domino.py` reproduces the exhaustive count, and `domino_certificates.py` checks every physical arrangement.

References

[1] daw, *Number of arrangements of dominos into squares with equal edge sum*, Mathematics Stack Exchange, question 5149342.