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ALL 97 CHAOS TILES DUOS IN A SINGLE 66-TILE ARRANGEMENT

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Abstract. Ed Pegg Jr. asks whether 50 Veks and 40 Kays suffice to realize all 97 legal two-tile contacts in one connected arrangement. We exhibit such an arrangement using 35 Veks and 31 Kays. A numbered diagram, assembly instructions and a certificate identifying each of the 97 contacts specify the construction.

1. A connected arrangement

Theorem 1.1. *There is a legal sequence of 66 placements of Chaos Tiles that realizes all 97 duos in Pegg's illustrated catalogue. Its 35 Veks and 31 Kays determine 104 unordered pairs of tiles sharing at least one complete side.*

The construction gives an upper bound of 66 on the minimum total. Mike Hirschhorn discovered the Vek. Ed Pegg Jr. described its companion pentagon, the Kay, and published the challenge, rules, and catalogue [1, 2, 3, 4]. Tile numbers indicate the order of placement.

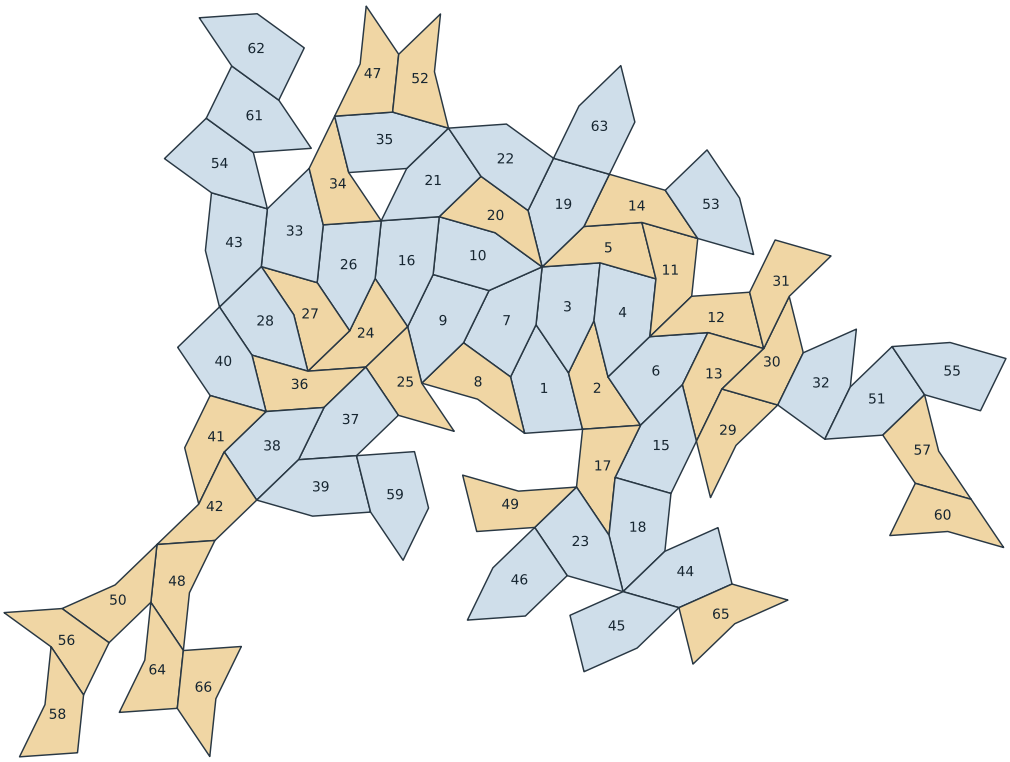


Figure 1. The Veks are blue and the Kays are gold.

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2. Exact assembly

Give every edge unit length. Number the vertices from 0 to 4 in the order of the following interior angles. Edge j runs from vertex j to vertex $j+1$ with indices taken modulo 5.

	0	1	2	3	4
V	80°	160°	60°	140°	100°
K	40°	200°	60°	100°	140°

The sign + means that the vertices run counterclockwise, and − means clockwise. Start tile 1 with vertex 0 at the origin and edge 0 pointing right. In each subsequent row, place edge v of the new tile onto edge u of the indicated parent. The directed edges point in opposite directions when the signs agree and in the same direction when the signs differ. These data determine all vertices exactly.

No.	Tile	Parent	u	v	No.	Tile	Parent	u	v	No.	Tile	Parent	u	v
1	V+	–	–	–	23	V−	17	4	0	45	V+	44	2	2
2	K−	1	0	3	24	K−	16	2	2	46	V+	23	3	4
3	V−	1	1	2	25	K+	9	2	1	47	K+	35	0	2
4	V+	2	0	1	26	V−	16	3	0	48	K+	42	4	2
5	K+	3	4	0	27	K+	24	0	2	49	K+	23	4	4
6	V+	2	1	4	28	V−	27	0	0	50	K+	48	3	4
7	V+	1	2	0	29	K+	13	0	3	51	V+	32	0	1
8	K−	1	3	4	30	K+	13	1	1	52	K−	35	1	2
9	V−	7	3	0	31	K+	12	2	4	53	V−	14	4	3
10	V−	7	2	2	32	V+	30	3	3	54	V+	43	4	1
11	K−	4	3	0	33	V+	26	3	0	55	V+	51	4	2
12	K+	6	2	0	34	K−	26	4	2	56	K+	50	2	3
13	K+	6	1	3	35	V−	21	0	2	57	K+	51	3	2
14	K−	5	3	1	36	K+	24	4	0	58	K+	56	1	4
15	V−	6	0	1	37	V+	25	3	4	59	V+	39	4	0
16	V−	9	3	1	38	V+	36	3	2	60	K−	57	4	3
17	K+	2	2	2	39	V−	37	2	3	61	V−	54	3	3
18	V+	15	4	4	40	V−	28	3	0	62	V−	61	0	0
19	V−	5	4	1	41	K+	38	3	1	63	V−	19	4	4
20	K−	10	0	1	42	K−	38	4	2	64	K+	48	4	4
21	V+	16	4	2	43	V+	28	4	2	65	K−	44	3	3
22	V−	19	3	4	44	V+	18	2	1	66	K+	64	3	3

Exact checks. With $z = e^{i\pi/9}$, the eighteen directed unit edges are z^j for $0 \le j < 18$. All endpoints belong to $\mathbb{Z}[z]$, where $z^6 - z^3 + 1 = 0$. Orientation and intersection tests reduce to signs in the real field generated by $s = 2\cos(\pi/9)$. The minimal polynomial is $s^3 - 3s - 1$, with a unique root in $(1, 2)$, so rational bisection determines the sign of every nonzero reduced polynomial. We used these tests on all pairs of tiles. At shared vertices we also checked the occupied sectors, in steps of 20°, after each placement.

3. Witnesses for the 97 published duos

Read Pegg's sheet from left to right, then top to bottom. The first eight rows have eleven drawings apiece, and the ninth has nine. The entries below are tile numbers from Figure 1. Each pair reproduces the indicated drawing up to a rigid motion or reflection.

Duo	Tiles	Duo	Tiles	Duo	Tiles	Duo	Tiles
1	3, 7	26	30, 31	51	24, 26	76	2, 3
2	1, 3	27	41, 42	52	3, 5	77	20, 21
3	51, 55	28	24, 25	53	6, 13	78	21, 22
4	48, 50	29	38, 41	54	30, 32	79	23, 46
5	5, 14	30	2, 4	55	35, 47	80	15, 18
6	13, 29	31	13, 15	56	1, 7	81	18, 23
7	10, 20	32	1, 8	57	9, 16	82	11, 12
8	25, 37	33	40, 41	58	37, 39	83	5, 11
9	27, 33	34	61, 62	59	16, 21	84	9, 25
10	17, 23	35	43, 54	60	57, 60	85	26, 27
11	26, 34	36	9, 10	61	11, 14	86	7, 8
12	21, 35	37	13, 30	62	2, 6	87	5, 19
13	39, 59	38	12, 31	63	17, 18	88	14, 19
14	44, 45	39	29, 30	64	36, 38	89	54, 61
15	56, 58	40	4, 5	65	19, 20	90	7, 10
16	50, 56	41	27, 28	66	38, 42	91	3, 4
17	42, 48	42	20, 22	67	32, 51	92	47, 52
18	6, 12	43	23, 49	68	37, 38	93	2, 17
19	34, 35	44	36, 40	69	6, 15	94	33, 34
20	14, 53	45	16, 26	70	10, 16	95	1, 2
21	8, 9	46	4, 6	71	64, 66	96	36, 37
22	51, 57	47	19, 63	72	24, 27	97	16, 24
23	18, 44	48	12, 13	73	15, 17		
24	7, 9	49	48, 64	74	4, 11		
25	33, 43	50	24, 36	75	44, 65		

Coverage check. Fix one tile. For each choice of two edge labels and relative handedness, attach the second tile along the selected edge. Discard interior overlaps and 20° gaps, and identify congruent pairs, allowing reflections. If the same two tiles share two edges, their union contributes one duo. This enumeration gives 97 types. The construction contains every one. To index the table we matched the two pentagon outlines in each published drawing to their ordered angle lists. The drawing-to-type correspondence is one-to-one.

4. Verification of the theorem

Proof. For each numbered placement in Section 2 the parent has a lower number, so the full-edge attachment builds a connected arrangement. The table uses 35 convex and 31 concave pieces. For the geometry, put $z = \exp(\pi i/9)$. Each directed side is a power of z and every vertex lies in $\mathbb{Z}[z]$. Write its coordinates in the basis $1, z, \ldots, z^5$, reducing by $z^6 = z^3 - 1$. This represents point equality by six integer comparisons.

For a directed edge and a test point, rotate the edge onto the positive real axis. The sign of the imaginary coordinate of the resulting displacement determines its side of the supporting line. Dividing by the positive $\sin(\pi/9)$ gives a polynomial in $s = 2\cos(\pi/9)$ by the sine recurrence. Reduce modulo $s^3 - 3s - 1$. Zero is then exact. For a nonzero polynomial, rational bisection in $(1, 2)$ eventually separates its value from zero. The corresponding cosine recurrence gives the same procedure for projections along an edge.

For every pair of polygons the checker excludes proper edge crossings, vertices strictly inside the other polygon, vertices in the interior of a side, and overlapping angular sectors at coincident vertices. These tests exhaust interior intersections of two simple polygons here. Collinear overlaps of distinct unit sides either put an endpoint strictly inside another side or make the two sides identical. At each assembly step the occupied sectors at a vertex form a subset of the 18 sectors of $20°$. A vacant sector with occupied sectors on both sides would give a forbidden gap. The test finds none.

An adjoining pair is identified by the labels of its common sides and by whether the two pieces have the same handedness. The five interior angles of each tile are distinct. An edge choice together with relative handedness therefore fixes the second tile when the first is fixed, up to a common isometry. Collecting all common edges into a single signature also counts a pair sharing two sides once. The legal signature counts are 30 Vek–Vek, 44 Vek–Kay, and 23 Kay–Kay. The 97 witnesses in Section 3 give different signatures. Thus every signature occurs. Across the entire assembly 104 unordered pairs share at least one edge. □

The supplementary file `verify_chaos_tiles.py` includes the placements, the catalogue signatures in sheet order, and the contact witnesses. Run it with Python 3.

```
python verify_chaos_tiles.py
```

Codex computed the arrangement, implemented the checker, and prepared the figure.

References

[1] E. Pegg Jr., *The Prize Page*, Chaos Tiles challenge.
[2] E. Pegg Jr., *Chaos Tiles rules.*
[3] E. Pegg Jr., *The 97 Chaos Tiles two-tile combinations*, illustrated catalogue.
[4] E. Pegg Jr., *Chaos Tiles*, account of the two pentagons and their discovery.