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The Assumption Package of the Yang–Mills Construction: Formal Definitions and Derivations

Companion to *From Four-Dimensional Wilson Lattice Gauge Theory to Quantum Yang–Mills Theory, with a Conditional Mass-Gap Theorem*

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Abstract

This companion extracts, from the Lean statement glossary of the validated formalization (Appendix A of the validation document [1]), the *mathematical definitions* behind each external input of the construction—(RG1)–(RG5), (UV1), (PH1)–(PH3), (AN1), (Y1)–(Y4), (B1), (C1)—and records, for those inputs that are derivable from more primitive data, the derivations in full: the one-loop recursion of the marginal coupling, the growth bound of the inverse RG transport, the one-loop beta-function constant $b_0 = \frac{11C_{\mathfrak{g}}}{48\pi^2}$ from the loop sum, and the short-distance reduction of the trace two-point function. Statements already proved in the main text are not reproved; they are cited by number.

Conventions

The main text is cited as [M]: Lemma 5.8, Theorem 9.9, etc. Everything defined here is a definition extracted from the formalization unless a derivation is explicitly given; nothing here upgrades an assumption to a theorem. The one-loop constant $b_0 = \frac{11C_{\mathfrak{g}}}{48\pi^2}$ is fixed throughout.

1 The renormalization-group package

1.1 (RG1): symmetry data

Definition 1.1 (Lattice symmetry group)**.** Γ_{lat} is the finite group of the hypercubic lattice generated by translations by one lattice unit, the signed coordinate permutations (hyperoctahedral group, order $2^4 \cdot 4! = 384$), and the reflection $\theta : (x_0, \vec{x}) \mapsto (-x_0, \vec{x})$.

Construction 1.2 (Equivariant blocking kernel)**.** A blocking kernel C_ρ coarsening level m (spacing $a(m)$) to level $m-1$ (spacing $a(m-1) = b_m a(m)$) is a Markov kernel on field configurations assigning to each coarse edge a local average of its b_m-dilated preimage with kernel profile ρ of compact support. Symmetry-preserving gauge fixing (averaging over the coset of a fixed axial gauge; the invariance is the finite-group averaging of [M, Lemma 5.2]) makes C_ρ Γ_{lat}-equivariant.

1.2 (RG2): one-step estimates

Definition 1.3 (Effective action decomposition)**.** At level m the effective action has the form

$$S_m(U) = \beta_m \sum_{\square} \Big(1 - \frac{1}{m} \operatorname{Re} \operatorname{Tr} U_{\square}\Big) + J_m(U),$$

where β_m is the marginal (plaquette) coefficient and J_m lies in the finite-dimensional space of irrelevant local terms (plaquette correlators of diameter ≥ 2), normed by $\|J_m\|_m = \sum_\tau |c_\tau| b_m^{-\mathrm{diam}(\tau)}$.

Definition 1.4 (One-step estimates)**.** With $\varrho \in (0,1)$, $K > 0$, $\beta_* > 0$ and the dominance $2b_0 \log 3 > K/\beta_*$:

$$\|J_{m+1}\|_{m+1} \leq \varrho \|J_m\|_m + K\beta_m^{-1}, \qquad \left|\beta_{m+1} - \beta_m - 2b_0 \log b_m\right| \leq K\beta_m^{-1}, \qquad \beta_m \geq \beta_*,$$

uniformly in the volume index p.

Derivation 1.5 (The marginal recursion)**.** *The second estimate is the discretization of the one-loop beta function. With $g^2 = 1/\beta$,*

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\, g^{-2}(\mu) = -\frac{2\beta(g)}{g^3} = 2b_0 + O(g^2)$$

([M, Lemma 9.9]). Integrating over one blocking step $\mu_{m-1} = \mu_m / b_m$ gives

$$\beta_{m+1} - \beta_m - 2b_0 \log b_m = \int_{\log \mu_{m-1}}^{\log \mu_m} O(g^2(\mu))\, \mathrm{d}\log\mu.$$

The two-loop term $\beta(g) = -b_0 g^3 + \beta_2 g^5 + \cdots$ contributes $O(g_m^2) = O(\beta_m^{-1})$ to the integral, since $\log b_m$ is schedule-controlled ($b_m = \lceil (\log \hat{\beta}_m)^2 \rceil$ type growth, validation document Appendix D) so that $\int O(g^2) \leq K\beta_m^{-1}$. The same schedule control gives the weighted Cesàro form of the remainder recorded in [M, Lemma 5.8]. □

Remark 1.6 (Status). The contraction of irrelevant directions is a property of the blocked polymer gas (kernel-averaging); it is certified in the repository (YM.RG.Program, YM.RG.Calendar.Fits). No derivation from bare definitions is given here.

1.3 (RG3): the insertion readout

Definition 1.7 (Transport map and readout)**.** For a degree-d label with coefficient vector c at level N, the RG transport $T_{N\to r}$ is the linear map of coefficient vectors induced by applying the blocking kernel $N - r$ times to the insertion [M, eq. (4.7)]. The readout $\mathcal{R}(\Pi) = c|_{r(d)}$ is evaluation of the transported coefficient at the reference level $r(d) \leq N$.

Derivation 1.8 (Growth of the inverse)**.** *Each blocking step acts on a degree-d insertion with a norm factor at most b_m^d (dilation of the support pattern); hence*

$$\|T_{N\to r}^{-1}\| \leq \prod_{m=r+1}^{N} b_m^d = D_N^d = a_N^{-d},$$

using $D_N = \prod_{m<N} b_m = a_N^{-1}$ [M, Lemma 4.1]. The p-independence follows from the shared kinetic paths ([M, Lemma 4.15(4)]). □

1.4 (RG5) and (UV1): kernel bounds

Definition 1.9 (Kernel seminorms)**.** For a finite label list v and compact $K \subset \Omega_n$, the seminorm is

$$\|C_{N,p}(v;\cdot)\|_{\alpha,K} := \sup_{x\in K} \left|\partial_x^\alpha K_{N,p}(v;x)\right|,$$

with $K_{N,p}$ the summation kernel of [M, Remark 5.15].

(RG5) is the statement $\sup_{N,p} \|C_{N,p}(v;\cdot)\|_{\alpha,K} < \infty$; (UV1) is the corresponding statement for the limits S_p together with a modulus of continuity. The mechanism (reflection positivity, chessboard estimates, transfer operator, Cauchy estimates) is derived in [M, Lemma 6.2]; here only the definitions are recorded.

2 The phase algebra

2.1 (PH1): word operators

Construction 2.1 (Slot operators)**.** For a word $A = (v_1, f_1)\cdots(v_k, f_k)$ with distinct points in the slab $0 < t < s$, the operator $B_{s,A}$ on the OS Hilbert space shifts existing positive-time insertions forward by s and prepends the insertions of A; $M_A = P_0 B_{s,A}|_{\mathcal{H}_0}$ and $\mathcal{A} = C^*(\{M_A, M_A^*\})$.

(PH1) is the conjugation law $B_{s,A} = T_s B_{0,A} T_{-s}$ with $P_0 T_s = P_0$; its consequence, s-independence of M_A, is proved in [M, Lemma 7.5]. (PH2) is the statement that $\mathcal{A}$ is commutative (the mechanism is recorded in [M, §7.2]); (PH3) is faithfulness of Ω_0.

2.2 (AN1): the OS axioms

Definition 2.2 (OS axioms)**.** For a hierarchy of Euclidean correlations S_n:

E0
each S_n is a tempered distribution on $\mathcal{S}_0((\mathbb{R}^4)^n)$ (diagonal-flat Schwartz tests);

E0′
arity growth $\|\mathcal{S}_n(f)\| \le \beta^n (n!)^A \|f\|_{(ns,ns)}$;

E1
invariance under the Euclidean group $E(4)$;

E2
reflection positivity: for every finite family (f_j, g_j) supported at positive/negative times, $\sum_{j,k} S_{n_j+n_k}(\theta f_j \otimes g_k) \ge 0$;

E3
invariance under simultaneous permutation;

E4
clustering: $S_{n+m}(f \otimes g_a) - S_n(f)S_m(g) \to 0$ as the separating translation $a \to \infty$;

normalization
$S_0 = 1$;

analyticity
each S_n is the boundary value of a function holomorphic in the tube $\mathrm{Im}\,\zeta_j \in V^+$, $\zeta_j = z_{j+1} - z_j$.

(AN1) is the statement that every phase S_ξ satisfies this list; the continuum Wightman reconstruction from it is [M, Theorem 8.10].

3 The Yang–Mills identification package

3.1 (Y1): the stress tensor

Definition 3.1 (Candidate space and normalization)**.** Within $\mathcal{F}_4$, consider symmetric rank-two tensors $T_{\mu\nu}$ that are fixed by field conjugation, centered with respect to S_{mix}, transform as rank-two tensors under coordinate permutations and sign changes, and satisfy $\partial_\mu T_{\mu\nu} = 0$ in $\mathcal{F}_5$. (Y1) states this space is one-dimensional; the normalization is fixed by the Ward identity [M, Proposition 9.4]. The representative is $T_{\mu\nu} = \mathrm{Tr}(F_{\mu\alpha}F_{\nu\alpha}) - \frac{1}{4}\delta_{\mu\nu}\,\mathrm{Tr}(F_{\alpha\beta}F_{\alpha\beta})$; the one-dimensionality of the candidate space is derived in [M, Lemma 9.2].

3.2 (Y2): the two laws

Definition 3.2 (Calculus and flux laws)**.** For every label list and distinct points, with χ a bump equal to one near x_0 and supported away from the other insertions,

$$S_\xi(D_b v_0, v_1, \dots, v_n) = \partial_b^{x_0} S_\xi(v_0, v_1, \dots, v_n), \qquad -\sum_a \int \partial_a \chi \, S_\xi(T_{ab}, v_0, \dots, v_n)\, \mathrm{d}^4 y = S_\xi(D_b v_0, v_1, \dots, v_n).$$

For translation-invariant Gaussian hierarchies both laws are derived in [M, Proposition 10.7]; for the constructed theory they are construction inputs (phase.flux, phase.calculus).

3.3 (Y3): the trace two-point reduction

Definition 3.3 (Trace asymptotic input)**.** With $g_r = g(1/r)$, $\beta(g) = \mu\,\mathrm{d}g/\mathrm{d}\mu$, and $Q = F^a_{\mu\nu}F^a_{\mu\nu}$,

$$K_\Theta(r) = \Big(\frac{\beta(g_r)}{2g_r}\Big)^2 \big\langle Q(\tfrac{r}{2}e_0) Q(-\tfrac{r}{2}e_0)\big\rangle_{\mathrm{free},c}\,(1 + o(1)) \qquad (r \downarrow 0).$$

Derivation 3.4 (Why the anomaly coefficient appears)**.** *The trace of the stress tensor is the response of the measure to a scale transformation: formally $\Theta = \frac{\beta(g)}{2g} F^2_{\mathrm{ren}}$, the renormalized F^2 being the only marginal scalar at weight four (the marginality is what transmits the one-loop running into the residue). Squaring the insertion gives the factor $(\beta(g_r)/2g_r)^2$; asymptotic freedom [M, Theorem 9.18] sends the renormalized Q-two-point function to its free Wick value at separated points, whose coefficient $\frac{48\dim_{\mathbb{R}}\mathfrak{g}}{\pi^4}$ is computed in [M, Lemma 9.7]; assembly is [M, Theorem 9.9].* □

3.4 (Y4): the one-loop constant

Derivation 3.5 ($b_0 = \frac{11C_{\mathfrak{g}}}{48\pi^2}$)**.** *We record the standard loop sum with the conventions of [M, §2.1]. The beta function at one loop receives three contributions.* Gauge-boson loop*: the two gluon vacuum-polarization diagrams give $\frac{10}{3}C_{\mathfrak{g}}$ (seagull and tadpole included); the Faddeev–Popov ghost loop contributes the remaining $\frac{1}{3}C_{\mathfrak{g}}$ (the complex anticommuting adjoint scalar with its sign); together*

$$b_0 = \frac{1}{(4\pi)^2}\Big(\frac{10}{3} + \frac{1}{3}\Big)C_{\mathfrak{g}} = \frac{1}{(4\pi)^2}\cdot\frac{11}{3}C_{\mathfrak{g}} = \frac{11C_{\mathfrak{g}}}{48\pi^2},$$

with no scalar or fermion loops; the tadpole/ghost bookkeeping is a certified rational/Casimir identity (YM.Pert.Asymptotics.b0, YM.Gauge.casimir), not a Feynman-diagram expansion. □

Remark 3.6. The derivation is the textbook loop computation; in the formalization it is a certified rational/Casimir identity, not a Feynman-diagram expansion.

4 Field content and finite law

4.1 (B1): the dictionary objects

Definition 4.1 (Register and comparison space)**.** The register is the graded invariant polynomial algebra Labels.Pol G with its invariant subspace; a label of weight d is a finite complex combination of invariant contractions. The comparison space Maxwell-shell-sector(G, d) is the space of gauge-invariant weight-d polynomials in the free sourceless curvature data ([M, Definition 2.10]); the dictionary map ψ_d and its kernel are [M, Definition 2.7, Lemma 2.11]. The six-step audit asserting $N_d = \ker \psi_d$ and its phase transport are [M, Proposition 2.12, §9.6].

4.2 (C1): the finite-law comparison

Definition 4.2 (Two finite laws and the conversion)**.** The submission's Wilson law on $\Lambda_{N,p} = (\mathbb{Z}/(L_p D_N)\mathbb{Z})^4$ with normalized Haar and density $e^{-\beta E}$ is compared, lattice point by point, with the independent formalization [LGT]; the coupling conversion is $\beta_{\text{submission}} = m\,\beta_{\text{lgt}}$. The adapter reindexes sites, links, and plaquettes and proves equality of the normalized laws and finite moments; it is record-only and enters no derivation of the main text ([M, Proposition 4.15]).

5 Volume-uniform difference estimates

Throughout this section $f \in \mathcal{T}_n = C_c^\infty(\Omega_n)$ is an admissible test function in the sense of [M, Definition 3.1], and $K := \operatorname{supp} f$. Let

$$p_0(K) := \min\{p :\ L_p > 2\ \operatorname{diam}(K) \text{ and no periodic image of } K \text{ meets } K\},$$

so that for every $p \geq p_0(K)$ the support of f lies strictly inside $((-L_p/2, L_p/2)^4)^n$ and avoids all periodic images. This is the same threshold as in [M, Assumption 6.1 (UV1)].

Lemma 5.1 (Riemann-sum localization)**.** *Let $f \in \mathcal{T}_n$, $K = \operatorname{supp} f$, and $p \geq p_0(K)$. Then for every level N, with $|K|$ the $4n$-dimensional Lebesgue measure of K,*

$$a_N^{4n} \sum_{x_1,\dots,x_n \in \Lambda_{N,p}} \big|f(a_N x_1, \dots, a_N x_n)\big| \ \leq\ \|f\|_{L^1(K)} + c_n a_N \|\nabla f\|_\infty |K| + O(a_N \|\nabla f\|_\infty |\partial K|)\,, \quad \text{(C.1)}$$

where $c_n = \sqrt{4n}/2$. In particular the right-hand side is independent of L_p.

Proof. On each cell $\mathcal{C}_{N,x}$ of side a_N centered at $a_N x$ one has $|f(a_N x) - \bar{f}_{N,x}| \leq c_n a_N \|\nabla f\|_\infty$. Only cells meeting K contribute; their number is $|K| a_N^{-4n} + O(|\partial K| a_N^{-4n+1})$. Multiplying by a_N^{4n} and using $a_N^{4n} \sum_x \bar{f}_{N,x} = \int_K f = \|f\|_{L^1(K)}$ gives (C.1). For $p \geq p_0(K)$ the set of contributing cells is exactly the same for every admissible volume, hence no L_p appears. □

Remark 5.2 (Numerical check of (C.1))*.* With $\varphi(x) = \prod_i (1 - x_i^2)_+$, $n = 2$, $\int f = (4/3)^8 = 9.98872123$: the left side of (C.1) is identical in all three volumes $L_p = 8, 32, 256$ once $L_p/2 > 1$ (values 5.96046, 8.80632, 9.68081, 9.91095, 9.96923 at $a_N = 1/2, 1/4, 1/8, 1/16, 1/32$), with error $\propto a_N$ and independent of L_p. The global bound $L_p^{4n} = L_p^8$ overestimates by about 10^{19} at $L_p = 256$.

Lemma 5.3 (p-independence of the cumulant constant)**.** *In the proof of [M, Lemma 5.13], the constant B bounding the level-dependent part of the first cumulant by $B\|J_N\|_N$ may be taken of the form $B = B(v, f, K)$, independent of N and of p for all $p \geq p_0(K)$.*

Proof. We inspect each source of B:

(i) By (RG3) the RG transport from level N back to $r(d)$ is invertible and independent of N and p ([M, Lemma 5.13]); its inverse obeys $\|T_{N\to r}^{-1}\| \le D_N^d = a_N^{-d}$ by the Derivation of §1.3, with p-independence supplied by [M, Lemma 4.15(4)] (shared kinetic paths).

(ii) $\|J_N\|_N \le \varrho^N \|J_0\|_0 + K/(\beta_*(1-\varrho))$ by [M, Lemma 5.10], which records "all constants are p-independent by (RG2)".

(iii) (RG5) already carries the quantifier $\sup_{N,p}\sup_{x\in K}|\partial_x^\alpha K_{N,p}(v;x)| \le M_{\alpha,K,v}$.

(iv) The only remaining possible p-dependence is whether $\operatorname{supp} f$ fits inside the finite-volume box; [M, Definition 3.1] turns this into the finitely many exceptions $p < p_0(K)$.

Hence $B = B(v, f, K)$ for $p \ge p_0(K)$. □

Lemma 5.4 (Volume-uniform (RG4); Lemma 5.13′)**.** *Assume (RG1)–(RG3), (RG5) and [M, Definition 3.1]. There is a constant*

$$A(v,f,K) = B'(v,f,K)\Big(\|f\|_{L^1(K)} + c_n a_0 \|\nabla f\|_\infty |K|\Big) < \infty$$

such that for every N and every $p \ge p_0(K)$,

$$\big|C_{N+1,p}(v;f) - C_{N,p}(v;f)\big| \;\le\; A(v,f,K)\,\varrho^N. \tag{5.4′}$$

Moreover $\sup_p A_p(v,f) < \infty$.

Proof. Reproduce the proof of [M, Lemma 5.13] verbatim up to the bound

$$\big|C_{N+1,p}(v;f) - C_{N,p}(v;f)\big| \le B'\varrho^N\, a_N^{4n} \sum_{x_1,\dots,x_n\in\Lambda_{N,p}} \big|f(a_N x_1,\dots,a_N x_n)\big|.$$

In the original lemma the sum is estimated by [M, Lemma 4.12], giving the volume factor $a_N^{4n}|\Lambda_{N,p}|^n = L_p^{4n}$. Instead, apply Lemma 5.1 and Lemma 5.3. For the finitely many volumes $p < p_0(K)$, compactness of $\Omega_{N,p}$ and (RG5) give finite constants $A_p(v,f)$; taking the maximum of these with $A(v,f,K)$ yields $\sup_p A_p < \infty$. □

Corollary 5.5 (Uniform UV limit with rate)**.** $\displaystyle\sup_{p\ge p_0(K)} \big|S_p(v;f) - C_{N,p}(v;f)\big| \;\le\; \frac{A(v,f,K)}{1-\varrho}\,\varrho^N.$
Consequently the N-limit and the p-limit may be interchanged at the level of the N-direction with a geometric rate; this closes the "no rate" clause of [M, Remark 5.21] on the N side.

Corollary 5.6 (Localized quadrature error)**.** *Equation (5.12) of [M, Remark 5.15] should read*

$$|\mathcal{E}_{N,p}(f)| \le 2a_N \|\nabla f\|_\infty\, |K| \sup_{x\in K} |K_{N,p}(v;x)|, \tag{5.12′}$$

with the global factor L_p^{4n} replaced by the support measure $|K|$. Otherwise the (RG5)/(UV1) mechanism retains a spurious volume dependence.

Remark 5.7 (What Lemma 5.4 does *not* do). Lemma 5.4 is a UV statement. It does not discharge (B-c) of [M, Proposition 10.26], which is exactly $\Delta_{N,p} = -a_N^{-1}\log\|\mathcal{T}_{N,p}|_{\Omega^\perp}\| \ge \varepsilon_0$, i.e. (MG2″). Indeed, take $\rho_p = (1-\varepsilon_p)\delta_1 + \varepsilon_p\rho_{\mathrm{IR}}$ with $\varepsilon_p = 10^{-3}2^{-p}$ and $\rho_{\mathrm{IR}} = \sum_{k\le 40} 2^{-k}\delta_{2^{-k}}$; the infrared modification is independent of N, so (5.4′) holds with a p-independent constant, $\sup_p\sup_t|G_p(t) - e^{-t}| \le 5\cdot 10^{-4}$, yet $\inf\operatorname{supp}\rho_p\setminus\{0\} = 2^{-40}$ and no volume has a gap. This is [M, Proposition 10.8] together with the "effective readability of $\mu_p((0,\varepsilon))$ for large t" of [M, Remark 10.16]. Uniqueness of the infinite-volume limit (O1/O9) is not needed for the gap ([M, Theorem 10.17, Remark 10.18]).

6 Summary table

Input	Content here	Status
(RG1)	Definitions 1.1	construction
(RG2)	Definition + derivation	recursion derived; contraction certified
(RG3)	Definition + derivation	inverse growth derived
(RG5)/(UV1)	Definitions	mechanism in [M, Lemma 6.2]
(PH1)–(PH3)	Constructions	[M, §7]
(AN1)	Definition	per-phase OS list
(Y1),(Y2)	Definitions	[M, Lemma 9.2, Prop. 10.7]
(Y3)	Definition + derivation	reduction derived
(Y4)	Derivation	loop sum
(B1),(C1)	Definitions	audits in [M]
(RG4′)	§5	derived (volume-uniform form of (RG4))

Appendix: audit statements (not new theorems)

The following three items are recorded for completeness. Consistently with the abstract of this companion, none of them upgrades a registered external input to a theorem.

(A) Iteration closure of (RG2). From $\beta_m \geq \beta_*$ one gets $K\beta_m^{-1} \leq K/\beta_* =: \kappa$, hence $u_{m+1} \leq \varrho u_m + \kappa$ for $u_m = \|J_m\|_m$ and

$$u_m \leq \varrho^m u_0 + \frac{\kappa}{1-\varrho} \leq \|J_0\|_0 + \frac{K}{\beta_*(1-\varrho)}, \qquad \beta_m \geq \bar{\beta} + m\delta', \quad \delta' = 2b_0 \log 3 - K/\beta_* > 0.$$

This is [M, Lemma 5.10] and is p-uniform. The contraction rate ϱ itself is a certificate of the blocked polymer gas (`YM.RG.Program`, `YM.RG.Calendar.Fits`); Remark 1.6 of this companion states that no derivation from bare definitions is given.

(B) Mechanism behind (RG5)/(UV1). Reflection positivity (E2) ⇒ OS form ⇒ Cauchy–Schwarz plus permutation symmetry (E3) ⇒ $|S_p(v;x)| \leq \prod_i \langle A_i, A_i\rangle^{1/2}$ ⇒ chessboard bounds by finite-volume two-point functions ⇒ local boundedness by (RG5); equicontinuity follows from analyticity at separated points and the Cauchy integral formula on a polydisc of radius δ, giving $\omega_K(r) = CM_K r/\delta$. This expands [M, Lemma 6.2]; (UV1) remains an external input. As noted in [M, Lemma 6.6], these bounds are local on compact sets and therefore cannot supply the $s \to \infty$ rate required by (MG).

(D) Audit statement of (C1). The adapter is a bijection on sites, links and plaquettes of $\Lambda_{N,p} = (\mathbb{Z}/(L_p D_N)\mathbb{Z})^4$; normalized Haar measures agree pointwise; the conversion $\beta_{\text{submission}} = m\beta_{\text{lgt}}$ equates the normalized finite laws; all finite moments then agree. By [M, Proposition 4.16] the internal proof of the adapter *is* input (C1) (record only, enters no derivation), so no equivalence of the two independent formalizations is proved here.

Boundary statement

All statements of this companion belong to the ultraviolet/definition layer. The counterexample family of [M, Proposition 10.7] satisfies (AN1) E0–E4, vacuum uniqueness, (B1), (Y1), (Y2), the short-distance constant [M, Theorem 9.9], the OPE data, *and* the volume-uniform (RG4′) of §5; its gap is still 0. Hence deriving A–D in full leaves the registered input set compatible with both gap and no-gap. The remaining new mathematics is unchanged: a volume-uniform log-Sobolev

inequality with $C_{\mathrm{LS}} \sim 1/a_N$, a dynamical-to-Hamiltonian connection theorem with the a_N factor, and a multiscale UV-summability/nonperturbative mass-generation identity (the missing dilation Ward / Callan–Symanzik intermediate term of [M, Proposition 10.9]). The mass gap is not proved.

References

[1] Yang–Mills Shepherding Committee, *Validation of the OpenAI Yang–Mills construction*, release of November 10, 2026, repository `yang-mills-group/YM` (Appendix A: Lean statement glossary).

[2] C. Zeng, *From Four-Dimensional Wilson Lattice Gauge Theory to Quantum Yang–Mills Theory, with a Conditional Mass-Gap Theorem*, companion submission, 2026.

[3] Lattice gauge theory formalization, `mrdouglasny/lgt`, pinned revision.