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CONSECUTIVE PRIMES AT DISTANCES 123,662 AND 125,318

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Abstract. We exhibit consecutive-prime gaps of exactly 123 662 and 125 318, with endpoints of 8 416 and 6 454 decimal digits, respectively. Covering congruences certify most interior integers as composite. For the second gap, the remaining integers are certified by proper divisors or Fermat witnesses. Elliptic-curve certificates prove all four endpoints prime. We give the elementary arguments that turn these finite arithmetic checks into a proof. The proof is computer-assisted.

1. The two gaps

For $h \in \{123\,662, 125\,318\}$, let p_h be the integer displayed in Appendix A or B, respectively.

Theorem 1.1. *For each of these two values of h, the integers p_h and $p_h + h$ are consecutive primes. Both endpoints have 8 416 digits when $h = 123\,662$ and 6 454 digits when $h = 125\,318$.*

The full covering tables, additional compositeness witnesses, and four primality chains form the supplementary arithmetic appendix. Their mathematical format and the required checks are described below.

For each gap, the covering table consists of pairs (q, b_q) with distinct prime moduli q and $b_q \not\equiv 0, -h \pmod q$. Define

$$W_h = \prod q, \qquad 0 \le a_h < W_h, \qquad a_h \equiv b_q \pmod q.$$

The Chinese remainder theorem determines a_h uniquely, and the displayed starting integers satisfy $p_h = a_h + m_h W_h$, with the following parameters.

h	m_h	Number of moduli	Largest modulus	Digits of W_h
123 662	1 523 845	2 216	32 587	8 410
125 318	5 046 505	1 754	14 983	6 448

These progressions were obtained by optimizing the covering congruences. Sieving and endpoint tests then found the displayed multipliers.

2. Compositeness of the interior

If $1 \le j < h$ and $j \equiv -b_q \pmod q$, then $q \mid p_h + j$. Since $1 < q < p_h + j$, this is a proper divisor. The exact covering counts are

h	Covered by congruences	Additional divisors	Fermat witnesses
123 662	123 661	0	0
125 318	124 526	175	616

For the second gap, the remaining 791 offsets are listed in the arithmetic appendix. At 175 of them, an integer d satisfies $1 < d < p_h + j$ and $d \mid p_h + j$. At each of the other 616, direct modular exponentiation gives

$$2^{p_h + j - 1} \not\equiv 1 \pmod{p_h + j}.$$

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Fermat's little theorem shows that these integers are composite. The three columns account for every interior offset, proving the required compositeness in both cases.

3. Primality of the endpoints

We use the elliptic-curve criterion underlying ECPP [1].

Lemma 3.1. *Let $N > 3$ be coprime to 6 and let $E : y^2 = x^3 + Ax + B$ satisfy $\gcd(4A^3 + 27B^2, N) = 1$. Suppose an affine point R satisfies $[r]R = O$, where r is prime and $r > (N^{1/4} + 1)^2$. If every division in the computation has denominator coprime to N, then N is prime.*

Proof. If N were composite, it would have a prime divisor $\ell \le \sqrt{N}$. Reduction modulo ℓ gives a nonsingular elliptic curve and a nonzero point of order r. Hasse's bound gives $r \le \#E(\mathbb{F}_\ell) \le (\sqrt{\ell} + 1)^2 \le (N^{1/4} + 1)^2$, a contradiction. □

Each endpoint certificate is a chain of tuples $(N_i, t_i, s_i, A_i, X_i, Y_i)$. Set $N_{i+1} = (N_i + 1 - t_i)/s_i$, $P_i = (X_i, Y_i)$, and $B_i \equiv Y_i^2 - X_i^3 - A_i X_i \pmod{N_i}$. At every step the exact checks are

$$\begin{gathered} N_i > 3, \quad s_i > 0, \quad N_{i+1} \in \mathbb{Z}, \quad \gcd(N_i, 6) = \gcd(4A_i^3 + 27B_i^2, N_i) = 1, \\ N_{i+1} > (N_i^{1/4} + 1)^2, \qquad R_i = [s_i]P_i \neq O, \qquad [N_{i+1}]R_i = O. \end{gathered} \tag{1}$$

The points R_i are affine, and all divisions in these group operations have denominators coprime to N_i. The four chains end as follows.

Starting integer	Steps	Terminal prime
$p_{123\,662}$	679	7 440 766 535 021
$p_{123\,662} + 123\,662$	671	636 419 037 730 097
$p_{125\,318}$	538	450 463 843 321
$p_{125\,318} + 125\,318$	537	403 307 424 311 829 421

For completeness, the arithmetic appendix gives elementary order certificates for the four terminal primes. Each row specifies N, a complete factorization of $N - 1$, and an integer g such that

$$g^{N-1} \equiv 1 \pmod{N}, \qquad \gcd\bigl(g^{(N-1)/q} - 1, N\bigr) = 1 \quad (q \text{ prime},\ q \mid N - 1).$$

For every prime divisor ℓ of N, these checks force g to have order $N - 1$ modulo ℓ. Thus $N - 1 \mid \ell - 1$, so $\ell \ge N$ and N is prime. The factor primes are certified recursively in increasing order, starting with primes below 100, which are checked by trial division.

Proof of Theorem 1.1. Starting from the terminal primes and applying Lemma 3.1 backwards along each chain proves all four endpoints prime. Section 2 proves every integer between the corresponding endpoints composite. The displayed decimal expansions and addition of h give the stated digit counts. □

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References

[1] A. O. L. Atkin and F. Morain, *Elliptic curves and primality proving*, Math. Comp. **61** (1993), no. 203, 29–68.

Appendix A. The starting prime for $h = 123\,662$

The following lines concatenate to $p_{123\,662}$. Spaces separate blocks of ten digits. The right endpoint is obtained by adding 123 662.

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1978533019 1033653481 5536674032 5124196129 8851048675 9715711486 7089520013 2794096239 6866825311 1422429041 3682734817 7817220447
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9865300046 1294199498 4026084184 6749988697 3762950016 3082003277 6153838005 1679679976 8092530840 6230099904 9643743672 5611218287
2815090479 351597
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Appendix B. The starting prime for $h = 125\,318$

The following lines concatenate to $p_{125\,318}$. Spaces separate blocks of ten digits. The right endpoint is obtained by adding 125 318.

```
5177351185 4459854632 7695956438 5729587744 1335282968 5638645816 7233145552 3593544663 3379922985 5373108128 0426918899 1827237135
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