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CONSECUTIVE PRIMES AT DISTANCE 122,290

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Abstract. We exhibit two consecutive primes whose difference is exactly 122 290. Both primes have 8 262 decimal digits. Every integer strictly between them has a prime divisor at most 32 467. We give the covering argument and the elliptic-curve criterion that certify the result. The proof is computer-assisted and uses finite arithmetic checks to establish endpoint primality and the compositeness of every interior integer.

1. The result and the covering argument

A prime gap of length h is a pair of primes $p, p+h$ such that every integer between them is composite. We prove the following statement.

Theorem 1.1. *Let p be the integer written in Appendix A. Then p and $p + 122\,290$ are consecutive primes, each with 8 262 decimal digits. Moreover, every integer n with $p < n < p + 122\,290$ has a prime divisor at most 32 467.*

The full residue table and the two primality chains form the supplementary arithmetic appendix. Their mathematical format is specified below.

A covering congruence supplies one small divisor for each interior integer.

Lemma 1.2. *Let $h \geq 2$, let $\mathcal{Q}$ be a finite set of primes, and choose a residue b_q modulo each $q \in \mathcal{Q}$. Put $W = \prod_{q\in\mathcal{Q}} q$, and let $0 \leq a < W$ satisfy $a \equiv b_q \pmod q$ for every $q \in \mathcal{Q}$. Suppose that*

$$\{1, \dots, h-1\} \subseteq \bigcup_{q\in\mathcal{Q}} \{j : j \equiv -b_q \pmod q\}.$$

If $m \geq 1$, $p = a + mW > \max \mathcal{Q}$, and $p, p+h$ are prime, then they are consecutive primes.

Proof. For $1 \leq j < h$, choose $q \in \mathcal{Q}$ with $j \equiv -b_q \pmod q$. Since $p \equiv a \equiv b_q \pmod q$, we have $q \mid p + j$. The inequalities $1 < q < p + j$ show that $p + j$ is composite. □

For the integer p in Theorem 1.1, the residue table has 2 180 pairs (q, b_q) and satisfies

$$b_q \equiv p \pmod q, \qquad b_q \not\equiv 0, -122\,290 \pmod q.$$

Every q is prime and $q \leq 32\,467$. The Chinese remainder construction is $p = a + 530\,520W$, where $W = \prod q$, $0 \leq a < W$, and $a \equiv b_q \pmod q$. The integer W has 8 256 decimal digits.

To check the interior directly, start with $S = \{1, \dots, 122\,289\}$. For each prime $q \leq B$, remove the offsets $j \equiv -p \pmod q$. The successive numbers of remaining offsets are

B	Remaining	B	Remaining
2	61 144	14 983	818
97	14 577	19 997	126
997	8 665	24 989	26
4 999	4 673	29 989	3
9 973	2 342	32 467	0

Thus every interior integer has a proper prime divisor at most 32 467.

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2. Endpoint primality and completion of the proof

We use the following form of the elliptic-curve primality criterion [1]. All group operations in the statement are performed modulo N.

Lemma 2.1. *Let $N > 3$ be coprime to 6, and let $E : y^2 = x^3 + Ax + B$ satisfy $\gcd(4A^3 + 27B^2, N) = 1$. Suppose that r is prime, $r > (N^{1/4} + 1)^2$, and an affine point R on E satisfies $[r]R = O$. If the computation uses only denominators coprime to N, then N is prime.*

Proof. If N were composite, it would have a prime divisor $\ell \le \sqrt{N}$. Reduction modulo ℓ gives a nonsingular elliptic curve, and R reduces to an affine point. This point has order r, since it is nonzero and is annihilated by the prime r. Hasse's bound therefore gives

$$r \le \#E(\mathbb{F}_\ell) \le (\sqrt{\ell} + 1)^2 \le (N^{1/4} + 1)^2,$$

a contradiction. □

For each endpoint, the certificate is a finite chain of integers N_i and tuples $(t_i, s_i, A_i, X_i, Y_i)$, with

$$N_{i+1} = \frac{N_i + 1 - t_i}{s_i}, \qquad B_i \equiv Y_i^2 - X_i^3 - A_i X_i \pmod{N_i}.$$

For $0 \le i < k$, $N_i > 3$, $s_i > 0$, and N_{i+1} is an integer. At each step we have

$$\begin{aligned} \gcd(N_i, 6) = \gcd(4A_i^3 + 27B_i^2, N_i) &= 1, \\ N_{i+1} &> (N_i^{1/4} + 1)^2, \\ R_i = [s_i](X_i, Y_i) \neq O, \qquad [N_{i+1}]R_i &= O. \end{aligned} \tag{1}$$

The point R_i is affine and every division in its computation, and in that of $[N_{i+1}]R_i$, has denominator coprime to N_i. These are exact modular arithmetic checks. The two chains have the following endpoints.

Starting integer	Number of steps	Terminal integer
p	647	77 611 968 613
$p + 122\,290$	659	78 278 019 539

Both terminal integers are prime. For a direct order check, their factorizations and bases are

$$\begin{aligned} 77\,611\,968\,613 - 1 &= 2^2 \cdot 3^4 \cdot 937 \cdot 255\,649, \quad g = 6, \\ 78\,278\,019\,539 - 1 &= 2 \cdot 389 \cdot 5\,531 \cdot 18\,191, \quad g = 2. \end{aligned}$$

Every displayed factor is prime by trial division. In each row, $g^{N-1} \equiv 1 \pmod{N}$ and $\gcd(g^{(N-1)/r} - 1, N) = 1$ for every prime $r \mid N-1$. Consequently g has order $N-1$ modulo every prime divisor ℓ of N, so $N-1 \mid \ell-1$ and $\ell \ge N$. This proves the claim.

Proof of Theorem 1.1. The finite modular checks (1), starting from the two terminal primes and applying Lemma 2.1 backwards along each chain, prove that p and $p + 122\,290$ are prime. The covering calculation in Section 1 proves every intermediate integer composite. Thus the primes are consecutive. The decimal representation of p and addition of 122 290 give 8 262 digits in both cases. □

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References

[1] A. O. L. Atkin and F. Morain, *Elliptic curves and primality proving*, Math. Comp. **61** (1993), no. 203, 29–68.

Appendix A. The starting prime

The decimal expansion of p is displayed below. The lines are concatenated and spaces separate blocks of ten digits. The integer $p + 122\,290$ is then determined by addition.

2524630355 5392095200 6849031818 2436336455 8705070412 1679587043 2398027747 9814213567 5465079894 0493256636 4749901363 7823525575
4209929759 9738020881 7752842148 3859541993 4209196681 4556241831 2586384700 5663906315 1839543596 7260996320 6698569079 2712486548
1453992286 7309146566 4147352403 1034075473 0932486503 7771948675 0538956940 2856290174 5774135130 8828043496 4853408946 0162269581
4061115880 3828627806 0403256665 7074998453 7470356071 3668275390 8061279014 2506972619 0119303941 2589965001 5584117686 9475615343
6449533093 2910451236 1220951615 2184713873 2797330447 6485527212 2577310486 5208352493 4033071492 6223893449 5446224729 9128489789
0064275766 6889877248 3321933151 8539283669 5896459721 0204974914 1353610729 4232034593 9209438523 9061891081 3827634985 7887852391
0418366629 1542255791 1958492222 4598665039 5249055786 2846777088 8395643767 5937390640 5292674126 2373670525 4827389933 0732408098
6939913760 1274338263 7517970302 1828395992 8627815024 5649001329 3631552984 9105155093 4575288391 0177794671 8860046765 0792351466
3655060518 8431946824 8646326914 5199183796 4304679065 2821823293 0006079496 1699777199 0824081861 5185650275 0386743802 5538811771
5051917928 6453527121 4754385031 2847869816 3076786496 1782064058 3458729594 5550585916 0437474672 4563880450 4628050102 8610642121
9866783775 7677493203 4046536681 6758671044 9913134478 4319417191 2897639301 3038574757 3431939458 9607257413 4622799655 1751800158
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1564506423 4457934027 8485098886 4962041472 1612544564 9886958936 1334402470 7084549558 7622917919 2961450426 9004069963 9871721950
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7723035108 1543450869 0659923574 6056699848 3716076746 7548415275 1995726963 8028406666 9346759480 9127972023 3445401568 2522725532
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2597790757 0002133120 6040566400 9617913407 3320368260 5373421482 0759207570 6039643092 7389100660 3990780772 0589393831 0872494082
9493766287 8062899260 0224761029 2289257069 8480178527 8538183760 6213842880 0583915487 0359119158 0992251426 6982867334 3661419324
3273448404 6903443079 1595562780 6628207496 5421765530 5557100799 6196644907 8553956444 8839092547 3088067064 0750855438 0992248519
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