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Double-Pulsar Drag Bounds Dissipative Sources of Diósi–Penrose Decoherence

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Abstract

In the proposal of Diósi and Penrose a mass in a superposition of two positions loses coherence at the rate $\Gamma = CE_{\mathrm{G}}/\hbar$, first order in Newton's constant. We ask what could produce this rate for a static superposition of equal masses. A bath in its ground state cannot: its noise equals its absorption, which vanishes at zero frequency when the response is continuous there. For the free linearized gravitational field and a 1 μg body the loss of coherence stays below 10^{-2} at all times, for separations up to a millimetre, in a scalar proxy normalized to E_{G} (it tends to a few times 10^{-3}), and it is reversible. These results are standard, at second cumulant. A bath that is not in a ground state can supply the rate, but if it couples to the mass density through a Hamiltonian, the noise that decoheres a body also takes momentum from it. We show that stationary, white, passive noise of Diósi–Penrose form drags a moving body at a rate of at least $(2\pi/3)(C\hbar G/k_{\mathrm{B}}T_{\mathrm{eff}})$ times its density, with T_{eff} the fluctuation–dissipation temperature. If the noise keeps this form and one temperature out to the kilometre wavelengths that a neutron star resolves, as it does by construction in the linear-friction dissipative Diósi–Penrose model, the orbital decay of the double pulsar PSR J0737−3039A/B requires $T_{\mathrm{eff}}/C > 3.3 \times 10^{16}$ K, up to strong-field corrections of order one. This is more than 5×10^{27} above the laboratory bound on that model and excludes the proposed $T_\beta \approx 3$ K by 10^{16}. Baths that differ between kilometre and laboratory scales escape. Otherwise what survives is noise without back-action: Diósi's original model with Gaussian mass-density smearing R_0. Published data allow $R_0 > 4.5$ Å $C^{1/3}$ with no upper bound and do not fix C. For a uniform sphere of radius $a \gg R_0$ the rate is independent of R_0 to $1.7\,R_0/a$ at every separation d, so one visibility measurement, with other decoherence channels subtracted, determines C; for 1 μg of silica $\tau = 65$ ps$/C$ at $d = 1$ mm and 0.35 μs$/C$ at $d = 1$ μm.

1 Introduction

Diósi [1, 2] and Penrose [3] proposed that a mass in a superposition of two locations loses coherence at the rate

$$\Gamma = \frac{C\,E_{\mathrm{G}}}{\hbar}, \qquad E_{\mathrm{G}} = \frac{G}{2}\iint \frac{\Delta\rho(\mathbf{x})\,\Delta\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\,d^3x\,d^3y, \qquad \Delta\rho = \rho_L - \rho_R, \tag{1}$$

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where ρ_L and ρ_R are the mass densities of the two branches and C is a dimensionless coefficient ($C = 1$ in Diósi's normalization). The rate is first order in G and refers to no environment. Reviews of gravitational decoherence [4, 5] survey the models that contain it and the experiments that test it.

The question. The models that contain the rate (1) postulate it. We ask what kind of physical system could produce it, in the simplest case: one body held at rest in a superposition of two positions, so that both branches carry the same mass.

Why it is open. Two candidates present themselves. The first is the quantized gravitational field. Master equations that treat gravitons as an environment have rates of first order in G, but they decohere superpositions of different *energies* [6, 7, 8]. For a spatial superposition of one body the field gives a bounded, reversible loss of coherence and no rate. The same holds for any bath in a ground state whose response is continuous at zero frequency (Section 4; these results are standard [9, 10, 11, 12]). The second candidate is a bath that is not in a ground state, for instance one at finite temperature. Dissipative Diósi–Penrose models [13, 14] postulate such a bath, with a temperature T_β as a new parameter, and laboratory data require only $T_\beta > 10^{-13}$ K, or 6×10^{-12} K for small smearing lengths [15]. Black-hole horizons are a different case: they decohere superpositions through free, on-shell quanta [16, 17, 18], which lack noise of Diósi–Penrose form.

The answer. A bath of the second kind is bounded by celestial mechanics. If it couples to the mass density through a Hamiltonian, the noise that decoheres a body also slows the body when it moves, at a rate fixed by the noise strength C, the fluctuation–dissipation temperature T_{eff} of the bath and the density of the body (Section 2, Theorem 5.1). The orbital decay of the double pulsar PSR J0737−3039A/B then requires

$$T_{\text{eff}}/C > 3.3 \times 10^{16} \text{ K} \tag{2}$$

(Corollary 5.3), more than 5×10^{27} above the laboratory bound on the linear-friction dissipative Diósi–Penrose model. The bound leaves the limit $T_{\text{eff}} \to \infty$, noise without back-action [2, 19, 20]: Diósi's original model with a smearing length R_0. Its rate for a uniform sphere does not depend on R_0 at any separation, so a visibility measurement determines C (Sections 6 and 7).

The mechanism, fluctuation–dissipation, is textbook. What is new here is the bound: Theorem 5.1 applied to binary pulsars, and Corollary 5.3. Oppenheim *et al.* remark that a friction term in classical–quantum gravity could conflict with the orbital decay of binary pulsars [21]; we know of no earlier quantitative bound from orbital decay on a collapse bath. Neutron-star *heating* bounds exist on collapse models [22] and on a hybrid classical–quantum model of Newtonian gravity [23], and binary-pulsar orbital decay has been used to bound the non-conservation of energy–momentum in unimodular gravity [24].

What is not shown. The bound is conditional. First, a laboratory rate tests the noise only at wavelengths up to the size of the body. That the noise keeps the Diósi–Penrose form, stays white and passive, and has one temperature at the kilometre wavelengths a neutron star resolves is an assumption. It holds by construction in a model that postulates its kernel at every scale

and damps momentum linearly, as that of Ref. [14] does. It does not hold for every reciprocal medium: baths that differ between the two scales escape (Section 5.4). Second, the published dissipative classical-channel models [25] are not bounded, because their friction does not act on the mean orbit (Section 5.3). Third, Eq. (2) is an order-of-magnitude statement: the stars are modelled as Newtonian uniform spheres, and strong-field corrections of order one are not computed. Fourth, the ground-state results hold at second cumulant, for an external classical source, and for gravity only for the free linearized field; they are not a theorem about full quantum gravity. Finally, nothing here fixes C or R_0, or shows that the rate exists.

2 The idea: noise that decoheres a body also drags it

A rate needs noise at zero frequency. A body in a superposition of two positions loses coherence when something acts differently on the two positions and fluctuates. Let that something be a field Φ that enters the Hamiltonian of the body through $\int \rho\,\Phi\, d^3x$, as a Newtonian potential does. A fluctuation of Φ turns the relative phase of the two branches. Fast fluctuations average out. The phase wanders steadily, and the interference fringes fade at a constant rate, only if the fluctuations have power at zero frequency. The Diósi–Penrose rate corresponds to one such noise: white in time, with the spatial correlation $C\hbar G/|\mathbf{x}-\mathbf{y}|$.

A bath in its ground state has none. A bath in its ground state can take energy from a probe but cannot give any. Its fluctuations at a frequency ω are then exactly as strong as its ability to absorb at ω. Absorption is odd in ω, so if it is continuous it vanishes at $\omega = 0$. Such a bath dresses the two branches differently, which costs a fixed, small amount of coherence, and it produces no rate. The quantized gravitational field in its vacuum is of this kind (Section 4).

A bath that is not in its ground state can have the noise, and then it drags. A Hamiltonian coupling acts in both directions: if the field pushes on the mass, the mass pushes on the field. A body that moves through the bath stirs it, the stirred bath pulls back, and the body slows. Einstein's relation for a grain suspended in water expresses this [10]. The molecular impacts that make the grain jitter are the same impacts that resist its motion, and the ratio of the two effects is the temperature: $D_p = M\gamma\, k_{\mathrm{B}}T$, where D_p is the momentum diffusion constant of the grain, M its mass, γ its friction rate and k_{B} Boltzmann's constant. A hot bath jitters much for the friction it exerts, and a cold bath drags much for its jitter. In our problem the jitter is given: it is the Diósi–Penrose noise, of strength C. The friction then follows up to one number, the temperature T_{eff} at which the fluctuations and the dissipation of the bath balance, and it grows as the bath gets colder. Moreover the kicks are gradients of Φ, so the factor k^2 of the gradients cancels the $1/k^2$ of the noise spectrum and D_p is proportional to $\int \rho^2 d^3x$. The friction rate of a body therefore depends on the body only through its density,

$$\gamma = \frac{2\pi}{3}\,\frac{C\hbar G}{k_{\mathrm{B}}T_{\mathrm{eff}}}\,\langle\rho\rangle, \qquad \langle\rho\rangle = \frac{\int \rho^2\, d^3x}{\int \rho\, d^3x}, \tag{3}$$

when the noise has this form at every wavelength the body resolves; Theorem 5.1 gives the lower bound that holds when only the long wavelengths are known.

Neutron stars in a binary are the test. The densest bodies whose motion can be timed are neutron stars, and the best timed are the two stars of the double pulsar. Friction shrinks an orbit and shortens its period. The measured shortening agrees with the emission of gravitational waves to 1.3×10^{-4}, which leaves room for a friction rate below 6.1×10^{-21} s^{-1}. At the density of a neutron star this requires $T_{\rm eff}/C > 3.3 \times 10^{16}$ K (Corollary 5.3). A bath at 3 K with Diósi–Penrose noise would drain the orbit 10^{16} times faster than the timing allows.

What is left, and where the argument can fail. As $T_{\rm eff} \to \infty$ at fixed noise the friction vanishes. The limit is noise without back-action: a classical random field that kicks matter and is not kicked in return. This is how Diósi wrote his model. There are then three cases. In a ground state there is no rate. At finite temperature there is a rate and a drag, and the pulsars bound it. At infinite temperature there is a rate without drag, and an interferometer must decide (Section 7). The weak point is the step from the laboratory to the star: an interferometer probes the noise at wavelengths of micrometres, a neutron star at kilometres. A bath whose noise is cut off, slow, or hotter at long wavelengths passes the pulsar test (Section 5.4).

3 Conventions: visibility, noise and the energy $E_{\rm G}$

The measured quantity. A rigid body of mass M is prepared in a superposition of two static positions a distance d apart. Both branches carry the same mass. The source is inertial and enters as an external c-number density. The two paths are recombined and the fringe visibility $|z(t)|$ is recorded as a function of the time t spent apart, where z is twice the off-diagonal element of the body's density matrix between the two branches. A rate is a constant Γ with $|z(t)| \propto e^{-\Gamma t}$ at late times.

The rate from noise. Let the mass density couple to a potential operator, $H_{\rm int} = \int \rho\, \Phi\, d^3x$. The branches differ through the which-path variable $B(t) = \int \Delta\rho(\mathbf{x})\, \Phi(\mathbf{x}, t)\, d^3x$. With $\delta B = B - \langle B \rangle$, define its symmetrized noise and its dissipative (absorptive) response,

$$S_B(\omega) = \frac{1}{2} \int d\tau\, e^{i\omega\tau} \langle \{\delta B(\tau), \delta B(0)\} \rangle, \qquad \chi''_B(\omega) = \frac{1}{2\hbar} \int d\tau\, e^{i\omega\tau} \langle [B(\tau), B(0)] \rangle. \tag{4}$$

For Gaussian fluctuations and pure dephasing,

$$-\ln |z(t)| = \frac{1}{2\pi\hbar^2} \int_{-\infty}^{\infty} d\omega\; S_B(\omega)\, \frac{1 - \cos \omega t}{\omega^2}. \tag{5}$$

Since $\int (1 - \cos \omega t)\, \omega^{-2} d\omega = \pi t$, the late-time slope is

$$\Gamma = \lim_{t \to \infty} \frac{-\ln |z(t)|}{t} = \frac{S_B(0)}{2\hbar^2}. \tag{6}$$

A rate requires a zero-frequency floor. If $S_B(\omega) = \mathcal{O}(\omega^2)$ as $\omega \to 0$, the integrand of Eq. (5) is bounded at small ω and the dephasing exponent $-\ln |z|$ saturates.

Diósi–Penrose noise. Classical white potential noise with

$$\langle \delta\Phi(\mathbf{x}, t)\, \delta\Phi(\mathbf{y}, s) \rangle = \frac{C\hbar G}{|\mathbf{x} - \mathbf{y}|}\, \delta(t - s), \qquad S_\Phi(\mathbf{k}, \omega) = \frac{4\pi C\hbar G}{k^2} \quad \text{for all } \omega, \tag{7}$$

gives $S_B(0) = C\hbar G \iint \Delta\rho\,\Delta\rho\,|\mathbf{x}-\mathbf{y}|^{-1} = 2C\hbar E_{\rm G}$, so Eq. (6) reproduces Eq. (1). At $C = 1$ this is the noise behind Diósi's master equation $\dot{\hat\rho} = -(G/2\hbar)\iint[\hat M(\mathbf{x}),[\hat M(\mathbf{y}),\hat\rho]]\,|\mathbf{x}-\mathbf{y}|^{-1}$ [1, 2], with $\hat\rho$ the density matrix and $\hat M(\mathbf{x})$ the mass-density operator. We measure C in these Diósi units throughout. The normalization used with Penrose's prescription in Refs. [26, 27] is larger by 8π, so it corresponds to $C = 8\pi$; Appendix B gives the master equation of each cited bound and derives the factor.

The energy. $E_{\rm G}$ in Eq. (1) contains the same-branch terms; they make $E_{\rm G} \geq 0$. For uniform spheres of radius a at centre separation d, with $s = d/a$,

$$E_{\rm G} = \frac{GM^2}{a}\,f(s), \qquad f(s) = \begin{cases} \dfrac{s^2}{2} - \dfrac{3s^3}{16} + \dfrac{s^5}{160}, & s < 2, \\[2ex] \dfrac{6}{5} - \dfrac{1}{s}, & s \geq 2, \end{cases} \tag{8}$$

(Appendix A); the two branches agree with their first three derivatives at contact, $s = 2$. The energy is tidal, $E_{\rm G} \simeq GM^2d^2/(2a^3)$, for $d \ll a$ and saturates at $(6/5)GM^2/a$ for $d \gg a$. The point-mass expression GM^2/d is the magnitude of the cross term alone; it is not a limit of Eq. (1) for any mass distribution.

4 Known results: a bath in its ground state gives no rate

The statements in this section are standard results or direct consequences of them; the mathematics is the zero-temperature fluctuation–dissipation theorem and the spectral support of Wightman functions. We state them with their hypotheses because the drag theorem of Section 5 addresses what they leave open. Throughout, the bath is kept to second cumulant and the static source enters as an external c-number density.

4.1 Any ground state: the noise is the absorption

Let $H \geq 0$ generate a time translation of the laboratory under which the bath state and both branch configurations are stationary, and let Ω be a ground state, $H\Omega = 0$. No spatial symmetry is assumed: the Earth, the apparatus and the source may break every spatial translation. The proposition says that the noise then equals the absorption at each frequency, and that a soft enough absorption bounds the loss of coherence for all time.

Proposition 4.1 (Ground state; standard)**.** *(i) $S_B(\omega) = \hbar|\chi_B''(\omega)|$ for $\omega \neq 0$, and $S_B(0) = \hbar\chi_B''(0^+)$ when the limit exists. (ii) The rate vanishes if and only if the Fejér mean of χ_B'' at $\omega = 0$ vanishes and the spectral measure of $\delta B\,\Omega$ has no atom at $\omega = 0$. (iii) If $\Lambda_2 \equiv \pi^{-1}\int_0^\infty \chi_B''(\omega)\,\omega^{-2}d\omega$ is finite, then*

$$-\ln|z(t)| \leq \frac{2\Lambda_2}{\hbar} \quad \textit{for all } t, \qquad -\ln|z(t)| \xrightarrow{t\to\infty} \frac{\Lambda_2}{\hbar}. \tag{9}$$

Proof. With $H\Omega = 0$ and $H \geq 0$, the spectral measure of $\delta B\,\Omega$ lives on $E \geq 0$, so $\langle\delta B(t)\delta B\rangle$ has its Fourier support at $\omega \geq 0$ and $\langle\delta B\,\delta B(t)\rangle$ at $\omega \leq 0$; (i) is the zero-temperature fluctuation–dissipation theorem [9, 10]. Then Eq. (5) reads $-\ln|z(t)| = (\pi\hbar)^{-1}\int_0^\infty \chi_B''(\omega)(1-\cos\omega t)\,\omega^{-2}d\omega$.

Substituting $x = \omega t$ in $-\ln|z|/t$ gives (ii); an atom at $\omega = 0$ would add a term growing as t^2. Bounding $1 - \cos\omega t \le 2$ and applying the Riemann–Lebesgue lemma gives (iii). □

In (ii), the Fejér mean of χ''_B at $\omega = 0$ is $\lim_{t\to\infty} \int_0^\infty \chi''_B(\omega)\, 2(1-\cos\omega t)/(\pi t\,\omega^2)\, d\omega$; it equals $\chi''_B(0^+)$ when that limit exists. The proposition means that a ground-state bath produces a rate only if its absorption stays finite down to zero frequency. Since χ''_B is odd in ω, that requires a response that is discontinuous at $\omega = 0$: a Diósi–Penrose rate would need $\chi''_B(0^+) = 2CE_{\rm G}$, that is, a response tail $\chi_B(t) \simeq 4CE_{\rm G}/(\pi t)$ of the bath to the which-path variable. This is the loophole that the proposition leaves. For $\chi''_B \propto \omega^s$ at small ω, a rate needs $s \le 0$; Ohmic baths, $s = 1$, give $-\ln|z| \propto \ln t$, and only $s > 1$ gives a bounded exponent [12, 28, 29].

A finite reorganization energy $\pi^{-1}\int_0^\infty \chi''_B\, \omega^{-1} d\omega$ suffices for (ii), but it does not bound decoherence on laboratory times: a χ''_B that is flat over a band $[\omega_{\rm IR}, \omega_{\rm UV}]$ and normalized to reorganization energy $E_{\rm G}$ mimics Eq. (1) with $C_{\rm eff} = \pi/[2\ln(\omega_{\rm UV}/\omega_{\rm IR})]$ for $1/\omega_{\rm UV} \ll t \ll 1/\omega_{\rm IR}$, which is 0.07 between 0.1 ps and 1 ms. The all-times bound (9) is the statement that excludes this.

4.2 The free linearized gravitational field

In a thermal equilibrium (Kubo–Martin–Schwinger, KMS) state at inverse temperature β, the fluctuation–dissipation theorem gives $S_B(\omega) = \hbar\chi''_B(\omega)\coth(\beta\hbar\omega/2)$, so

$$S_B(0) = \frac{2}{\beta}\lim_{\omega\to 0}\frac{\chi''_B(\omega)}{\omega}\,: \tag{10}$$

a floor at finite temperature needs an Ohmic response. The vacuum restricted to a Rindler wedge [30], or for a conformal field to a causal diamond [31], is KMS for its modular flow; describing the vacuum in modular time adds no excitations. A free massless field obeys $\Box\hat\phi = 0$, so its commutator, and hence χ''_B, is a c-number supported on $\omega = \pm c|\mathbf{k}|$, the same in every state. Inhomogeneity of the laboratory changes the propagator only at order $GM_\oplus/(R_\oplus c^2) = 7\times 10^{-10}$. Weight at $\omega \to 0$ comes only from $k \to 0$. For two equal spheres with form factor $\tilde\rho(k)$,

$$\big\langle|\Delta\tilde\rho_{\mathbf{k}}|^2\big\rangle = 2\,\tilde\rho(k)^2\Big(1-\frac{\sin kd}{kd}\Big) \xrightarrow{kd\to 0} \tfrac{1}{3}M^2k^2d^2. \tag{11}$$

Take as a scalar proxy a propagating field normalized so that its reorganization energy, $\pi^{-1}\int_0^\infty \chi''_B\,\omega^{-1}d\omega$, equals $E_{\rm G}$ (Appendix A). Then

$$\chi''_B(\omega) = \frac{G\omega}{c}\big\langle|\Delta\tilde\rho_{\omega/c}|^2\big\rangle \;\simeq\; \frac{GM^2d^2}{3c^3}\,\omega^3 \qquad (\omega d/c \ll 1), \tag{12}$$

which is super-Ohmic. Only a difference of branch *energies* makes it Ohmic. That is the channel of the graviton master equations [6, 7, 8], whose rates are first order in G for superpositions of different energies and vanish for the equal-mass spatial superposition considered here. Linearized general relativity has less infrared response than the proxy. Its Newtonian potential is fixed by the constraint, an instantaneous real kernel with $\chi'' = 0$ that fluctuates only through the matter that sources it, at order G^2. Its radiative tensor modes couple through the trace-free quadrupole difference of the branches, $\chi''_{\rm TT} = (G/5c^5)\,|\Delta Q^{\rm TF}|^2\,\omega^5$, which for $|\Delta Q^{\rm TF}|^2 = \frac{2}{3}M^2d^4$ (quadrupoles about one branch, the origin of the proper-detector frame; about an origin at distance D the difference grows by terms of order $M^2D^2d^2$, still $\propto\omega^5$) is $\frac{2}{5}(\omega d/c)^2$ times Eq. (12),

less than the proxy for $\omega d/c \ll 1$. The response of the field is therefore too soft at low frequency for a floor, in the vacuum and at any temperature:

Proposition 4.2 (Field channel)**.** *For a static inertial source with equal branch masses, $S_B(0) = 0$ and the dephasing exponent saturates in the vacuum and in every thermal state of the free linearized field, including states that are KMS for modular flow. In a stationary Gaussian state with mode occupation $n(\omega)$, $S_B(0) = 0$ if and only if $\lim_{\omega\to 0^+} n(\omega)\,\chi_B''(\omega) = 0$, and the exponent saturates if and only if $\int_0 n(\omega)\,\chi_B''(\omega)\,\omega^{-2} d\omega < \infty$, for example if $n\chi_B'' = \mathcal{O}(\omega^{1+\delta})$ with $\delta > 0$.*

Proof. For a free field $S_B = \hbar\chi_B''\,[1 + 2n(\omega)]$ with χ_B'' state-independent and $\propto \omega^3$ or softer at small ω. The vacuum, $n = 0$, is Proposition 4.1. A thermal state has $2n \simeq 2/(\beta\hbar\omega)$, so $S_B = \mathcal{O}(\omega^2)$. For the vacuum and thermal states $S_B(\omega)/\omega^2$ is therefore integrable at $\omega = 0$ and Eq. (5) converges as $t \to \infty$; in general $S_B(0) = 0$ is the stated limit and integrability is the stated condition. □

The free field gives a static equal-mass superposition no rate, whatever its temperature. The occupation condition is not automatic. A statistically homogeneous stochastic gravitational-wave background with strain spectrum $S_h \propto \omega^{-4}$ violates it and gives a white floor at first order in G, but $\propto M^2 d^4$, not of Diósi–Penrose form. In the tensor channel such a background has an infrared-divergent energy density, so it can only be band-limited. At closure density at 1 Hz its rate is 3×10^{-8} of Eq. (1) for the benchmark of Section 7 at $d = 1$ mm.

For the vacuum, Proposition 4.1(iii) makes the statement quantitative. For 1 μg of silica the proxy (12) gives $\Lambda_2/\hbar = 1.1 \times 10^{-3}$ at $d = 2a$ and 4.2×10^{-3} at $d = 1$ mm, of order $GM^2/(\hbar c) = 2.1 \times 10^{-3}$; for $\omega d/c \ll 1$ the tensor channel gives less. By Eq. (9) the coherence loss $-\ln|z(t)|$ stays below $2\Lambda_2/\hbar$, which is 2.2×10^{-3} at $d = 2a$ and 8.3×10^{-3} at $d = 1$ mm, at all times; it tends to $\Lambda_2/\hbar$ and is reversible: it is the overlap of the field states dressing the two branches, not a rate. Direct calculations agree. In the graviton vacuum, the decoherence functional of a spatial superposition that is opened and closed again approaches a constant at late times rather than growing linearly [32]. The which-path information radiated by a superposition is set by the time taken to recombine it, not by the time it is held apart [33], and in the inertial Minkowski vacuum it does not grow with the holding time [16].

4.3 The covariant case, and what these results leave open

If the state is invariant under spacetime translations and the energy–momentum spectrum lies in the forward cone, the zero follows without computing any response:

Proposition 4.3 (Spectrum condition; standard)**.** *Let Ω be invariant under spacetime translations $U(a) = e^{iP\cdot a}$, let the joint spectrum of P^μ lie in the closed forward cone $\bar{V}_+$, let $\Phi(x) = U(x)\Phi(0)U(x)^\dagger$ (local or gravitationally dressed) have a locally finite connected two-point spectral measure μ, and let the source be rigid. Then $S_B(0) = 0$.*

Proof. In this proof $\hbar = 1$: $p = (p^0, \mathbf{p})$ is a wave four-vector and $\mathbf{p}$ is the wavevector called $\mathbf{k}$ elsewhere. With $P|\Omega\rangle = 0$, $\langle\Omega|\Phi(x)\Phi(0)|\Omega\rangle_c = \int e^{-ip\cdot x}\, d\mu(p)$ with $\operatorname{supp}\mu \subset \bar{V}_+$. The zero-frequency noise of B is the density at $p^0 = 0$ of $|\Delta\tilde{\rho}(\mathbf{p})|^2 d\mu(p)$ and its mirror image, $S_B(0) = c_0 \lim_{\varepsilon\to 0} \varepsilon^{-1} \int_{0\le p^0\le\varepsilon} |\Delta\tilde{\rho}|^2\, d\mu$, with c_0 a convention-dependent constant. On the

support $|\mathbf{p}| \le \varepsilon/c$, and a rigid body has $|\Delta\tilde{\rho}|^2 = 4|\tilde{\rho}|^2 \sin^2(\mathbf{p}\cdot\mathbf{d}/2) \le M^2|\mathbf{p}|^2 d^2$, so

$$S_B(0) \le c_0 \lim_{\varepsilon\to 0} \frac{M^2 d^2}{c^2}\, \varepsilon\, \mu(\{p^0 \le \varepsilon\}) = 0 \tag{13}$$

by local finiteness. □

Since the cone implies $P^0 \ge 0$, Proposition 4.3 is the special case of Proposition 4.1 in which the cone and the dipole suppression of an equal-mass source supply $\chi_B''(0^+) = 0$. An atom of μ at $p = 0$ is weighted by $|\Delta\tilde{\rho}(0)|^2 = 0$, so it needs no separate assumption. This is a standard consequence of the spectral support of Wightman functions [11, 34] combined with the dipole suppression of an equal-mass source. The Lorentz-cone support forces, for any covariant field coupled to a source with vanishing monopole difference, the conclusion that the theory of dephasing reaches for Ohmic and super-Ohmic baths [12, 28, 29]. In asymptotically flat gravity the cone would follow from positivity of the total energy, which we do not derive.

The propositions leave open whatever breaks one of their hypotheses.

- *The state is not a ground state.* In a thermal, stationary non-equilibrium or infinite-temperature state the generator of time translations that leaves the state invariant is not bounded below; for a thermal state of an infinite system its spectrum is all of $\mathbb{R}$. This is the class of Section 5. Relativistic collapse models escape the same way: a mediating field at each spacetime point [35], a scalar field whose energy spectrum takes all real values [36], a clock field that fixes the foliation [37], or classical noise with a Lorentz-invariant correlation, which as a classical field need not have a positive generator. Why such noise is hard to reconcile with a stable vacuum is the subject of a long literature [38, 39, 40, 41, 42, 43, 44]. Gundhi, Diósi and Carlesso obtain a finite energy rate for Lorentz-invariant, non-Markovian noise [45]; their dynamics still excites the vacuum, as they note, and the finite rate comes from a normal-ordering prescription applied to the evolved observables, which is not a positive map on states. The model therefore supplies no stationary positive-energy bath with a floor, and its kernel is not of Diósi–Penrose form.

- *Killing horizons* replace the positive generator of laboratory time by a boost generator with spectrum $\mathbb{R}$. They decohere at rates set by horizon data that vanish for an inertial laboratory without a horizon [16, 17, 46, 47].

- *Non-inertial laboratories.* The generator of the co-rotating frame is not bounded below. A source fixed on the rotating Earth radiates branch-dependently at a rate of order 10^{-49} of Eq. (1) in the scalar proxy, and less in general relativity, where the dipole does not radiate.

- *A quantized source.* Internal dynamics that diffuse the source's trajectory lie outside the c-number treatment [48]. So does the source's own Gauss-law sector, where boosts are spontaneously broken; we conjecture that its corrections are of higher order in GM than the leading term, but have not shown it.

- *Beyond second cumulant*: higher cumulants of the bath are not controlled by these propositions.

- *Non-unitary dynamics*: Penrose-type objective reduction [3] and collapse models whose record is a sequence of jumps with a stable vacuum [49, 50] lie outside the bath framework.

5 The drag theorem and the double-pulsar bound

This section makes the argument of Section 2 precise for a bath with a zero-frequency floor. Theorem 5.1 is the Einstein relation, the classical content of the fluctuation–dissipation theorem [9, 10] and of the link between decoherence and friction in quantum Brownian motion [51], applied to Diósi–Penrose noise. What is new is the bound from binary neutron stars and the identification of $T_{\rm eff}/C$ as the parameter that the orbital decay bounds.

5.1 The drag theorem: friction from the noise and one temperature

Let $H_{\rm int} = \int \rho\,\Phi\, d^3x$ with Φ a bath operator, and let $\Phi_{\mathbf{k}}$ be its spatial Fourier transform. Per unit volume define the symmetrized noise and the dissipative response

$$S(\mathbf{k},\omega) = \frac{1}{2}\int dt\, e^{i\omega t}\langle\{\Phi_{\mathbf{k}}(t),\Phi_{-\mathbf{k}}\}\rangle, \qquad \chi''(\mathbf{k},\omega) = \frac{1}{2\hbar}\int dt\, e^{i\omega t}\langle[\Phi_{\mathbf{k}}(t),\Phi_{-\mathbf{k}}]\rangle. \tag{14}$$

Fix a wavenumber k_* and a speed $v_{\rm max}$, and assume:

(H) *Reciprocity:* the operator ρ couples to is the one that carries the noise, through a Hamiltonian (or Schwinger–Keldysh) coupling, so S and χ'' come from the same correlator.

(S) *Stationarity:* the bath state is stationary, homogeneous and isotropic in a rest frame, at scales $\geq 1/k_*$.

(DP) *Diósi–Penrose noise:* $S(k,0) = 4\pi C\hbar G/k^2$ for $k \leq k_*$.

(W) *Whiteness over the drift band:* $\chi''(k,\omega)/\omega$ equals its $\omega \to 0$ value for $|\omega| \leq k_* v_{\rm max}$, $k \leq k_*$.

(F) *Finite positive temperature:* $T_{\rm eff}(k) \equiv S(k,0)/(2k_{\rm B}\,\partial_\omega\chi''(k,0)) \in (0,\infty)$ for $k \leq k_*$, with $T_{\rm eff} \equiv \sup_{k<k_*} T_{\rm eff}(k) < \infty$; a KMS state at temperature T has $T_{\rm eff} \equiv T$.

(P) *Passivity:* $\omega\chi''(k,\omega) \geq 0$ at every k and ω. This holds in every KMS state at $T > 0$; for other states it is a separate assumption.

(L) *Linear response:* the bath responds linearly to the body's density, so the friction is linear in the velocity for all $|\mathbf{v}| \leq v_{\rm max}$.

$T_{\rm eff}$ is a fluctuation–dissipation parameter, not the thermodynamic temperature of any identified degrees of freedom. The theorem bounds the friction on a moving body from below, using only the noise at wavenumbers below k_*.

Theorem 5.1 (Drag)**.** *Under (H), (S), (DP), (W), (F), (P) and (L), a body moving with velocity* $\mathbf{v}$*,* $|\mathbf{v}| \leq v_{\rm max}$*, relative to the rest frame feels* $\mathbf{F} = -M\gamma\mathbf{v}$ *with*

$$\gamma \;\geq\; \frac{2\pi}{3}\,\frac{C\hbar G}{k_{\rm B}T_{\rm eff}}\,\rho_{k_*}, \qquad \rho_{k_*} \equiv \frac{1}{M}\int_{k<k_*}\frac{d^3k}{(2\pi)^3}\,|\rho_{\mathbf{k}}|^2. \tag{15}$$

Proof. Uniform motion gives $\rho_{\mathbf{k}}(t) = \rho_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{v}t}$, so the body drives mode $\mathbf{k}$ at $\omega = \mathbf{k}\cdot\mathbf{v}$. By (H), (S) and (L), $\langle\Phi_{\mathbf{k}}\rangle = -\chi(\mathbf{k},\mathbf{k}\cdot\mathbf{v})\rho_{\mathbf{k}}$, and the self-force is

$$\mathbf{F} = -\int \rho\,\nabla\langle\Phi\rangle\, d^3x = -\int\frac{d^3k}{(2\pi)^3}\,\mathbf{k}\,\chi''(\mathbf{k},\mathbf{k}\cdot\mathbf{v})\,|\rho_{\mathbf{k}}|^2, \tag{16}$$

because χ' is even in ω and drops out against the odd factor $\mathbf{k}$. The power dissipated in mode $\mathbf{k}$, $(\mathbf{k}\cdot\mathbf{v})\chi''(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|\rho_{\mathbf{k}}|^2$, is non-negative by (P), so restricting the integral to $k<k_*$ can only lower the drag. There, (W) gives $\chi''(\mathbf{k},\mathbf{k}\cdot\mathbf{v})=(\mathbf{k}\cdot\mathbf{v})\,\partial_\omega\chi''(k,0)$; isotropy replaces k_ik_j by $\delta_{ij}k^2/3$; and (F) with (DP) gives

$$\partial_\omega\chi''(k,0)=\frac{S(k,0)}{2k_{\mathrm{B}}T_{\mathrm{eff}}(k)}\geq\frac{2\pi C\hbar G}{k_{\mathrm{B}}T_{\mathrm{eff}}\,k^2}. \tag{17}$$

The factor k^2 cancels, and Eq. (15) follows. □

The theorem says that the friction rate is the noise strength divided by the temperature, times the density of the body within the band $k<k_*$. No other property of the bath enters.

Remark 5.1 (Reading the bound). (i) In classical form $\gamma=(4\pi/3)G\rho_{k_*}/\kappa_{\mathrm{FDR}}$ with the fluctuation–dissipation rate $\kappa_{\mathrm{FDR}}\equiv 2k_{\mathrm{B}}T_{\mathrm{eff}}/(C\hbar)$, in which $\hbar$ cancels. The laboratory rate fixes C; the drag bounds T_{eff}/C. If $T_{\mathrm{eff}}(k)$ is unbounded on $k<k_*$, (F) fails and the bound degenerates; this is escape (E4) of Section 5.4. (ii) Writing the momentum diffusion as $D_p=\frac{1}{2}\int k_ik_j\,S(k,0)|\rho_{\mathbf{k}}|^2$, Eq. (15) is the Einstein relation $\gamma=D_p/(Mk_{\mathrm{B}}T_{\mathrm{eff}})$. A Markovian model whose centre-of-mass momentum relaxes to a Maxwellian at T_{eff} is covered, whether or not a bath is written down, provided its friction is linear in the velocity up to orbital speeds and its noise satisfies (DP), (W) and (P) at $k\leq k_*$. (iii) At $k_*=10/R$, $\rho_{k_*}/\bar\rho=0.909$ for a uniform sphere of radius R and mean density $\bar\rho$, 1.42 for $\rho\propto 1-r^2$ and 1.64 for an $n=1$ polytrope. The uniform sphere is the conservative case. (iv) Treating a body as rigid with linear friction requires its centre-of-mass motion not to be thermal in the bath, $Mv^2\gg k_{\mathrm{B}}T_{\mathrm{eff}}$, so that the fluctuating force is negligible against the drag. Many-body versions of dissipative collapse can differ: collective pair friction in a Fock-space formulation gives a stationary per-particle temperature of order $2T_\beta/N$ at collapse temperature T_β [52]. Under the stated condition such effects only lower the centre-of-mass temperature, and the bound tightens. (v) By Kramers–Kronig, (P), (W) and (F) also imply a static response $\chi'(k,0)\geq(2/\pi)\int_0^{k_*v_{\max}}\chi''\,\omega^{-1}d\omega$, a renormalization of Newton's constant with $\delta G/G\geq C\hbar k_*v_{\max}/(\pi k_{\mathrm{B}}T_{\mathrm{eff}})$, which is 5×10^{-26} at the bound below: the bath does not otherwise modify Newton's law observably.

The next lemma converts a friction rate into the observable of pulsar timing, the rate of change $\dot P_b$ of the orbital period P_b.

Lemma 5.2 (Orbital decay)**.** *Two bodies of masses M_1,M_2 with drags γ_1,γ_2 in a bound orbit feel a drag $-\Gamma_{\mathrm{orb}}\mathbf{v}$ on their relative velocity, $\Gamma_{\mathrm{orb}}=(\gamma_1M_2+\gamma_2M_1)/(M_1+M_2)\geq\min\gamma_i$; the orbital binding energy grows at the relative rate $2\Gamma_{\mathrm{orb}}$, and $\dot P_b/P_b=-3\Gamma_{\mathrm{orb}}$.*

Proof. Write $M=M_1+M_2$, $\mu=M_1M_2/M$ and $\mathbf{v}$ for the relative velocity. In the centre-of-mass frame $\mathbf{v}_1=(M_2/M)\mathbf{v}$ and $\mathbf{v}_2=-(M_1/M)\mathbf{v}$, so the drag on the relative velocity is $-\Gamma_{\mathrm{orb}}\mathbf{v}$. The velocity $\mathbf{w}$ of the centre of mass relative to the rest frame enters only as the constant differential acceleration $-(\gamma_1-\gamma_2)\mathbf{w}$, which does no secular work on the orbital energy at linear order. With $\dot E=-\mu\Gamma_{\mathrm{orb}}v^2$ and Kepler's law, $\dot a=-2\Gamma_{\mathrm{orb}}a$ for the semi-major axis, and $P_b\propto a^{3/2}$ gives $\dot P_b/P_b=-3\Gamma_{\mathrm{orb}}$, independent of the eccentricity to this order. □

5.2 The double-pulsar bound on T_{eff}/C

By the lemma, a limit on any unexplained shortening of the period is a limit on the friction. PSR J0737−3039A/B has $P_b=0.1022515593$ d and observed $\dot P_b=-1.247920(78)\times10^{-12}$; after

subtracting the external terms its orbital decay agrees with the quadrupole formula to 1.3×10^{-4} at 95% confidence [53].

Any additional decay therefore obeys

$$\Gamma_{\rm orb} < \gamma_{\max} = \frac{1.3 \times 10^{-4}\,|\dot{P}_b|}{3P_b} = 6.1 \times 10^{-21}\ {\rm s}^{-1}. \tag{18}$$

Both bodies are neutron stars, of measured masses 1.338 and 1.249 $M_\odot$ [53]. We model each as a uniform sphere of $M = 1.2\,M_\odot$ and $R_{\rm NS} = 14$ km, lighter and larger than the real stars and therefore of lower density, $\bar{\rho} = 2.1 \times 10^{17}\ {\rm kg\,m^{-3}}$. Each star moves about the centre of mass at 300–325 ${\rm km\,s^{-1}}$, so $v_{\max} = 10^3\ {\rm km\,s^{-1}}$ also covers a centre-of-mass velocity of several hundred ${\rm km\,s^{-1}}$ relative to the bath. With $k_* = 10/R_{\rm NS}$, (W) and (L) are needed only up to $k_* v_{\max} = 714\ {\rm s}^{-1}$ at kilometre wavelengths. Along the orbit the velocity turns at $2\pi/P_b = 7.1 \times 10^{-4}\ {\rm s}^{-1}$, six orders below this band, so the theorem applies instantaneously along the orbit, with corrections of relative order 10^{-6}.

Corollary 5.3 (Pulsar bound)**.** *For the Newtonian uniform-sphere model of the two stars described above, every bath obeying the hypotheses of Theorem 5.1 at $k_* = 10/R_{\rm NS}$ and $v_{\max} = 10^3$ km s^{-1} satisfies*

$$\frac{T_{\rm eff}}{C} > \frac{2\pi\hbar G\,(0.909\,\bar{\rho})}{3k_{\rm B}\gamma_{\max}} = 3.3 \times 10^{16}\ {\rm K}, \tag{19}$$

equivalently $k_{\rm B}T_{\rm eff}/C > 2.8$ TeV or $\kappa_{\rm FDR} > 8.6 \times 10^{27}\ {\rm s}^{-1}$.

Hypothesis (DP) is a statement at $k \le 10/R_{\rm NS} = 7 \times 10^{-4}\ {\rm m}^{-1}$. A laboratory rate does not test it there: for $d \ge 2a$ the small-k part of $E_{\rm G}$ is suppressed by $|\Delta\tilde{\rho}|^2 \propto k^2$, so the rate fixes the kernel only at $k \gtrsim 1/a$. The corollary therefore bounds models whose kernel and single temperature are postulated at every k, such as the linear-friction dissipative Diósi–Penrose model of Section 5.3, and not every reciprocal medium; baths that differ between the two scales are the escapes of Section 5.4.

The bound is an order-of-magnitude statement. Strong-field corrections (compactness $GM/Rc^2 = 0.13$ for the model star, 0.15–0.16 for the measured masses at 12 km) are of order one; a realistic stellar profile raises ρ_{k_*} (Remark 5.1) and strengthens it. With the measured masses and a radius of 12 km the bound is 5.6×10^{16} K. A drag only adds decay, so a one-sided limit from the measured ratio $\dot{P}_b^{\rm GW}/\dot{P}_{b,{\rm GR}}^{\rm GW} = 0.999963(63)$ [53] is about twice as strong; we quote the two-sided value. Free fields at $T > 0$ are not constrained: their spectral weight sits on $|\omega| = ck$, so $S(k,0) = 0$ and they fail (DP). Binary-pulsar timing has been used as a probe of friction from a medium for dark matter [54], and the finite-temperature question for gravitational decoherence was raised for the classical-channel models [25].

The granular part. If the bath couples to the full density with Gaussian smearing R_0, the discreteness of the matter adds to ρ_{k_*} a piece from $k \sim 1/R_0$, $m_n \bar{S}/(8\pi^{3/2}R_0^3)$, where m_n is the nucleon mass and $\bar{S}$ is the static structure factor of the stellar matter averaged with weight $k^2 e^{-k^2R_0^2}$; it requires (W) up to the frequencies of the matter's own density fluctuations at $k \sim 1/R_0$. For uncorrelated nucleons ($\bar{S} = 1$) it would be $0.18\,\bar{\rho}$ at $R_0 = 1$ fm, but neutron-star matter is a degenerate liquid with a spacing of 2 fm: at $R_0 = 1$ fm the free-Fermi-gas structure factor and the nucleon's mass form factor give $\bar{S} \approx 0.4$ and a piece of about $0.07\,\bar{\rho}$; at $R_0 = 4.5$ Å the density fluctuations are those of a nearly incompressible liquid with sound

speed c_1, $\bar{S} \approx \hbar k/(2m_n c_1) \sim 10^{-6}$, and the piece is of order 10^{-6} kg m^{-3}. It only strengthens the bound.

5.3 Which published models the bound reaches

Dissipative Diósi–Penrose. The Diósi–Penrose model needs a short-distance cutoff to keep its energy increase finite [55]. Bahrami, Smirne and Bassi introduced dissipation to tame that increase, and found that it does not reconcile the model with data unless the cutoff is kept artificially large or the model is applied only to massive systems [13]; the analogous dissipative and coloured extensions of continuous spontaneous localization are bounded in Refs. [56, 57]. The linear-friction model of Ref. [14] adds friction at a collapse temperature T_β to the kernel $D_k = 4\pi\hbar G\, e^{-R_0^2 k^2}/k^2$. Its centre-of-mass damping $\Gamma = \hbar^2\eta/(2Mk_{\mathrm{B}}T_\beta)$, with η the k^2-weighted Diósi–Penrose diffusion [15], is Eq. (15) at $C = 1$, $T_{\mathrm{eff}} = T_\beta$. Corollary 5.3 gives $T_\beta > 3.3\times10^{16}$ K for $R_0 \ll R_{\mathrm{NS}}$. We have made this identification only for the linear-friction model; another dissipative construction is bounded if it meets the conditions of Remark 5.1(ii). Levitated-mechanics data exclude $T_\beta < 10^{-13}$ K for $R_0 < 1$ μm and $T_\beta < 6\times10^{-12}$ K for $R_0 < 10^{-8}$ m [15]; the pulsar bound is stronger by 3×10^{29} and 5.5×10^{27}, respectively. The second range contains the X-ray floor of Section 6 and the point $R_0 = 10^{-15}$ m discussed next, so the improvement is more than 5×10^{27}. An earlier levitated-magnet test bounds the related collisional model [58], not T_β of this one. The pulsar bound excludes the parameter point $R_0 = 10^{-15}$ m, $T_\beta \approx 3$ K (the temperature of the cosmic microwave background) discussed in Ref. [15] by 10^{16}. At $T_\beta > 3\times10^{16}$ K the dissipative model is plain Diósi–Penrose for every laboratory observable, so the X-ray bounds of Section 6 apply to it unchanged.

Classical-channel and classical–quantum gravity. The models of Kafri, Taylor and Milburn [59] and of Tilloy and Diósi [19] treat the Newtonian field as classical and sourced by a measurement record; the feedback formulation of Ref. [60] is of the same kind. Without friction the response of the potential to the signal is instantaneous and real, $\chi'' = 0$, and there is no drag. Dissipative generalizations that thermalize to a finite effective temperature exist [25], but Corollary 5.3 does not bound them as published. In the dissipative Tilloy–Diósi model the friction acts on the deviation of each body's momentum from its classical trajectory, so the bath in effect moves with each body and the mean orbit is not damped; this is the co-moving escape at the end of Section 5.4. The dissipative Kafri–Taylor–Milburn model is formulated for two trapped oscillators and does not describe an orbit. A variant whose friction acted linearly on the full momentum would have Diósi–Penrose noise at $C = 1$, would fall under Theorem 5.1 through the Einstein relation, and would require $T_{\mathrm{eff}} > 3.3\times10^{16}$ K; we know of no published model of this kind. The absence of drag also holds for the Newtonian limit of classical–quantum gravity [61, 62], where the potential is classical and diffusive and friction terms are possible but not included; a friction term would set a temperature scale and break Lorentz invariance [21]. A frictionless classical–quantum model sits at $T_{\mathrm{eff}} = \infty$: the drag vanishes, and the trade-off of that framework between decoherence and diffusion [21] bounds the diffusion of the potential, not a temperature.

5.4 The escapes: no back-action, or a scale-dependent bath

Corollary 5.3 leaves open the limit $T_{\rm eff} \to \infty$ at fixed noise. In that limit $\chi'' \to 0$ while S stays finite, so the drag vanishes. It is noise without back-action: a classical potential field acting on matter, whose reduced dynamics is the random-unitary Diósi–Penrose master equation with Hermitian Lindblad operators [2]; in the classical-channel construction it is the floor set by the principle of least decoherence [20]. For a bath with a finite-dimensional or type-I algebra the limit is its tracial state; a quantum field algebra of type III has no tracial state, and only the limit, or a classical noise field, realizes it. Classical stochastic gravitational fields acting on matter belong to this class [63]. If $T_{\rm eff}$ were the thermodynamic temperature of ordinary degrees of freedom, a bath at the bound would carry an energy density of order $a_{\rm rad}T^4 \sim 10^{51}\ \mathrm{J\,m^{-3}}$, with $a_{\rm rad}$ the radiation constant; the survivor is therefore a non-thermodynamic noise field. Its non-reciprocal noise heats each pulsar's centre-of-mass motion at $3\gamma k_{\rm B}T_{\rm eff} = 2\pi C\hbar G\rho_{k_*} = 8\times10^{-27}$ W at $C = 1$, independent of $T_{\rm eff}$, so the double pulsar does not constrain it. Noise with and without back-action are thus not two kinds of model but the two ends of one family, parametrized by $T_{\rm eff}$, and the pulsars allow only $T_{\rm eff}/C > 3.3\times10^{16}$ K. This end of the family, with every bath above that temperature, is escape (E1).

It is not the only escape. A bath at ordinary temperature can still hide where its kilometre-scale behaviour differs from its laboratory-scale behaviour:

(E2) *an infrared cut:* the kernel departs from $1/k^2$ below some $k_{\rm IR} \gg 1/R_{\rm NS}$. The coherent drag then comes only from the star's density structure above $k_{\rm IR}$. For a sharp-edged uniform sphere that fraction is $0.955/(k_{\rm IR}R_{\rm NS})$, 6.8×10^{-7} at $k_{\rm IR} = 100\ \mathrm{m^{-1}}$; a smooth envelope lowers it further, and we have not modelled the crust. The granular part does not rescue the bound: at the X-ray floor $R_0 = 4.5\ \text{Å}\, C^{1/3}$ of Section 6 the matter's structure factor at $k \sim 1/R_0$ is of order 10^{-6}, so it gives only $T_{\rm eff} \gtrsim 10^{-7}\, C^{-1/3}$ K, and only if the bath is white up to $c_1/R_0 \sim 10^{17}\ \mathrm{s^{-1}}$. An infrared cut leaves no useful bound;

(E3) *a slow bath:* a correlation time longer than $R_{\rm NS}/(10v_{\rm max}) = 1.4$ ms at kilometre wavelengths violates (W) and suppresses the drag, while the bath can still be white at laboratory k;

(E4) *a scale-dependent temperature:* $T_{\rm eff}(k)$ finite at laboratory k but above $3.3\times10^{16}\, C$ K at $k \lesssim 10/R_{\rm NS}$;

(E5) *active modes:* a non-KMS state with $\omega\chi'' < 0$ at $k > k_*$ violates (P) and can cancel the drag from $k < k_*$.

A bath that co-moves with each body (non-linear entrainment, violating (L) and (S) locally) also escapes, at the price of having no rest frame. These escapes are a class, not a corner: any bath whose correlation length or temperature differs between 1 μm and 1 km passes.

6 The survivor: Diósi's model, and the region the data allow

The limit $T_{\rm eff} = \infty$ needs a precise statement before it can be compared with data: which density the noise couples to, and how the density is smeared.

Definition 6.1 (Surviving rate clause)**.** The full mass density of matter, nuclei and electrons, taken as the number-conserving (non-relativistic) density $\hat{O} = \sum_a m_a\psi_a^\dagger\psi_a$, so that $\hat{O}\,\Omega = 0$ on

the vacuum, and convolved with a normalized Gaussian $\propto e^{-r^2/2R_0^2}$, sees classical, symmetric, white potential noise with the correlation (7) and does not act back on it ($T_{\rm eff} = \infty$). The master equation is Diósi's with coefficient C, and $\Gamma = CE_{\rm G}/\hbar$ with $E_{\rm G}$ evaluated on the smeared density.

Here ψ_a is the field operator of the particle species a, of mass m_a. This is Diósi's original model with a Gaussian cutoff [1, 2], not a new model; the number-conserving density is the one it uses, and it leaves the vacuum unheated in every noise realization. Densities with pair terms are excluded: white noise coupled to them excites the vacuum without bound. What the preceding sections add is the bound on the model's dissipative relatives. There is no friction and no detailed-balance factor, so every bound on Diósi–Penrose noise applies directly. Minimal-noise results for non-quantized Newtonian gravity bound from the other side what such a channel must add [64, 65]. Two parameters remain, C and R_0. No argument here fixes C; taking it of order one is a naturalness assumption, not an outcome.

Why a density smearing. The smearing must give the momentum diffusion a finite k^2 moment. Smearing the full density by $e^{-k^2R_0^2/2}$ per factor gives the kernel $(4\pi/k^2)\,e^{-k^2R_0^2}$ and, for any constituent of mass m, nucleus or electron,

$$\frac{d\langle p^2\rangle}{dt} = 4\pi C\hbar G\, m^2 \int \frac{d^3k}{(2\pi)^3}\, e^{-k^2R_0^2} = \frac{C\hbar G\, m^2}{2\sqrt{\pi}\, R_0^3}, \qquad \frac{P}{M} = \frac{1}{2m^2}\frac{d\langle p^2\rangle}{dt} = \frac{CG\hbar}{4\sqrt{\pi}\, R_0^3}, \tag{20}$$

a heating rate per unit mass independent of m. A cutoff of the kernel instead, $(1 - e^{-r/R_0})/r$ with Fourier transform $4\pi/[k^2(1 + k^2R_0^2)]$, falls only as k^{-4}; for constituents of size r_N its k^2 moment is $2R_0/r_N$ times the Gaussian one and diverges for point electrons. The same divergence appears if only nuclei are smeared. We therefore smear the full mass density with a Gaussian, the form used in the published bounds.

Converting the bounds. To leading order, spontaneous radiation and heating are both proportional to C/R_0^3. A lower bound $R_{\rm pub}$ published at normalization $C_{\rm pub}$ becomes

$$R_0 > R_{\rm pub}\,(C/C_{\rm pub})^{1/3}. \tag{21}$$

Table 1 lists the bounds at $C = 1$; Appendix B derives each $C_{\rm pub}$ from the cited paper's own formula. As an independent check, Eq. (20) reproduces the heating figures of Ref. [66], and the XENONnT statement that it improves on MAJORANA by a factor of five [67] holds only for the corrected MAJORANA value [68] read at $C = 8\pi$, where the ratio is 5.2. The LISA Pathfinder row is the analysis of Ref. [69], 285.5 fm; the earlier analysis of the 2016 data gave 40.1 fm [70].

Because every bound scales in the same way with C, one search sets the floor on R_0 at every C, and because more smearing means less noise, no null result sets a ceiling:

Proposition 6.2 (Allowed region)**.** *The surviving rate clause is consistent with the data of Table 1 if and only if*

$$R_0 > 4.5 \times 10^{-10}\ \text{m}\ C^{1/3}, \tag{22}$$

set by XENONnT at every C at which Eq. (21) holds. No published noise observable bounds R_0 from above.

Table 1. *Published lower bounds on R_0 (95% C.L. for MAJORANA and XENONnT, 0.95 Bayesian probability for Gran Sasso; the heating, force-noise and neutron-star rows are the authors' stated bounds, without a confidence level), converted to $C = 1$ by Eq. (21). The Gran Sasso, MAJORANA and XENONnT searches look for spontaneous X- and γ-ray emission from germanium or xenon; the others bound heating or force noise.*

Search	published R_0 (m)	C_{pub}	R_0 at $C = 1$ (m)
XENONnT, 1–140 keV, 95% [67]	4.5×10^{-10}	1	4.5×10^{-10}
MAJORANA, 19–100 keV, 95% [27, 68]	2.54×10^{-10}	8π	8.7×10^{-11}
Gran Sasso, 1–3.8 MeV, $P = 0.95$ [26]	0.54×10^{-10}	8π	1.8×10^{-11}
Cryostat heat leak, 10 pW/kg [66]	4.6×10^{-12}	1	4.6×10^{-12}
Neptune internal heat, 20 pW/kg [66]	3.7×10^{-12}	1	3.7×10^{-12}
LISA Pathfinder force noise [69]	2.9×10^{-13}	1	2.9×10^{-13}
Neutron-star heating [22]	$\gtrsim 10^{-13}$	1/2	$\gtrsim 1.3 \times 10^{-13}$

Proof. At leading order all bounds scale as Eq. (21), so their order does not depend on C, and XENONnT is the largest at $C = 1$, by a factor 5.2 in R_0 over MAJORANA. For the upper side: the kernel $e^{-k^2R_0^2}/k^2$ is pointwise non-increasing in R_0, so every positive functional of the noise, E_{G}, heating, force noise and emission, is non-increasing in R_0. A null result for such an observable bounds R_0 only from below. □

The data therefore constrain only the combination C/R_0^3, and only from one side. At the edge of the region the heating is $P/M = G\hbar/[4\sqrt{\pi}\,(4.5\ \text{Å})^3] = 1.1 \times 10^{-17}\ \mathrm{W\,kg^{-1}}$, independent of C and six orders below the cryostat floor. The leading power law R_0^{-3} is used down to atomic scales; XENONnT itself rescales a fixed spectral shape by R_0^{-3} from its best fit at 9.1 Å, and a change in the atomic emission rate moves Eq. (22) by the cube root of that change. For $C \ll 1$, where Eq. (22) falls below atomic size, the conversion is not validated. In particular, Penrose's own suggestion of a nuclear-size smearing, $R_0 \approx 10^{-15}$ m, is excluded at leading order unless $C < 1.1 \times 10^{-17}$. Two candidate upper bounds do not apply. Inverse-square-law tests constrain a smearing of the Newtonian *force*; here the mean field is that of general relativity and only the noise is smeared. A requirement that the noise also make macroscopic bodies classical within a fixed time [71] would cap R_0; the survivor is a decoherence clause, not a solution of the measurement problem, and we do not impose it. Figure 1 shows the region.

7 The prediction: a visibility measurement gives C

To leading order every non-interferometric bound of Table 1 measures C/R_0^3 and cannot separate C from the free R_0. An interferometer can, because E_{G} is dominated by wavelengths of the size of the body, which a smearing $R_0 \ll a$ does not affect. The proposition makes this quantitative for a sphere.

Proposition 7.1 (Rate at all separations)**.** *For a uniform sphere of mass M, radius a and density ρ, with interatomic spacing $\lesssim R_0 \ll a$, in a superposition with separation d,*

$$\Gamma = \frac{C}{\hbar}\,\frac{GM^2}{a}\,f\Big(\frac{d}{a}\Big)\,[1+\varepsilon], \tag{23}$$

with f of Eq. (8) and $|\varepsilon| \le (3/\sqrt{\pi})R_0/a$ at every d. To leading order, $\varepsilon = -\frac{9}{4}\frac{R_0^2}{a^2}\frac{s(1-s^2/12)}{f(s)}$ for $R_0 \ll d < 2a$ (hence $-\frac{9}{2}R_0^2/(ad)$ for $R_0 \ll d \ll a$), $\varepsilon \to -(3/\sqrt{\pi})R_0/a$ for $d \ll R_0$, and

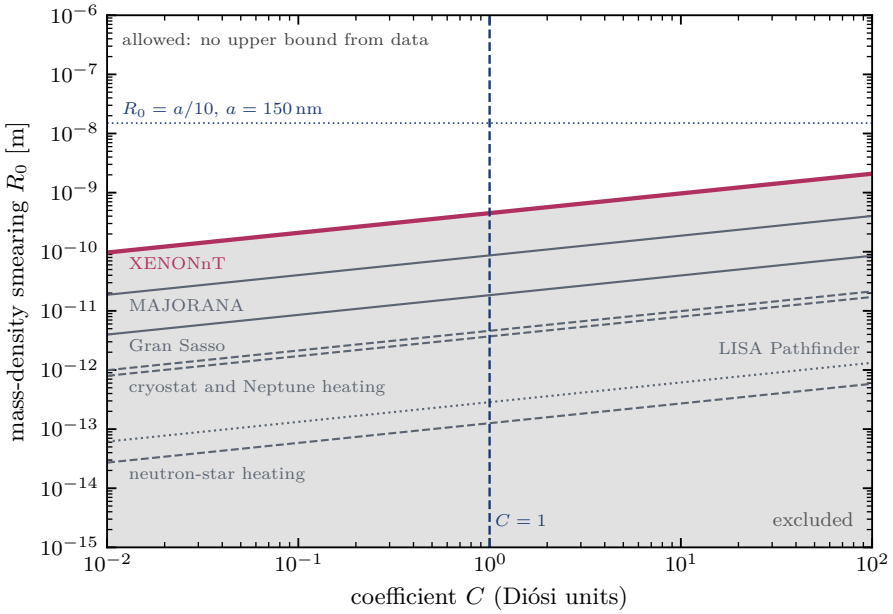


Figure 1. *The (C, R_0) plane of the surviving rate clause. Each sloping line is a published lower bound on R_0 converted with the leading-order rule (21): radiation searches (solid), heating (dashed) and force noise (dotted). XENONnT (thick) binds at every C shown, and the grey region is excluded. Nothing bounds R_0 from above. The vertical dashed line marks Diósi's normalization $C = 1$. Below the horizontal dotted line, $R_0 < a/10$ for a sphere of radius $a = 150$ nm, the rate of Section 7 depends on R_0 by at most 4% for $d \geq 2a$ and 17% at any d.*

$\varepsilon = -3R_0^2/(a^2 f)$ *for* $d \geq 2a$. *For* $d \geq 2a$ *the rate saturates,*

$$\Gamma_{\mathrm{sat}} \equiv \lim_{d\to\infty} \Gamma = \frac{6C}{5\hbar}\, G\Big(\frac{4\pi\rho}{3}\Big)^{1/3} M^{5/3}. \tag{24}$$

To this accuracy the rate does not depend on R_0.

Proof. Equation (8) and Appendix A give f. Smearing multiplies the kernel by $e^{-k^2R_0^2} = 1 - k^2R_0^2 + \ldots$. The $k^2R_0^2$ term is the contact term $-4\pi R_0^2\,\delta^3(\mathbf{x}-\mathbf{y})$, so $\delta E_{\mathrm{G}} = -2\pi G R_0^2 \int(\Delta\rho)^2 d^3x = -2\pi G R_0^2 \rho^2 V_{\triangle}(d)$, with $V_{\triangle}$ the volume of the symmetric difference of the two balls, $2\pi a^2 d(1 - d^2/12a^2)$ for $d < 2a$ and $8\pi a^3/3$ beyond. Dividing by $E_{\mathrm{G}} = GM^2 f/a$ gives the three leading forms. When $d \lesssim R_0$ the density difference is a dipole layer thinner than R_0, the expansion in R_0^2 fails, and the exact correction saturates at $-(3/\sqrt{\pi})R_0/a$; $|\varepsilon|$ grows monotonically as d decreases, which gives the bound at every d. The expansion is not analytic in R_0: the step edges of the balls add terms of order $(R_0/a)^3$, $+(6/\sqrt{\pi})(R_0/a)^3\, GM^2/a$ for disjoint balls. Between disjoint balls the cross term changes only by terms exponentially small in $(d-2a)/R_0$; at contact, $d = 2a$, the smeared tails overlap only within a region of volume of order aR_0^2 around the contact point, so the change is of higher order in R_0/a than the self-term correction. With $a = (3M/4\pi\rho)^{1/3}$ the limit $d \to \infty$ gives Γ_{sat}. □

The proposition means that an interferometer does not see R_0 as long as $R_0 \ll a$: it measures C. Once the spheres no longer overlap the rate is set by the self-energy $\frac{6}{5}GM^2/a$ minus the cross term GM^2/d: $E_{\mathrm{G}}(d)/E_{\mathrm{G}}(\infty) = 1 - 5a/6d$ is 0.583 at $d = 2a$ and within 10% of saturation for $d \geq 8.33\,a$. The degeneracy between C and R_0 reopens only for $R_0 \sim a$. In the allowed region (22) R_0 exceeds the interatomic spacing of about 2.5 Å for $C > 0.17$, so the continuum density applies. Uncorrelated point nuclei would add at most $G\sum_i m_i^2[1/(\sqrt{\pi}R_0) - \mathrm{erf}(d/2R_0)/d]$

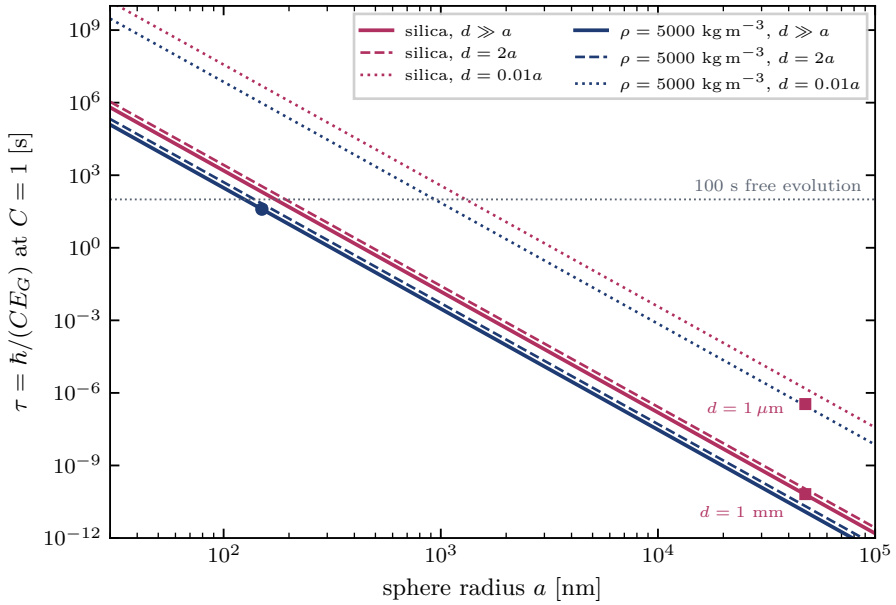


Figure 2. *Decoherence time $\tau = \hbar/(CE_G)$ at $C = 1$ against sphere radius, for silica and for $\rho = 5000$ kg m^{-3}, at $d \gg a$ (solid), $d = 2a$ (dashed) and $d = 0.01\,a$ (dotted), for $R_0 \ll a$, where it depends on R_0 by at most $1.7\,R_0/a$. The squares mark 1 μg of silica at $d = 1$ mm and at $d = 1$ μm, the circle the MAQRO-class sphere at $d \gg a$. The horizontal line is a free-evolution time of 100 s. The time scales as $1/C$.*

to E_G; for silica ($\sum m_i^2/\sum m_i = 21.6$ u) this is at most 4×10^{-3} of E_G at $R_0 = 4.5$ Å and below 10^{-4} for $d \geq 10$ nm.

Benchmarks. For 1 μg of silica ($\rho = 2200$ kg m^{-3}, $a = 47.7$ μm) at $d = 1$ mm, $E_G = 24.2\,GM^2/d$ and the decoherence time is

$$\tau = \frac{\hbar}{CE_G} = \frac{65.4\text{ ps}}{C}. \tag{25}$$

In the overlapping regime the time grows as $1/d^2$, $C\tau = 2a^3\hbar/(GM^2d^2)$ for $d \ll a$, which gives $\tau = 0.343$ μs$/C$ at $d = 1$ μm (0.346 μs$/C$ with the full f). Appendix A lists the values at other separations and the size of the smearing corrections. Proposed nanosphere superpositions have $d \lesssim a$ [72], so the overlapping branch of f is the one they test. A space platform of the MAQRO class [73, 74], with free evolution of order 100 s, reaches $d \sim a$: a sphere of $a = 150$ nm and $\rho = 5000$ kg m^{-3} has $\tau = 39.5$ s$/C$ for $d \gg a$. A silica sphere needs $a \geq 173$ nm $C^{-1/5}$ for $\Gamma \times 100$ s ≥ 1 at $d \gg a$. Figure 2 shows the decoherence time against radius.

What a measurement determines. At any separation $C = \hbar\Gamma_{\text{grav}}/E_G$, with E_G known from M, a and d, where Γ_{grav} is the measured visibility decay rate after the known decoherence channels (residual-gas collisions, black-body emission, scattering and absorption [72, 75], trap and recoil noise) have been subtracted. If the residual is the survivor's and $R_0 \ll a$, one measurement determines C. The saturated regime $d \geq 2a$ needs no knowledge of d; the overlapping regime needs d calibrated, since $\tau \propto d^{-2}$, and in exchange gives the steepest signal. Scanning d across $d \sim a$ over-determines C, since f has no free parameter. A rate below the prediction means either a smaller C or $R_0 \gtrsim a$; the two are separated by the radius dependence, $\Gamma \propto a^5\rho^2$ at fixed C in the saturated regime against a suppression once R_0 approaches a. A null residual excludes

the survivor at the tested (C, R_0).

8 Discussion: what an observed rate would mean

A rate of Diósi–Penrose form observed for an isolated, inertial, equal-mass spatial superposition would not confirm a derivation from quantized linearized gravity or from a thermal medium whose noise extends to kilometre scales. By Proposition 4.1 and Corollary 5.3 it would signal one of the following: noise without back-action, or a bath at $T_{\rm eff}/C > 3.3 \times 10^{16}$ K (E1); a bath that differs at kilometre scales (E2–E5) or co-moves with each body; a horizon; physics outside the hypotheses of Section 4; or non-unitary reduction. If it is the survivor's, its size gives C (Section 7). The survivor itself is not derived here from anything. Closing the escapes E2–E5 needs the high-k structure of neutron stars, which we have not modelled, or bodies resolved at scales between the laboratory and the star.

Emergent-gravity programmes. In programmes where the Einstein equation is an equilibrium condition of diamond entanglement [76, 77], the vacuum gives no Gaussian white floor (Proposition 4.1), and a medium at finite temperature that meets the hypotheses of Theorem 5.1 drags (Corollary 5.3). A rate linear in G in such a programme is therefore an additional postulate of the Diósi type, with C not fixed, unless the medium uses one of the escapes E2–E5.

Appendix A: The energy $E_{\rm G}$ of two uniform spheres

With $\int d^3k\,(2\pi)^{-3}\,4\pi k^{-2} f(\mathbf{k}) = (2/\pi)\int_0^\infty dk\,\langle f\rangle$ for any angle average $\langle f\rangle$, Eq. (1) reads

$$E_{\rm G} = \frac{G}{\pi}\int_0^\infty dk\,\langle|\Delta\tilde\rho_{\mathbf{k}}|^2\rangle, \tag{26}$$

with the angle average (11) and the form factor of a uniform sphere, $\tilde\rho(k) = 3M(\sin ka - ka\cos ka)/(ka)^3$. The proxy response (12) has reorganization energy $\pi^{-1}\int_0^\infty \chi_B''\,\omega^{-1}d\omega$ equal to (26), which fixes its normalization; its small-ω expansion gives the ω^3 law, and $\Lambda_2/\hbar = (G/\pi\hbar c)\int_0^\infty\langle|\Delta\tilde\rho_{\mathbf{k}}|^2\rangle\,k^{-1}dk$ gives the values quoted in Section 4. In position space, with $\iint\rho\rho\,|\mathbf{x}-\mathbf{y}|^{-1} = (6/5)M^2/a$ for each ball (twice the self-energy $\frac{3}{5}GM^2/a$ divided by G) and $\iint\rho_L\rho_R\,|\mathbf{x}-\mathbf{y}|^{-1} = M^2/d$ for disjoint balls (Newton's shell theorem),

$$E_{\rm G} = \frac{G}{2}\Big[2\cdot\frac{6M^2}{5a} - \frac{2M^2}{d}\Big] = GM^2\Big(\frac{6}{5a} - \frac{1}{d}\Big) \qquad (d \ge 2a). \tag{27}$$

For overlapping balls, $s = d/a < 2$, the cross term is the Newtonian potential, at distance d, of the autocorrelation of one ball, $\rho^2\pi(4a + r)(2a - r)^2/12$ for $r < 2a$, which has total mass M^2; the shell theorem then gives

$$E_{\rm G} = \frac{GM^2}{a}\Big(\frac{s^2}{2} - \frac{3s^3}{16} + \frac{s^5}{160}\Big) \xrightarrow{s\to 0} \frac{GM^2d^2}{2a^3}. \tag{28}$$

Both branches agree at $s = 2$, where $E_{\rm G} = 0.7\,GM^2/a$, with first derivative $1/4$, second $-1/4$ and third $3/8$; the fourth jumps by $9/4$.

Numerical values. For the benchmark of Section 7, 1 μg of silica with $a = 47.7$ μm and $R_0 \ll a$, Eq. (23) gives the decoherence time $\tau = \hbar/(CE_{\rm G})$ at separation d as

d	10 nm	100 nm	1 μm	10 μm	$2a$	1 mm
$C\tau$	3.43 ms	34.3 μs	0.346 μs	3.72 ns	108 ps	65.4 ps

and 62.8 ps/C as $d \to \infty$. The smearing correction at $R_0 = 4.5$ Å is $(5/2)(R_0/a)^2 = 2 \times 10^{-10}$ for $d \to \infty$ and 2×10^{-8} at $d = 1$ μm. The sphere of $a = 150$ nm and $\rho = 5000$ kg m^{-3} has $\tau = 283$ s/C at $d = 100$ nm, 67.8 s/C at $d = 2a$ and 39.5 s/C for $d \gg a$.

Appendix B: Normalization of the published bounds

The conversion (21) needs the normalization $C_{\rm pub}$ of each bound in Table 1, measured against Diósi's master equation,

$$\dot{\hat{\rho}} = -\frac{C\,G}{2\hbar} \iint d^3x\, d^3y\, \frac{[\hat{M}(\mathbf{x}), [\hat{M}(\mathbf{y}), \hat{\rho}]]}{|\mathbf{x}-\mathbf{y}|}, \qquad C = 1 \text{ (Diósi)}, \tag{29}$$

with $\hat{M}$ the Gaussian-smeared mass density. Equivalently, the noise is that of Eq. (7) and the energy functional of Eq. (1) carries the prefactor $G/2$.

Penrose normalization, $C_{\rm pub} = 8\pi$. Donadi *et al.* [26] write the master equation with prefactor $4\pi G/\hbar$ in place of $G/(2\hbar)$, a ratio 8π, and their Supplementary Information states that their rates agree with Diósi's apart from that factor. The MAJORANA analysis [27, 68] uses the same convention, which we check from its own emission formula. At $C = 1$ the acceleration $\mathbf{a} = -\nabla\Phi$ of a point charge in the smeared noise is white, with $\langle a_i a_j \rangle = D\,\delta_{ij}\delta(t-s)/3$ and

$$D = \int \frac{d^3k}{(2\pi)^3}\, k^2\, \frac{4\pi\hbar G}{k^2}\, e^{-k^2R_0^2} = \frac{\hbar G}{2\sqrt{\pi}\, R_0^3}. \tag{30}$$

Larmor emission from this acceleration gives, per charge e,

$$\frac{d\Gamma}{dE} = \frac{e^2 D}{6\pi^2\varepsilon_0 c^3 \hbar E} = \frac{1}{12\pi^{5/2}}\, \frac{e^2 G}{\varepsilon_0 c^3 R_0^3 E}. \tag{31}$$

The corresponding coefficient in MAJORANA's rate formula, per Z^2, is $2/(3\pi^{3/2})$, which is 8π times $1/(12\pi^{5/2})$; the factor-of-five statement checked in Section 6 confirms the reading.

Diósi normalization, $C_{\rm pub} = 1$. XENONnT [67] states that it follows the Diósi as opposed to the Penrose convention, which differ by 8π. Vinante and Ulbricht [66] give the heating per particle as $G\hbar m/(4\sqrt{\pi}R_0^3)$, which is Eq. (20) at $C = 1$. For a uniform sphere at large separation, Howl, Penrose and Fuentes write the Penrose time as $5\hbar R/(6GM^2)$ [78], which is $\hbar/E_{\rm G}$ with Eq. (24) at $C = 1$. For the LISA Pathfinder row we take $C_{\rm pub} = 1$; recomputing the bound of Ref. [70] from its force-noise formula and data with that reading reproduces its 40.1 fm. The force-noise rows sit three orders of magnitude below XENONnT, so a factor 8π would not change the binding bound.

Tilloy and Stace, $C_{\mathrm{pub}} = 1/2$**.** Ref. [22] uses the heating coefficient $G\hbar m/(8\sqrt{\pi}r_c^3)$, half of Eq. (20). Its bound is an estimate for the conservative surface temperature of PSR J1840−1419 with unit emissivity, not a confidence limit.

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