hexagon:2611.00391v2

SORTING NETWORKS ON 13 INPUTS NEED AT LEAST 44 COMPARATORS

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Notice. This paper has been merged into the paper *Sorting networks on 13 inputs need 45 comparators*, version 6 [1] (doi:10.5281/zenodo.24523931), which supersedes it. Every result of versions 1 to 6 of this paper is proved there. The values $P(2, N) = \lceil \log_2 F(N) \rceil$ for $3 \le N \le 32$ are Theorem 1.2 of [1]. In Section 1 of [1], the bound $S(13) \ge 44$ follows from that theorem and Harder's $S(11) = 35$, and Theorem 1.1 of [1] improves it to $S(13) \ge 45$; with Juillé's network this gives $S(13) = 45$. Section 14 of [1] proves the twenty lower bounds of Corollary 1.3 of this paper ($13 \le N \le 32$) without using Theorem 1.1 of [1]. Corollary 1.3 of [1] raises each of them by one.

The programs, logs, independent checks and Lean proofs of this paper are archived as [2]. Their README files use the numbering of results and sections shared by versions 5 and 6, and the table below shows where each numbered item appears in [1]. Versions 1 to 6 of this paper stay available on Zenodo; version 6 is doi:10.5281/zenodo.23730883.

in this paper	in [1]
Theorem 1.1 ($S(13) \ge 44$)	Section 1, "Two values in the suffix"; improved to 45 by Theorem 1.1; in Lean, Appendix A.6
Theorem 1.2	Theorem 1.2
Corollary 1.3	Corollary 1.3, which raises each bound by one; its proof in Section 14 also gives the bounds of this paper without Theorem 1.1
Definition 2.1; Lemmas 2.2, 2.3 and 2.4; Proposition 2.5; Table 1	Definition 2.2; Lemmas 2.3, 2.4 and 2.5; Proposition 2.6; Table 1
Section 3, results 3.1 to 3.12	Section 3, the same numbers; Corollary 3.11 writes $\nu(N)$ for $\beta(N)$
Sections 4.1 to 4.4; Proposition 4.1; Lemmas 4.2 to 4.4	the same numbers
Section 5; Theorem 5.1; Table 2	Section 4.5; Theorem 4.5; Table 2
Section 6	Section 4.6 and Appendix B
Table 3 (in Section 6 of version 5, in Appendix A of version 6)	Table 6
Section 7	Appendices A.1, A.2 and A.6
Section 8; Table 4 (version 5 only)	Section 14; Table 4
Appendix A (version 6)	Appendix B

This notice is version 7 of this paper (doi:10.5281/zenodo.24523936; all versions, doi:10.5281/zenodo.23453292). The text is under CC BY 4.0.

References

[1] T. S. Tomar, *Sorting networks on 13 inputs need 45 comparators*, version 6, Zenodo, 2026, doi:10.5281/zenodo.24523931; all versions, doi:10.5281/zenodo.23504358.

[2] T. S. Tomar, *Programs and logs for "Sorting networks on 13 inputs need at least 44 comparators"*, version 6, Zenodo, 2026, doi:10.5281/zenodo.24299105; repository `https://github.com/tejasvi/sortnet13-two-value`, commit `198b4237afe8cffd55c5fb1470912ec83de57d07`.

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