THE HANKEL DETERMINANTS OF THE BAUM–SWEET SEQUENCE ARE 2-AUTOMATIC

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Abstract. Let b be the Baum–Sweet sequence and $H_n = \det(b_{i+j})_{0\le i,j<n}$ its Hankel determinants over $\mathbb{F}_2$. We prove that the sequence $\chi_0(n) = [H_n \neq 0]$ is 2-automatic: $\chi_0(n+1)$ is computed by an explicit automaton with 8 states reading the binary expansion of n, and the minimal automaton of χ_0 reading least significant digits first has 74 states. The same holds for the shifted determinants $\det(b_{i+j+1})$, with the same 8-state graph and 68 states. This contrasts with Mkaouar's theorem that the partial quotients of the Baum–Sweet series are not automatic. On the way we show that $\Theta = tf^2$, a root of Z^3+Z+t^3 in $\mathbb{F}_2((t^{-1}))$, has all its partial quotients in $\{t, t^3, t^3+t\}$, given by an explicit coding of the Mills–Robbins expansion, and that the linear complexity profile of b satisfies $-2 \le 2L(b,N) - N \le 3$ for all N, all six values occurring. The proofs reduce to finite verifications, which an ancillary script performs.

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1. Introduction

The Baum–Sweet sequence $b = (b_n)_{n\ge0}$ over $\mathbb{F}_2$ is defined by $b_0 = 1$ and, for $n \ge 1$, $b_n = 1$ if and only if the binary expansion of n has no block of zeros of odd length; equivalently $b_{2n+1} = b_n$, $b_{4n} = b_n$, $b_{4n+2} = 0$. Its generating series $f = \sum_{n\ge0} b_n t^{-n}$ is the unique root in $\mathbb{F}_2((t^{-1}))$ of

$$tf^3 + f + t = 0.$$

Baum and Sweet [BS76] showed that all partial quotients of f have degree 1 or 2. Mills and Robbins [MR86] gave the expansion explicitly (Section 3), and Mkaouar [Mk95] proved that the sequence of partial quotients is not automatic, although it is the image of a fixed point of a non-uniform substitution on 20 letters.

For $c \ge 0$ let $H_n^{(c)} = \det(b_{c+i+j})_{0\le i,j<n}$ (with $H_0^{(c)} = 1$) and

$$\chi_c(n) = \begin{cases} 1 & \text{if } H_n^{(c)} \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Hankel determinants of automatic sequences have been studied since Allouche, Peyrière, Wen and Wen [APWW98]; Hu and Han [HH19] proved automaticity for the reduced Hankel determinants modulo 2 of a family of ±1 sequences, and Allouche, Han and Niederreiter [AHN20, Section 7] ask whether the Hankel determinants of a d-automatic sequence are d-regular or, reduced modulo some integer, d-automatic. For the Baum–Sweet sequence and reduction modulo 2 (the determinants of the $0,1$ matrices modulo 2 are the determinants over $\mathbb{F}_2$) the answer is positive.

Theorem 1.1. *For $c \in \{0,1\}$ the sequence χ_c is 2-automatic. More precisely, let $\mathcal{A}$ be the automaton with states $0,\dots,7$, initial state 0, and transitions*

state	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
digit 0	*0*	*2*	*4*	*6*	*1*	*7*	*6*	*5*
digit 1	*1*	*3*	*5*	*0*	*4*	*5*	*7*	*6*
output ω_0	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
output ω_1	*1*	*1*	*0*	*1*	*1*	*0*	*1*	*1*

reading the binary expansion of n from the most significant digit. Then $\chi_c(n+1)$ is the output ω_c of the state reached, for every $n \ge 0$; and $\chi_c(0) = 1$. The minimal automaton of χ_0 (resp.

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χ_1) reading the most significant digit first has 15 *(resp.* 8*) states, and the one reading the least significant digit first has* 74 *(resp.* 68*) states.*

Let $\Theta = tf^2$. From $tf^3 + f = t$ one gets $\Theta^3 + \Theta = t(tf^3 + f)^2 = t^3$, so Θ is a root of $Z^3 + Z + t^3$ in $\mathbb{F}_2((t^{-1}))$, and $f/t = 1/(1 + \Theta)$. Since $\Theta = t^3/(1 + \Theta^2)$, Θ is hyperquadratic in the sense of Lasjaunias [La00]. As in Section 3 we write $X = t$ when t is used as a partial quotient.

Theorem 1.2. *$\Theta = [X; a_1, a_2, \dots]$ with every $a_i \in \{X, X^3, X^3 + X\}$. Explicitly, cut the Mills–Robbins word Γ_∞ of Section 3 into blocks consisting of one letter of degree* 1 *or of a letter of degree* 2 *followed by a letter of degree* 1*; then $a_i = \psi(i\text{-th block})$, where ψ maps a block of degree* 1 *to X, a block beginning with X^2 to X^3, and a block beginning with $X^2 + 1$ to $X^3 + X$.*

Corollary 1.3. *Let $L(b, N)$ be the N-th linear complexity of b over $\mathbb{F}_2$. For every $N \geq 1$, $2L(b, N) - N \in \{-2, -1, 0, 1, 2, 3\}$, and each of these six values occurs.*

This refines the bounds $N/3 \leq L(b, N) \leq (2N + 1)/3$ of Mérai and Winterhof [MW18, Corollary 3]. Some bound $|2L(b, N) - N| \leq C$ follows from the boundedness of the partial quotients of f/t; the point is the exact window and the fact that it is attained. Allouche, Han and Niederreiter [AHN20, Section 7] ask for which automatic sequences the profile can be computed exactly, and Theorem 1.1 gives it by an automaton. We also obtain the transfer relation $\chi_0(n) = \chi_1(n)\chi_1(n - 1)$ for $n \geq 1$ (Proposition 5.2).

The proofs reduce every infinite statement to a finite one: an induction on the Mills–Robbins recursion closed by finitely many boundary words (Lemma 4.1), a transducer with finitely many configurations (Theorem 5.1), and a self-similarity of the dyadic blocks (Proposition 3.2) that a finite automaton is shown to satisfy (Section 6). The finite verifications are carried out by the ancillary script, described in Section 8.

2. Continued fractions and Hankel determinants

For $\alpha = \sum_{n \geq 1} s_{n-1} t^{-n} \in \mathbb{F}_2((t^{-1}))$ with convergents P_j/Q_j, one has $\det(s_{i+j})_{0 \leq i,j < n} \neq 0$ if and only if $n \in \{\deg Q_j : j \geq 0\}$; this is the classical link between Hankel determinants and the Padé table, see e.g. [AHN20]. We apply it to

$$\alpha_0 = \sum_{n \geq 0} b_n t^{-n-1} = f/t, \qquad \alpha_1 = \sum_{n \geq 1} b_n t^{-n} = f + 1,$$

so that χ_0 and χ_1 are the indicator functions of $\Sigma_0 = \{\deg Q_j(\alpha_0)\}$ and $\Sigma_1 = \{\deg Q_j(\alpha_1)\}$. If $\alpha = [0; q_1, q_2, \dots]$ then $\deg Q_j = \deg q_1 + \cdots + \deg q_j$. By Niederreiter's theorem [Ni88, Theorem 1], the linear complexity of $s_0, \dots, s_{N-1}$ is $\deg Q_j$, where j is determined by $\deg Q_{j-1} + \deg Q_j \leq N < \deg Q_j + \deg Q_{j+1}$.

3. The Mills–Robbins expansion and its dyadic structure

Write $T = \{X, X + 1, X^2, X^2 + 1\}$ (with $X = t$), let τ exchange $X \leftrightarrow X + 1$ and $X^2 \leftrightarrow X^2 + 1$, and for a word w let $\widetilde{w}$ be its reversal. Put $\Gamma_0 = X + 1$, $\Gamma_1 = X + 1, X^2 + 1$, $\Gamma_2 = X + 1, X^2 + 1, X, X^2, X + 1$ and, for $n \geq 3$,

$$\Gamma_n = \Gamma_{n-1}\, \tau'(\widetilde{\Gamma}_{n-3})\, \Lambda_n\, \Gamma_{n-3},$$

where τ' applies τ to the first letter only, and

$$\Lambda_n = X, X{+}1, X^2, (X, X^2)^k, X{+}1, X, \qquad k = (2^{n-1} - 4)/3,\ n \text{ odd},$$
$$\Lambda_n = X{+}1, X^2{+}1, (X, X^2)^k, X, X^2{+}1, X{+}1, \qquad k = (2^{n-1} - 5)/3,\ n \text{ even}.$$

Each Γ_n is a prefix of Γ_{n+1}; let Γ_∞ be the limit. Mills and Robbins proved [MR86], see [Mk95, p. 363], that

$$f = [1; X, \Gamma_\infty].$$

For a word w over T let $D(w)$ be the sum of the degrees of its letters, and let φ be one of the codings φ_1, φ_0 defined by $\varphi_e(c) = 1$ if $\deg c = 1$ and $\varphi_1(c) = 01$, $\varphi_0(c) = 00$ if $\deg c = 2$. Both preserve lengths: $|\varphi(w)| = D(w)$.

Lemma 3.1. *$D(\Gamma_n) = 2^{n+1} - 1$ for all $n \geq 0$; for $n \geq 3$, $D(\Lambda_n) = 2^{n-1} + 2$ and Λ_n is a palindrome.*

Proof. Λ_n has $(2^n+7)/3$ letters, of which $(2^{n-1}-1)/3$ have degree 2, if n is odd, and $(2^n+5)/3$ letters, of which $(2^{n-1}+1)/3$ have degree 2, if n is even; in both cases $D(\Lambda_n) = 2^{n-1}+2$. Since τ and reversal preserve degrees, $D(\Gamma_n) = D(\Gamma_{n-1}) + 2D(\Gamma_{n-3}) + 2^{n-1} + 2$, and induction from $D(\Gamma_0) = 1$, $D(\Gamma_1) = 3$, $D(\Gamma_2) = 7$ gives the claim. □

Put $A_n = \varphi(\Gamma_n)$, $B_n = \varphi(\widetilde{\Gamma}_n)$, $L_n = \varphi(\Lambda_n)$ and, for $m \geq 5$,

$$G_m = B_{m-3}\, L_m\, A_{m-3}.$$

By the recursion, the palindromy of Λ_n and $\deg \tau(c) = \deg c$, we have $A_n = A_{n-1}G_n$ and $B_n = G_n B_{n-1}$ for $n \geq 5$, and $|G_m| = 2^m$ by Lemma 3.1.

Proposition 3.2. *Let $\omega = 1\,\varphi(\Gamma_\infty)$ (an infinite word indexed from 0). Then, for every $m \geq 5$, $\omega[2^m, 2^{m+1}) = G_m$; moreover $\omega[1, 2^{n+1}) = A_n$ for all n. Consequently, for $m \geq 8$, $q = 2^{m-2}$ and $0 \leq x < 2^m$:*

(R1) *if $x \geq 3q+1$, then $\omega(2^m + x) = \omega(x - 3q)$;*
(R2) *if $q - 2^{j+1} \leq x < q - 2^j$ with $5 \leq j \leq m-3$, then $\omega(2^m + x) = \omega(2^j + x - q + 2^{j+1})$;*
(R3) *if $q - 32 \leq x \leq q - 2$, then $\omega(2^m + x) = B_4[x - q + 32]$;*
(R4) *if $q - 1 \leq x \leq 3q$, then $\omega(2^m + x) = L_m[x - q + 1]$.*

Proof. $\omega = 1A_4G_5G_6\cdots$ with $|1A_4| = 32$, and $|G_m| = 2^m$; this gives the first two assertions. In $G_m = B_{m-3}L_mA_{m-3}$ the three factors have lengths $q-1$, $2q+2$, $q-1$, which gives (R4), and (R1) since $A_{m-3}[i] = \omega(i+1)$. For $m \geq 8$, $B_{m-3} = G_{m-3}G_{m-4}\cdots G_5B_4$, in which G_j occupies the positions $[q - 2^{j+1}, q - 2^j)$ and B_4 the positions $[q-32, q-1)$; with $G_j = \omega[2^j, 2^{j+1})$ this gives (R2) and (R3). □

In binary, with $n = 2^m + x$, the four cases read: (R1) $n = 111y$ with $y \neq 0$, and then $\omega(n) = \omega(y)$; (R2) $n = 100\,1^a\,0\,z$ with $|z| = j \geq 5$, and then $\omega(n) = \omega(1z)$; (R3) $n = 100\,1^{m-7}w$ with $|w| = 5$, $w \neq 11111$, and then $\omega(n) = B_4[\mathrm{val}(w)]$; (R4) all remaining n, namely 1001^{m-2}, $101u$, $110u$ and 1110^{m-2}. In (R4), L_m is explicit: it is φ of the palindrome Λ_m, whose middle part $\varphi((X, X^2)^k)$ is periodic of period 3.

4. The factors of length three

Lemma 4.1. *The set F of factors of length 3 of Γ_∞ has 20 elements and is closed under reversal. In Γ_∞ no two letters of degree 2 are adjacent.*

Proof. Let F be the set of factors of length 3 of Γ_{16}; it has 20 elements and is closed under reversal. We show $\mathrm{Fac}_3(\Gamma_n) \subseteq F$ for all $n \geq 3$ by induction; for $3 \leq n \leq 8$ this is a direct check. Let $n \geq 9$. A factor of length 3 of $\Gamma_n = \Gamma_{n-1}\,\tau'(\widetilde{\Gamma}_{n-3})\,\Lambda_n\,\Gamma_{n-3}$ either lies in one of the four blocks or meets a junction. Factors of Γ_{n-1} and Γ_{n-3} lie in F by induction, factors of $\widetilde{\Gamma}_{n-3}$ by closure under reversal, and factors of Λ_n by inspection: for $n \geq 5$, $\mathrm{Fac}_3(\Lambda_n)$ depends only on the parity of n, and it is contained in F. The remaining factors are the factor of $\tau'(\widetilde{\Gamma}_{n-3})$ containing its first letter and the six factors meeting the three junctions. They depend only on the first two letters of Γ_{n-3}, the last three letters of Γ_{n-1} and of Γ_{n-3}, and the first two and last two letters of Λ_n. The first two letters of Γ_k are $X+1, X^2+1$ for all $k \geq 1$; the last three letters of Γ_k for $k \geq 6$ are those of Γ_{k-3}, hence depend only on k mod 3; the ends of Λ_n depend only on the parity of n. So these seven factors depend only on n mod 6, and the six cases are checked to lie in F. The last statement is read off F. □

5. A transducer for t/f

For a matrix $M = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ over $\mathbb{F}_2[t]$ and a series y write $M(y) = (py+q)/(ry+s)$. To compute the continued fraction of $z = t/f$ from $f = [1; X, \Gamma_\infty]$ we keep a matrix M with $z = M(y)$, where y is the remaining tail of f. Reading a partial quotient a of f replaces M by

$M\begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix}$; emitting a partial quotient κ of z replaces M by $\begin{pmatrix} 0 & 1 \\ 1 & \kappa \end{pmatrix} M$. Both preserve $\det M$ up to sign, hence $\det M = t$ throughout (we start from $M = \begin{pmatrix} 0 & t \\ 1 & 0 \end{pmatrix}$).

If $y = a + 1/y'$ with $\deg a \geq 1$, $\deg y' \geq 1$, then $M(y) - p/r = \det M/(r(ry+s))$, and $\deg(ry+s) = \deg(ra+s)$ as soon as $\deg(ra+s) \geq \deg r$. Hence, if $\deg(ra+s) \geq \deg r$ and $\deg r + \deg(ra+s) > \deg \det M$ for every letter a that can follow, the polynomial part κ of p/r is the polynomial part of z, and if $\deg \kappa \geq 1$ it is a correct partial quotient of z: we *emit* it. We emit greedily as long as this holds for every letter allowed by F after the last two letters read.

Theorem 5.1. *Run the transducer on $1, X, \Gamma_\infty$. The triples (matrix, last two letters read, $E-D$), where D is the degree read in Γ_∞ and E the degree emitted, form a set of 12 configurations closed under reading every letter allowed by F. In each of them every emission is certain, and: reading a letter of degree 1 emits exactly one partial quotient, equal to X, X^3 or X^3+X according as the previous letter has degree 1, is X^2, or is X^2+1, after which $E = D+1$; reading a letter of degree 2 emits nothing, after which $E = D-1$. The first emissions are $X+1$ (after reading X) and X (after reading the first letter $X+1$ of Γ_∞).*

Proof. A finite computation over the 12 configurations and the letters allowed by F (Lemma 4.1), performed by the script. □

Proof of Theorem 1.2. By Theorem 5.1, $t/f = [X+1; a_1, a_2, \dots]$ with $a_i = \psi(i\text{-th block})$, and $\Theta = t/f + 1 = [X; a_1, a_2, \dots]$. □

Proposition 5.2. *$\chi_1 = 11\,\varphi_1(\Gamma_\infty)$ and $\chi_0 = 11\,\varphi_0(\Gamma_\infty)$. Consequently $\chi_0(n) = \chi_1(n)\chi_1(n-1)$ for $n \geq 1$.*

Proof. $\alpha_1 = f + 1 = [0; X, \Gamma_\infty]$, so Σ_1 consists of 0, 1 and the numbers $1 + D_i$, where D_i is the total degree of the first i letters of Γ_∞; this is $\chi_1 = 11\varphi_1(\Gamma_\infty)$. Next $\alpha_0 = f/t = [0; t/f]$, so Σ_0 consists of 0 and the cumulative degrees of the partial quotients of t/f. By Theorem 5.1 these are 1 and the numbers D_i+1 for which the i-th letter of Γ_∞ has degree 1; this is $\chi_0 = 11\varphi_0(\Gamma_\infty)$. For the last statement: $\chi_0(n) = 1$ means that position n ends a letter of degree 1, $\chi_1(n) = 1$ that it ends a letter; a letter of degree 2 is coded 01 by φ_1, so n ends a letter of degree 1 if and only if both n and $n-1$ end letters. □

Remark 5.3. Equivalently, $\Sigma_1 = \Sigma_0 \cup (\Sigma_0 - 1)$ (restricted to $n \geq 0$). One inclusion holds for every sequence: by the Desnanot–Jacobi identity $H^{(0)}_{n+1}H^{(2)}_{n-1} = H^{(0)}_n H^{(2)}_n - (H^{(1)}_n)^2$, if $H^{(0)}_n = H^{(0)}_{n+1} = 0$ then $H^{(1)}_n = 0$, that is $\Sigma_1 \subseteq \Sigma_0 \cup (\Sigma_0 - 1)$. The reverse inclusion is specific: it fails for most random binary sequences.

6. Proof of Theorem 1.1

By Proposition 5.2, $\chi_c(n+1) = \omega(n)$ with $\omega = 1\varphi_c(\Gamma_\infty)$ for $n \geq 0$. The word ω is determined by its first 256 letters and the rules (R1)–(R4) of Proposition 3.2, since each rule expresses $\omega(n)$, $n \geq 256$, through a smaller index or an explicit word. It therefore suffices to show that the output $\widehat{\omega}(n)$ of $\mathcal{A}$ on the binary expansion of n satisfies the same rules and agrees with ω on $[0, 256)$. Since $\delta(0,0) = 0$, leading zeros do not matter. Each rule is a statement about all words of a regular shape, and is checked in a finite product automaton:

- (R1): for all y with $|y| \geq 6$, $y \neq 0$, the states reached from $\delta(0, 111)$ and from 0 by reading y have the same output (reachability in $\mathcal{A} \times \mathcal{A} \times \{\text{length} \leq 6\} \times \{y = 0?\}$);
- (R2): the same for the states reached from $\delta(0, 1001^a 0)$, $a \geq 0$ (a finite set of states), and from $\delta(0,1)$, by words of length at least 5;
- (R3): $\widehat{\omega}(1001^c w) = B_4[w]$ for $c \geq 1$ and the 31 words w;
- (R4): $L_m[i]$ depends only on the parity of m, on i mod 3, and on whether i is one of the first four or last four positions; in the regions $101u$ and $110u$ the index i is $\mathrm{val}(u) + 1$ and $2^{|u|} + \mathrm{val}(u) + 1$, so the expected value is computed by a small automaton tracking $\mathrm{val}(u)$ mod 3, the parity of $|u|$ and whether $u \in 0^*$, 0^*1, 0^*10 (resp. 1^*, 1^*0, 1^*01); the two remaining words 1001^{m-2}, 1110^{m-2} have value 1.

All these checks succeed for $c = 0$ and $c = 1$ (script, Part 5). Hence $\widehat{\omega} = \omega$, and $\chi_c(n+1) = \widehat{\omega}(n)$ for all n.

From $\mathcal{A}$ one builds an automaton $\mathcal{B}_c$ for $n \mapsto \chi_c(n) = \widehat{\omega}(n-1)$ whose states are the pairs (state of $\mathcal{A}$ on n, state of $\mathcal{A}$ on $n-1$), plus an initial state for $n = 0$: reading a digit updates both, since $2n+1-1 = 2n$ and $2n-1 = 2(n-1)+1$. All automata here read the most significant digit first and accept leading zeros. Minimising $\mathcal{B}_c$ gives 15 and 8 states. For the other reading direction, the minimal automaton reading least significant digits first has one state per element of the 2-kernel $\{n \mapsto \chi_c(2^k n + r) : k \geq 0,\ 0 \leq r < 2^k\}$ (by the Myhill–Nerode theorem; see [AS03, Section 6.6]). The kernel element of (k, r) is the sequence $n \mapsto o(\delta(\sigma_n, w))$, where σ_n is the state of $\mathcal{B}_c$ on n and w the k digits of r. Every state of $\mathcal{B}_c$ is some σ_n, so two kernel elements are equal if and only if the maps $\sigma \mapsto o(\delta(\sigma, w))$ coincide on the states of $\mathcal{B}_c$. Enumerating these maps over all words w gives 74 and 68. □

7. Proof of Corollary 1.3

By Proposition 5.2 and Theorem 5.1, the consecutive elements of $\Sigma_0 = \{d_0 < d_1 < \dots\}$ differ by the degrees of the partial quotients of t/f, that is by 1 or 3, both occurring. By Niederreiter's theorem, $L = L(b, N) = d_j$ with $d_{j-1} + d_j \leq N < d_j + d_{j+1}$, so $2L - N \leq d_j - d_{j-1} \leq 3$ and $2L - N \geq 1 - (d_{j+1} - d_j) \geq -2$. If $d_j - d_{j-1} = 3$, the values $3, 2, 1, 0$ occur on this interval of N; if $d_{j+1} - d_j = 3$, the values $1, 0, -1, -2$ occur. □

8. Verification

The ancillary file `verify_baum_sweet_hankel.py` (Python 3, no dependency, exact arithmetic over $\mathbb{F}_2$, under a minute) performs all the finite checks used above: Lemma 4.1 (the set F, the base cases and the seven boundary factors of each class $n \bmod 6$); Theorem 5.1 (the 12 configurations, the certainty of every emission, the bookkeeping of $E - D$); and the rules (R1)–(R4) for $\mathcal{A}$, the base table, the minimal automata and the kernels. It also checks the statements against first principles: the Hankel determinants by Gaussian elimination for $n \leq 300$; the continued fractions of α_0 and α_1 by Euclid's algorithm, against Γ_∞ and against the automaton, for $n < 2^{16}$; and the linear complexity by the Berlekamp–Massey algorithm for $N \leq 3000$.

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