A CONTACT AND COEFFICIENT CORRECTION FOR KORENBLUM'S CONSTANT

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Abstract. Retaining an angular deficit in an annular coefficient estimate gives $C_{68} \geq 0.44667$ for Korenblum's constant in the unweighted Bergman space. The flux input is David Niedbala Giraudin's. A contact-moment estimate and an even outer factor retain a correction involving the first two coefficients; a rational measure then satisfies the modified moment conditions. The argument builds on Wang's coefficient approach and Wikström's moment integration. This note is unrefereed, with no independent human review or formalization.

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1. Statement and credit

Use normalized area measure on the unit disk $\mathbb{D}$, so $\| \sum a_k z^k \|_{A^2}^2 = \sum_{k\geq 0} |a_k|^2/(k+1)$. Let C_{68} be the supremum of $c \in (0,1)$ such that

$$|f(z)| \leq |u(z)| \quad (c < |z| < 1), \quad f, u \in A^2(\mathbb{D}) \quad \Longrightarrow \quad \|f\|_{A^2} \leq \|u\|_{A^2}.$$

Theorem 1. *The radius $c = 44667/100000$ is admissible. Thus $C_{68} \geq 0.44667$.*

David Niedbala Giraudin's PR 274 gives the Green-function flux input and the preceding 0.446 bound, with a supplementary certificate at 0.44605 [1]. The flux input is his, not a contribution of this note. The covering-strip coefficient approach continues Pinho's earlier construction; Wang supplies the coefficient-estimate precedent, and Wikström supplies moment integration and the atomic-plus-Lebesgue measure method [2, 3, 4]. The added input is a nonconstant angular deficit converted into a low-coefficient correction. The supplied review supports this distinction: the present measure fails the old uncorrected first-moment condition. No optimality or exhaustive priority is asserted.

2. The coefficient correction

For $w = -\log c$, set

$$A = \pi^2/w, \quad q = e^{-A}, \quad \Psi = \frac{(1+q)^2}{8q}, \quad \sigma(t) = \sin^2 \frac{\pi \log t}{2w}, \quad \kappa = \frac{3}{25},$$

$$D(t) = \frac{\Psi}{2\sigma(t)}, \quad H(t) = \sqrt{1 + D(t)}, \quad g(t) = \frac{2}{1 + H(t)},$$

$$\beta(t) = 1 - g(t), \qquad \epsilon(t) = \frac{7}{100} \frac{D(t)}{H(t)(1+H(t))^2}.$$

At $t = c^2, 1$, define $g = \epsilon = 0$ by continuity. The certificate checks the four scalar hypotheses

$$\operatorname{sech}^2(A/2) \leq 1/3, \qquad 16 \sinh^2(A/4) \leq 2(1 - 8\kappa)\Psi,$$
$$\tanh^2(A/8) \geq (1 + 8\kappa)\pi^2/64, \qquad \operatorname{sech}^2(A/8) \leq (1 - 4\kappa)/2.$$

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Niedbala Giraudin's flux estimate, in curvature -1 hyperbolic distance, implies for a disk-valued annular map ω and two points on radius r that

$$\cosh d(\omega(re^{i\theta}),\omega(re^{i\phi}))-1\le\frac{2\Phi(A(\theta-\phi)/(2\pi))}{\sigma(r^2)},$$

with angles reduced modulo 2π, where

$$\Phi(v)=\left(\operatorname{csch}^2 v+\operatorname{csch}^2(A-v)+2\operatorname{sech}^2(A/2)\right)^{-1}.$$

The four hypotheses give the Fourier majorant

$$\frac{2\Phi(A\theta/(2\pi))}{\Psi}\le\frac{1-\cos\theta}{2}-\kappa(1-\cos 2\theta).$$

Its proof splits $0\le v\le A/2$ into a small arc, a midpoint neighborhood, and the intervening arc, using respectively $\Phi(v)\le\sinh^2 v$, $\Phi(A/2+x)\le(\Psi/2)\operatorname{sech}^2 x$, and the four displayed inequalities. The complete derivation, including flux normalization and the arc comparisons, is supplied in `GREEN_FLUX_PROOF.md` and `ANGULAR_LEMMA.md`, Sections 1–2, in the analytic supplement [5]. Distinct Green-function lifts are used.

Choose the smallest enclosing hyperbolic disk of the image circle, center α, radius ρ. Properness gives existence. First variation and separation give a probability measure on at most three contact directions balancing at zero. Let its domain angles be θ_i, weights λ_i, and $m_j=\sum_i\lambda_i e^{ij\theta_i}$. The hyperbolic cosine rule, averaged over contacts, gives, with $h=\cosh\rho$ and $\zeta(\theta)=\cosh d(\omega(re^{i\theta}),\alpha)$,

$$H^2-\zeta(\theta)^2\ge D\max(S,T(\theta)),$$

$$S=|m_1|^2+2\kappa(1-|m_2|^2),\qquad T(\theta)=\Re(e^{i\theta}\overline{m}_1)+2\kappa(1-\Re(e^{2i\theta}\overline{m}_2)).$$

Indeed the two contact averages give $h^2\le H^2-DS$ and $\zeta^2\le h\zeta\le 1+2D\sum_i\lambda_i L(\theta-\theta_i)$, where $L=2\Phi/\Psi$. The Fourier majorant gives the other deficit DT.

Put $a=|m_1|$, $b=|m_2|$. After an angular shift,

$$\max(S,T(\theta))\ge h_{ab}(\theta):=\max\left(a^2+\frac{6}{25}(1-b^2),\ \frac{6}{25}(1-b)+\frac{12}{25}b\sin^2\theta-a\right).$$

For $a\ge 27/100$ or $b\le 21/25$, $S>7/100$ already. On the remaining rectangle, the scalar certificate proves $\exp(\int\log h_{ab}\,d\theta/(2\pi))\ge 7/100$. The positive, π-periodic weight has an even analytic outer factor C, with $|C|^2=h_{ab}$ and $|C(0)|^2\ge 7/100$. Hence $C_1=0$. Parseval applied to $CU(rz)$ gives

$$\int\max(S,T(\theta))|U(re^{i\theta})|^2\frac{d\theta}{2\pi}\ge\frac{7}{100}(|U_0|^2+r^2|U_1|^2).$$

For $a=0$, the explicit factor of $(252/625)\sin^2\theta$ handles the logarithmic zeros. The small-a proof and boundary limiting argument are given in `SPECTRAL_LEMMA.md` and `CONTACT_SPECTRAL_LEMMA.md`; both are in [5]. Only their scalar inequalities are delegated to computation.

Now write $f=\sum a_k z^k$, $u=\sum b_k z^k$, $x_k=(|a_k|^2-|b_k|^2)_+$, $y_k=(|b_k|^2-|a_k|^2)_+$, $X(t)=\sum x_k t^k$, $Y(t)=\sum y_k t^k$, including $k=0$. The quotient $\omega=f/u$ is holomorphic on the annulus by removal of common zeros. Constant quotients give the norm comparison immediately. Otherwise use $F=f-\alpha u$, $U=u-\overline{\alpha}f$, analytic on the whole disk even if the quotient has inner poles. On the circle, $|F|^2=(1-v)|U|^2$, where $v=2/(1+\zeta)$, and

$$v-g\ge\frac{D\max(S,T)}{H(1+H)^2},\qquad |U_k|^2-|F_k|^2=(1-|\alpha|^2)(|b_k|^2-|a_k|^2).$$

Moreover $|U_k|^2\ge(1-|\alpha|^2)y_k$. The preceding Parseval estimate therefore yields

$$X(t)\le\beta(t)Y(t)-\epsilon(t)(y_0+y_1t). \tag{1}$$

The inner endpoint follows by continuity. The checked inequality $\Psi>6$ implies $H>2$, $g<2/3$, and $0\le\beta-\epsilon\le 1$.

3. Moments, tails, and the finite packet

The exact certificate specifies a probability measure

$$\mu = \sum_{j=1}^{m} w_j \delta_{t_j} + \mathbf{1}_{[R,1]}\, dt, \quad c^2 = t_1 < \cdots < t_m = R < 1, \quad w_j > 0, \quad \sum_j w_j = R.$$

It suffices that

$$\int t^k \, d\mu \geq \frac{1}{k+1} \quad (k \geq 0), \qquad \int \beta(t) t^k \, d\mu \leq \frac{1}{k+1} \quad (k \geq 2),$$
$$\int (\beta - \epsilon) t \, d\mu \leq \frac{1}{2}.$$

The index-zero upper condition is automatic. Integrating (1) and applying Tonelli to its nonnegative coefficient weights gives $\sum x_k/(k+1) \leq \int X \, d\mu \leq \sum y_k/(k+1)$. Bergman Parseval proves the theorem.

For explicit finite conditions, put $b_j(1) = \beta(t_j) - \epsilon(t_j)$, $b_j(k) = \beta(t_j)$ for $k \geq 2$. The checked residuals are

$$\sum_j w_j t_j^k - \frac{R^{k+1}}{k+1} \geq 0, \qquad \frac{R^{k+1}}{k+1} + \int_R^1 g(t) t^k \, dt - \sum_j w_j b_j(k) t_j^k \geq 0.$$

Discarding the continuous-tail ϵ term at $k = 1$ weakens the sufficient condition. On each rational tail cell $[a, b]$, the proof derives $g(t) \geq \ell(1-t)$ using $-\log t \geq (1-t)(1+(1-b)/2)$ and $\sin x \geq x(1-U^2/6)$ for $0 \leq x \leq U < \sqrt{6}$. The resulting polynomial integrals are exact. The first infinite tail follows from $w_m(K_1 + 1) \geq R$. For the second, a retained $R \leq a < b < 1$ satisfies

$$\ell(1-b)(b-a)a^{K_2} > \sum_j w_j t_j^{K_2}.$$

Dividing by a^k, each atomic ratio $(t_j/a)^k$ is nonincreasing; the inequality persists for all $k \geq K_2$ and proves the second tail. The stored cutoffs are $K_1 = 6000$, $K_2 = 1800$.

The precise packet is `contact-spectral` in [5]: `certificate-contact-044667.json`, SHA-256

`aad4a43c5a7a72770dfced63b27466b66b4166dff5fad9ba1d86d8f0d184c83c`,

and `contact-cover.json`. The latter is a rational binary rectangle tree. Reconstruction checks coverage; each leaf uses a lower angular Riemann sum for $\log h_{ab}$. A separate analytic small-parameter estimate covers $0 < a \leq 1/10000$, so no singular endpoint is omitted. `verify_exact.py` and `verify_interval.py` check the scalar hypotheses, rectangle bounds, finite moments, and both tails. The retained Arb and interval receipts pass; a separate AI review also checked the analytic bridge and used another moment minorant. No verifier was rerun to prepare this note.

The certificate citation is pinned to a fixed public commit. The programs check numerical hypotheses, not complex analysis, hyperbolic geometry, or arbitrary prose.

This is an unrefereed preprint. There has been no independent human review and no formalization.

Analysis and proof drafting by AI agents under my direction; I reviewed the results at summary level. AI agents also prepared certificates, checking software, and publication materials. No line-by-line human verification is claimed.

References

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[3] F. Wikström, Moment duality and an improved lower bound for Korenblum's constant, Lemma 3.1 (moment integration); https://arxiv.org/abs/2607.17748.
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