

AN INTEGRAL-STRUCTURE INFLATION IN THE HULL OPERATION OF IUT COROLLARY 3.12

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ABSTRACT. This paper identifies and rigorously quantifies a third-class gap in the passage from Theorem 3.11 to Corollary 3.12 in S. Mochizuki's inter-universal Teichmüller theory (IUT). It occupies the same position as, but is of a different type from, and neither implies nor is implied by, the Scholze–Stix gap (non-commutativity of the j^2 -scaling) and the compatibility gap (9-1) recorded in the LANA project.

The holomorphic hull of IUT III, Remark 3.9.5, is the minimum over the constrained family $\{\lambda \cdot \mathcal{O}_{\mathcal{I}^\mathbf{Q}}\}$: each direct summand of $\mathcal{I}^\mathbf{Q} \cong \bigoplus_\alpha F_\alpha$ is scaled independently. This operation “boxes” the image of the tensor-product integral structure and introduces an index ρ . The main mathematical result of this paper is the exact closed form of this index,

$$\rho = |\mathfrak{d}_K|^{(n-1)d^{n-1}/2}, \quad d = [K : \mathbf{Q}_p],$$

proved as an identity of *positive integers* (not merely a p -adic valuation), where $\mathfrak{d}_K$ is the discriminant of $K/\mathbf{Q}_p$. Its p -adic corollary is $v_p(\rho) = \frac{(n-1)d^{n-1}}{2} v_p(\mathfrak{d}_K)$: the loss equals $\frac{n-1}{2} \times (\text{number of components}) \times (\text{the } p\text{-adic valuation of the local discriminant})$. The formula is uniform for tame and wild ramification, with no extra term; wild ramification only replaces the tame value $(d-1)$ by the actual $v_p(\mathfrak{d}_K)$.

Substituting IUT's own parameters ($n = j + 1$, $\ell^* = (\ell - 1)/2 \asymp h^{1/2}$, $v \in V^{\text{bad}}$), the inflation is doubly exponential: $\log \log \rho = \Theta(\sqrt{h})$, i.e. $\rho = \exp(\exp(\Theta(\sqrt{h})))$. Measured by the crossover scale $h^*(\kappa)$, the conclusion is robust: even if the target bound were as generous as h^{1000} , the $d = 2$ inflation overtakes it at $h \approx 4 \times 10^9$ and the $d = 4$ inflation at $h \approx 9 \times 10^8$.

This paper is *not* a verdict on IUT. What it proves is the narrower statement: even if (9-1) holds, layer-1 inflation alone already suffices to prevent Corollary 3.12 from yielding abc.

inter-universal Teichmüller theory; abc conjecture; holomorphic hull; local tensor products; discriminant; integral-structure mismatch
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0. NOTATION AND CONVENTIONS

- p is a fixed prime; $K/\mathbf{Q}_p$ is a finite *Galois* extension of degree $d = [K : \mathbf{Q}_p]$, $G = \text{Gal}(K/\mathbf{Q}_p)$ (so $|G| = d$).
- $\mathcal{O} = \mathcal{O}_K$; $\mathfrak{d}_K$ is the (field) discriminant, $|\mathfrak{d}_K|$ its absolute value; v_p is the normalized valuation.
- For $n \geq 1$ write $A_n = \bigotimes_{\mathbf{Q}_p}^n K$ (the n -fold tensor product as a $\mathbf{Q}_p$ -algebra) and $\mathcal{O}_n = \bigotimes_{\mathbf{Z}_p}^n \mathcal{O}$.

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- Write $B_n = \bigoplus_{g \in G^{n-1}} K$ with integral lattice $\mathcal{P}_n = \bigoplus_{g \in G^{n-1}} \mathcal{O}$.
- Elements of G^{n-1} are written $g = (g_2, \dots, g_n)$, with $g_1 = \text{id}$ fixed once and for all (normalization).
- Tr_K is the field trace; the traces on A_n and B_n are $\text{Tr}_{A_n}, \text{Tr}_{B_n}$.
- For a $\mathbf{Z}_p$ -lattice Λ , $\text{disc}(\Lambda) := \det(\text{Tr}_A(e_i e_j))$ is its discriminant (in the ambient algebra A).

A basic fact, used repeatedly: if $\Lambda' \subseteq \Lambda$ are $\mathbf{Z}_p$ -lattices of the same rank, then

$$(0.1) \quad \text{disc}(\Lambda') = [\Lambda : \Lambda']^2 \text{disc}(\Lambda).$$

1. INTRODUCTION

1.1. Background. The abc conjecture (Oesterlé–Masser, 1985) asserts that for every $\varepsilon > 0$ there are only finitely many primitive triples (a, b, c) with $a + b = c$, $\gcd(a, b, c) = 1$ and $c > \text{rad}(abc)^{1+\varepsilon}$. It is equivalent to the Szpiro conjecture and implies Fermat’s Last Theorem together with most other famous Diophantine problems.

In 2012 Shinichi Mochizuki released the four papers of inter-universal Teichmüller theory (IUT) [1, 2, 3, 4], claiming a proof of the abc conjecture. For over a decade the verification of the proof has been in a rare state: on the one hand no accepted refutation has been given; on the other hand, several teams of experts have reported steps that they could not close.

1.2. The gaps of the three camps. In the entire IUT argument, the passage from *Theorem 3.11* to *Corollary 3.12* is the unique load-bearing point: the hundreds of pages of construction before it (Hodge theater, Θ -link, log-link, multiradial algorithms) are all “identity machines”; every piece of structural work is cashed out in this final step as a single inequality. Three independent teams have identified a gap at the same position:

Camp	Location of the gap		Type			Status
Scholze–Stix [5]	j^2 -scaling	non-commutativity	diagram changed	not ex-		disputed
LANA (2026-07) [6]	q-pilot	double-construction	“not false, but no proof”	manifestly		undecided
this paper	hull’s boxing	inflation	magnitude (exactly computable)		quantified	

LANA §10 explicitly excludes the Scholze–Stix hexagon diagram (“*our analysis does not give rise to this particular diagram*”). The three gaps occupy the same position (all in 3.11 $\rightarrow$ 3.12), are of different types, and are mutually independent — a fact of decisive importance for the present paper’s orientation: this paper neither depends on nor disproves either of the other two.

1.3. Contributions of this paper.

- (1) **Exact closed form** (Theorem 5.1): $\rho_n = |\mathfrak{d}_K|^{(n-1)d^{n-1}/2}$, an identity of positive integers, with complete proof.

- (2) **Trace-preservation lemma** (Lemma 3.1): $\text{Tr}_{B_n}(\Phi(z)) = \prod_i \text{Tr}_K(z_i)$ — the pivot of the entire proof, and an identity valid for *every* Galois K and *every* n .
- (3) **Integrality/parity corollary** (Corollary 5.3): $(n-1)d^{n-1}v_p(\mathfrak{d}_K)$ is always even, proved by the Hilbert ramification-group formula; an internal consistency check on the closed form.
- (4) **Sparsity of the value set** (§8): $\text{rank}\langle\Theta\rangle \leq \min(\ell^*, [\mathbf{Q}_p(q) : \mathbf{Q}_p]) \leq d$ (the bound is saturated and attainable, not an identity; see B8), while the hull lives in dimension d^n ; and $\text{hull}(\Theta) = \text{hull}(\mathcal{O}_n)$ up to the normalization zero point — hence ρ is a strict lower bound.
- (5) **Robustness** (§10.3): crossover $h^*(\kappa)$ quantifies extreme insensitivity to assumptions on the target magnitude.
- (6) **Direction lock** (§8.4): $n_v = h_v \in \mathbf{Z}_{\geq 1}$ with $n_v \asymp h$ (IUT I Definition 3.1(c) + IUT IV Corollary 2.2) $\implies$ the layer-1 net effect is *permanently* inflationary; the contraction branch is unreachable in IUT's parameter space.
- (7) **Non-Galois treatment** (§14): compositum closed form $\rho_{\text{NG}}^2 = \rho_{\text{G}}^2/p^\Delta$, with $\Delta \leq \text{poly}(h)$ so the discount is asymptotically negligible.

1.4. Methodological statement. All conclusions of this paper carry reproducible numerical verification. §6.4 and Appendix B record in full the *ten errors* caught by the project's self-checks and their corrections — these errors once produced wrong conclusions (including a negative exponent and a directional misjudgment), and they are recorded as found. Two of them (B1, B10) carry methodological lessons, see Appendix B. In particular, §9 records the unique conceivable falsifier of this paper (procession normalization accidentally cancelling the low-rank degeneration) and gives the original-text check: the mechanism does not exist.

2. STANDARD DECOMPOSITION OF LOCAL TENSOR PRODUCTS

2.1. The standard isomorphism Φ . Define the $\mathbf{Q}_p$ -linear map

$$\Phi: A_n = \otimes^n K \longrightarrow B_n = \bigoplus_{g \in G^{n-1}} K$$

$$\Phi(x_1 \otimes \cdots \otimes x_n) := \left(x_1 \cdot \sigma_{g_2}(x_2) \cdots \sigma_{g_n}(x_n) \right)_{g=(g_2, \dots, g_n)}$$

Dimension check: $\dim A_n = d^n$; $\dim B_n = |G|^{n-1} \cdot d = d^{n-1} \cdot d = d^n$. ✓

2.2. Lemma 2.1: Φ is a $\mathbf{Q}_p$ -algebra isomorphism.

Lemma 2.1. Φ is an isomorphism of $\mathbf{Q}_p$ -algebras.

Proof. Multiplicativity. For pure tensors,

$$\begin{aligned} \Phi((x_1 \otimes \cdots \otimes x_n)(y_1 \otimes \cdots \otimes y_n)) &= \Phi(x_1 y_1 \otimes \cdots \otimes x_n y_n) \\ &= \left(\prod_i \sigma_{g_i}(x_i y_i) \right)_g = \left(\prod_i \sigma_{g_i}(x_i) \right)_g \left(\prod_i \sigma_{g_i}(y_i) \right)_g = \Phi(x) \Phi(y) \end{aligned}$$

(with $\sigma_{g_1} = \text{id}$). By bilinearity and linear extension, Φ is an algebra homomorphism.

Surjectivity (standard Galois descent, by induction). For $n = 1$, $A_1 = K = B_1$ and $\Phi = \text{id}$, which is clear.

For $n = 2$: since $K/\mathbf{Q}_p$ is Galois, $K \otimes K$ splits completely over its primitive idempotents into $\bigoplus_G K$. Concretely, the map

$$x \otimes y \longmapsto (x \sigma_g(y))_{g \in G}$$

is the classical isomorphism $K \otimes K \cong \text{Map}(G, K)$ (equivalently, $K \otimes_{\mathbf{Q}_p} K \cong \prod_G K$; see [7, III]). We recall the argument in detail, since the induction step reuses it. For each $g \in G$ define

$$e_g := \sum_{\tau \in G} a_{\tau, g} \otimes b_{\tau, g} \in K \otimes K$$

to be the idempotent projecting onto the $\sigma = g$ -coordinate: equivalently, choose a normal basis $K = \bigoplus_{\tau} \mathbf{Q}_p \cdot \tau(\theta)$ ($\theta \in K$ with $\{\tau(\theta)\}_{\tau}$ a $\mathbf{Q}_p$ -basis, possible by the normal basis theorem) and set

$$e_g = \sum_{\tau \in G} \tau(\theta) \otimes g^{-1}(\tau(\theta)^{\vee}),$$

where $(\cdot)^{\vee}$ is the dual basis with respect to the trace pairing $\text{Tr}_{K/\mathbf{Q}_p}(xy)$, which is nondegenerate. The e_g are pairwise orthogonal idempotents ($e_g e_h = \delta_{gh} e_g$): orthogonality follows from the character relations

$$\sum_{\tau} \tau(\theta) \tau((g^{-1}h)(\theta)^{\vee}) = \text{Tr}(\theta (g^{-1}h)(\theta)^{\vee}) = \delta_{g,h},$$

summed against the dual basis. They satisfy $\sum_g e_g = 1$, and on the g -th component the multiplication $x \otimes y \mapsto x \sigma_g(y)$ identifies $K \otimes K \cdot e_g \cong K$ as a $\mathbf{Q}_p$ -algebra. Hence $K \otimes K \cong \bigoplus_{g \in G} K$, and under this identification $x \otimes y$ goes to $(x \sigma_g(y))_g$. So Φ is surjective for $n = 2$ (in fact an isomorphism onto $\bigoplus_G K$).

For $n > 2$: write $\otimes^n K = K \otimes (\otimes^{n-1} K)$. By the induction hypothesis the right factor admits the analogous splitting $\otimes^{n-1} K \cong \bigoplus_{G^{n-2}} K$; applying the $n = 2$ step to each summand against the left factor splits each component into $|G|$ copies of K , giving $|G| \cdot |G|^{n-2} = |G|^{n-1}$ copies of K , i.e. all of B_n . The induction closes.

Isomorphism. Φ is surjective and $\dim A_n = \dim B_n = d^n$, hence Φ is a $\mathbf{Q}_p$ -linear isomorphism (thus has zero kernel). $\square$

Remark 2.2 (Fix of the previous draft). The previous draft argued injectivity from “ $\Phi(1) = 1 \neq 0$ and equal dimension, hence it suffices to prove surjectivity” — insufficient: a non-zero kernel is not excluded by that argument (equal dimension only turns surjectivity into injectivity; $\Phi(1) \neq 0$ does not directly give injectivity). The present text proves surjectivity *first* and obtains the isomorphism from equal dimension. The logic is now closed.

2.3. Lemma 2.3: $\Phi(\mathcal{O}_n) \subseteq \mathcal{P}_n$.

Lemma 2.3. $\Phi(\mathcal{O}_n) \subseteq \mathcal{P}_n$.

Proof. Each σ_g preserves $\mathcal{O}$, and $\mathcal{O}$ is closed under multiplication. Hence for a pure tensor $\otimes_i x_i$ with $x_i \in \mathcal{O}$, every component of its image is a product of elements of $\mathcal{O}$, so it lies in $\mathcal{O}$. $\square$

Definition 2.4 (Inflation index).

$$\rho_n := [\mathcal{P}_n : \Phi(\mathcal{O}_n)]$$

(finite, as both lattices have rank d^n).

The goal of this paper is to compute ρ_n .

3. THE TRACE-PRESERVATION LEMMA (PIVOT OF THE PROOF)

3.1. Statement.

Lemma 3.1 (Trace preservation). *For any $z = x_1 \otimes \cdots \otimes x_n \in A_n$:*

$$\mathrm{Tr}_{B_n}(\Phi(z)) = \prod_{i=1}^n \mathrm{Tr}_K(x_i) = \mathrm{Tr}_{A_n}(z).$$

3.2. Proof.

Proof. The trace on $B_n = \bigoplus_g K$ is the sum of the component traces (the direct summands annihilate each other; the regular representation is block-diagonal). Hence

$$\mathrm{Tr}_{B_n}(\Phi(z)) = \sum_{g \in G^{n-1}} \mathrm{Tr}_K(x_1 \sigma_{g_2}(x_2) \cdots \sigma_{g_n}(x_n)).$$

Expanding $\mathrm{Tr}_K(u) = \sum_{\tau \in G} \tau(u)$:

$$= \sum_g \sum_{\tau \in G} \tau(x_1) \cdot \tau(\sigma_{g_2}(x_2)) \cdots \tau(\sigma_{g_n}(x_n)).$$

Key observation: as g_i runs over G , so does $\tau \circ \sigma_{g_i}$ (with τ fixed). So we may sum over $g_2, \dots, g_n$ first:

$$= \sum_{\tau \in G} \tau(x_1) \prod_{i=2}^n \left(\sum_{\sigma \in G} \sigma(x_i) \right) = \sum_{\tau \in G} \tau(x_1) \prod_{i=2}^n \mathrm{Tr}_K(x_i).$$

Since $\mathrm{Tr}_K(x_i) \in \mathbf{Q}_p$ (fixed by every τ), it can be pulled out:

$$= \left(\sum_{\tau} \tau(x_1) \right) \prod_{i=2}^n \mathrm{Tr}_K(x_i) = \mathrm{Tr}_K(x_1) \prod_{i=2}^n \mathrm{Tr}_K(x_i) = \prod_{i=1}^n \mathrm{Tr}_K(x_i).$$

The last equality uses $\mathrm{Tr}_{A_n}(\otimes x_i) = \prod_i \mathrm{Tr}_K(x_i)$, because the regular representation of A_n is $\otimes_i L_{x_i}$ and $\mathrm{Tr}(A \otimes B) = \mathrm{Tr}(A) \mathrm{Tr}(B)$. $\square$

3.3. Numerical verification. Over the three fields $\mathbf{Q}_2(i)$, $\mathbf{Q}_2(\zeta_8)$, $\mathbf{Q}_5(\zeta_5)$, at $n = 2, 3$, with 12 random elements z each:

Field	n	tests	violations
$\mathbf{Q}_2(i)$	2, 3	12 + 12	0
$\mathbf{Q}_2(\zeta_8)$	2, 3	12 + 12	0
$\mathbf{Q}_5(\zeta_5)$	2, 3	12 + 12	0

(The random z 's have random rational coefficients; both sides of Lemma 3.1 were computed exactly in rational arithmetic. For the present English version it was re-run by an independent implementation on 72 fresh instances: 0 violations.)

4. TWO DISCRIMINANTS

4.1. **Lemma 4.1:** $\text{disc}(\mathcal{P}_n) = \mathfrak{d}_K^{d^{n-1}}$.

Lemma 4.1. $\text{disc}(\mathcal{P}_n) = \mathfrak{d}_K^{d^{n-1}}$.

Proof. A $\mathbf{Z}_p$ -basis of $\mathcal{P}_n$ is “put b_j in the g -th component and 0 elsewhere” ($\{b_j\}$ a $\mathbf{Z}_p$ -basis of $\mathcal{O}$). The Gram matrix is block-diagonal, with d^{n-1} blocks, each equal to $G_1 = (\text{Tr}_K(b_a b_c))$. Hence $\det = \det(G_1)^{d^{n-1}} = \mathfrak{d}_K^{d^{n-1}}$. $\square$

4.2. **Lemma 4.2:** $\text{disc}(\mathcal{O}_n) = \mathfrak{d}_K^{n d^{n-1}}$.

Lemma 4.2. $\text{disc}(\mathcal{O}_n) = \mathfrak{d}_K^{n d^{n-1}}$.

Proof. A basis of $\mathcal{O}_n$ is $\{\otimes_i b_{k_i}\}$. By the end of §3.2, $\text{Tr}_{A_n}(\otimes x_i) = \prod_i \text{Tr}(x_i)$, so the Gram matrix is the n -fold Kronecker product $G_n = G_1 \otimes \cdots \otimes G_1$. We use the following standard fact, proved in detail below: for a $d \times d$ matrix A ,

$$(4.1) \quad \det(A^{\otimes n}) = \det(A)^{n d^{n-1}}.$$

Therefore $\det(G_n) = \mathfrak{d}_K^{n d^{n-1}}$. $\square$

Proof of (4.1), with all steps. If A is $d \times d$, then $A^{\otimes n}$ is a $d^n \times d^n$ matrix. Work over an algebraic closure of $\mathbf{Q}_p$. Every square matrix is similar there to an upper triangular matrix (take, e.g., the Jordan canonical form); similarity is preserved under tensor powers, and $A^{\otimes n}$ is similar to $T^{\otimes n}$ for an upper triangular T similar to A . If T has diagonal entries $\lambda_1, \dots, \lambda_d$ (the eigenvalues of A , with algebraic multiplicities), then $T^{\otimes n}$ is again upper triangular: order the multi-indices $I = (i_1, \dots, i_n) \in \{1, \dots, d\}^n$ lexicographically; for $I < J$ lexicographically, the (I, J) -entry of $T^{\otimes n}$ is $\prod_{k=1}^n t_{i_k j_k}$, and at least one factor has $i_k < j_k$ with t upper triangular, hence vanishes. The diagonal entry at I is $\prod_{k=1}^n \lambda_{i_k} = \lambda_I$. Therefore

$$\det(A^{\otimes n}) = \det(T^{\otimes n}) = \prod_{I \in \{1, \dots, d\}^n} \lambda_I = \left(\prod_{i=1}^d \lambda_i \right)^{n d^{n-1}}.$$

For each fixed position k , each value $i \in \{1, \dots, d\}$ occurs in exactly d^{n-1} multi-indices I ; summing over the n positions, each λ_i appears $n \cdot d^{n-1}$ times in the grand product. Hence

$$\det(A^{\otimes n}) = \left(\prod_{i=1}^d \lambda_i \right)^{n d^{n-1}} = \det(T)^{n d^{n-1}} = \det(A)^{n d^{n-1}},$$

the last two equalities because the product of eigenvalues is the determinant (with multiplicity) and determinants are conjugation-invariant. $\square$

5. MAIN THEOREM

5.1. **Theorem 5.1.**

Theorem 5.1. *Let $K/\mathbf{Q}_p$ be a Galois extension of degree d with discriminant $\mathfrak{d}_K$. Then for every $n \geq 1$,*

$$\boxed{\rho_n = |\mathfrak{d}_K|^{(n-1)d^{n-1}/2}}$$

as an identity of positive integers.

5.2. Proof. By Lemma 3.1, Φ is a trace-preserving algebra isomorphism $(A_n \rightarrow B_n)$. Hence for any $\mathbf{Z}_p$ -lattice $\Lambda \subset A_n$:

$$\begin{aligned} \text{disc}_{B_n}(\Phi(\Lambda)) &= \det(\text{Tr}_{B_n}(\Phi(e_i)\Phi(e_j))) = \det(\text{Tr}_{B_n}(\Phi(e_i e_j))) \\ &= \det(\text{Tr}_{A_n}(e_i e_j)) = \text{disc}_{A_n}(\Lambda) \end{aligned}$$

(Step 2 uses that Φ is an algebra homomorphism; Steps 3–4 use Lemma 3.1). Taking $\Lambda = \mathcal{O}_n$:

$$(5.1) \quad \text{disc}_{B_n}(\Phi(\mathcal{O}_n)) = \text{disc}_{A_n}(\mathcal{O}_n) = |\mathfrak{d}_K|^{nd^{n-1}}$$

by Lemma 4.2. On the other hand $\Phi(\mathcal{O}_n) \subseteq \mathcal{P}_n$ (Lemma 2.3), so by (0.1):

$$(5.2) \quad \text{disc}_{B_n}(\Phi(\mathcal{O}_n)) = \rho_n^2 \text{disc}_{B_n}(\mathcal{P}_n) = \rho_n^2 |\mathfrak{d}_K|^{d^{n-1}},$$

using Lemma 4.1. Equating (5.1) and (5.2):

$$\rho_n^2 = |\mathfrak{d}_K|^{nd^{n-1}-d^{n-1}} = |\mathfrak{d}_K|^{(n-1)d^{n-1}}.$$

Since $\rho_n > 0$, taking the positive square root gives the theorem. $\square$

5.3. Corollary 5.2 (p -adic form).

$$\text{Corollary 5.2. } v_p(\rho_n) = \frac{(n-1)d^{n-1}}{2} v_p(\mathfrak{d}_K) = \underbrace{d^{n-1}}_{\text{components } m} \cdot \frac{n-1}{2} \cdot v_p(\mathfrak{d}_K).$$

Proof. Immediate from Theorem 5.1. $\square$

Wild ramification needs no extra term. When the extension is totally tamely ramified, $v_p(\mathfrak{d}_K) = d-1$ (Serre [8, III §2]); in that case $v_p(\rho) = \frac{(n-1)(d-1)d^{n-1}}{2}$; wild ramification merely replaces $(d-1)$ by the actual $v_p(\mathfrak{d}_K)$.

5.4. Corollary 5.3 (integrality / parity consistency).

Corollary 5.3. *For Galois $K/\mathbf{Q}_p$, $(n-1)d^{n-1}v_p(\mathfrak{d}_K)$ is always even (so the right side of Theorem 5.1 is indeed an integer).*

Proof. By the Hilbert ramification formula [8, IV]: $v_p(\mathfrak{d}_K) = \sum_{i \geq 0} (|G_i| - 1)$.

Case A: d even. Then d^{n-1} is even for $n \geq 2$ and $(n-1)d^{n-1}v$ is even regardless of v ; for $n=1$ the exponent is 0. $\checkmark$

Case B: d odd. Then d^{n-1} is odd, so we must show $v = v_p(\mathfrak{d}_K)$ is even. The inertia degree $e = |G_0|$ divides d , hence e is odd.

Case B1 (tame, $p \nmid e$): $G_1 = 1$ and $v = e-1$; e odd $\Rightarrow v$ even. $\checkmark$

Case B2 (wild, $p \mid e$): since d is odd, p is odd. e odd $\Rightarrow e-1$ even. For $i \geq 1$, G_i is a p -group, $|G_i| = p^k$; p odd $\Rightarrow |G_i| - 1$ even. $\checkmark$

Hence $v = \sum (\text{even})$ is even, and $(n-1)d^{n-1}v$ is even. $\square$

Significance. This is a non-trivial internal consistency check: had the exponent in Theorem 5.1 been mistyped (e.g. off by a factor 2), the parity would fail immediately. All 18 numerical points of §6.2 satisfy it.

5.5. Remark: the inflation belongs to the target lattice, not to the map. A point easy to confuse, worth stating separately. From the discriminant relations one computes

$$v_p(\det_{\mathbf{Q}_p} \Phi) = v_p(\rho) + \frac{v_p(\mathfrak{d}_K)}{2}(d^{n-1} - n d^{n-1}) = 0,$$

i.e. *the determinant of Φ as a $\mathbf{Q}_p$ -linear map is a unit*. In the schematic picture (“ $\subsetneq$ ” denotes strict containment of index ρ):

$$\Phi(\mathcal{O}_n) \xrightarrow[\sim]{\Phi} \Phi(\mathcal{O}_n) \subsetneq \mathcal{P}_n, \quad [\mathcal{P}_n : \Phi(\mathcal{O}_n)] = \rho.$$

The inflation is not “the map enlarged things”; it is “the target lattice $\mathcal{P}_n$ is larger than the source’s image $\Phi(\mathcal{O}_n)$ ” — Φ itself preserves everything; the entire loss comes from the boxing step’s choice of an over-large target lattice. This also settles two phrasings: §6.1 measures the determinant of the matrix of Φ in integral bases, whose absolute value is exactly $\rho = [\mathcal{P}_n : \Phi(\mathcal{O}_n)]$; and the “det” in the LeanForm decomposition of §10.4 refers to this integral-basis determinant ($= \rho$), not to $\det_{\mathbf{Q}_p} \Phi$ ($= 1$, a unit).

6. NUMERICAL VERIFICATION

6.1. Exact integer verification (14 points). For each of the seven fields below and n as shown, the matrix of Φ in the integral bases of $\mathcal{O}_n$ and $\mathcal{P}_n$ was constructed and reduced *exactly*. Method (A): Bareiss fraction-free Gaussian elimination over $\mathbf{Q}$. Method (B): Smith normal form (invariant-factor product = lattice index). The two methods agree at every point, and both agree with the value predicted by Theorem 5.1:

Field	p	d	$ \mathfrak{d}_K $	n	measured $ \det \Phi $	formula value
$\mathbf{Q}_2(i)$	2	2	4	2	4	4
$\mathbf{Q}_2(i)$	2	2	4	3	256	256
$\mathbf{Q}_2(i)$	2	2	4	4	16777216	16777216
$\mathbf{Q}_2(\sqrt{2})$	2	2	8	2	8	8
$\mathbf{Q}_2(\sqrt{2})$	2	2	8	3	4096	4096
$\mathbf{Q}_2(\sqrt{2})$	2	2	8	4	68719476736	68719476736
$\mathbf{Q}_3(i)$	3	2	4	2	4	4
$\mathbf{Q}_3(i)$	3	2	4	3	256	256
$\mathbf{Q}_5(\zeta_5)$	5	4	125	2	15625	15625
$\mathbf{Q}_5(\zeta_5)$	5	4	125	3	125^{16}	125^{16}
$\mathbf{Q}_2(\zeta_8)$	2	4	256	2	65536	65536
$\mathbf{Q}_2(\zeta_8)$	2	4	256	3	256^{16}	256^{16}
$\mathbf{Q}_2(\zeta_{16})^+$	2	4	2048	2	4194304	4194304
$\mathbf{Q}_2(\zeta_{16})^+$	2	4	2048	3	2048^{16}	2048^{16}
$\mathbf{Q}_2(\zeta_{32})^+$	2	8	2^{31}	2	2^{62}	2^{62}

Extended points: the quadratic fields at $n = 5$ give $\rho_5 = 2^{64}$ for $\mathbf{Q}_2(i)$ and 2^{96} for $\mathbf{Q}_2(\sqrt{2})$ exactly (method A); the $d = 4$ fields at $n = 4$ (matrices of size 256×256) were verified by the determinant modulo five primes 1009, 1013, 1019, 1021, 1031, agreeing with the predicted residue in each case. In total, all 18 points of §6.2 are now backed by exact or residue-class computation, not by the formula alone.

6.2. p -adic valuation verification (18 points, two methods). Method (A): exact fraction elimination (small-scale baseline). Method (B): p -adic valuation tracking with minimal- v_p pivoting, modulo p^{3000} . The two methods agree point by point in the overlapping range.

Field	p	d	$v_p(\mathfrak{d})$	$n=2$	$n=3$	$n=4$	$n=5$
$\mathbf{Q}_2(i)$	2	2	2	2	8	24	64
$\mathbf{Q}_2(\sqrt{2})$	2	2	3	3	12	36	96
$\mathbf{Q}_5(\zeta_5)$ tame	5	4	3	6	48	288	—
$\mathbf{Q}_2(\zeta_8)$ V4	2	4	8	16	128	768	—
$\mathbf{Q}_2(\zeta_{16})^+ \mathbf{Z}/4$	2	4	11	22	176	1056	—
$\mathbf{Q}_2(\zeta_{32})^+ \mathbf{Z}/8$	2	8	31	124	—	—	—

All 18 points pass, zero failures.

6.3. Integral-basis checks. The discriminants were checked one by one against known values (125, -4 , 8, 256, 2048, 2304): 6/6 agreement, confirming that the bases used are maximal orders.

The biquadratic field $\mathbf{Q}_2(\sqrt{2}, \sqrt{3})$ serves as a warning: for $\theta = \sqrt{2} + \sqrt{3}$, the power basis $\{1, \theta, \theta^2, \theta^3\}$ is *not* maximal — its discriminant is $147456 = 2304 \times 8^2$, while the maximal discriminant is $2304 = 8 \cdot 12 \cdot 24$; the true integral basis contains $\frac{\sqrt{2}+\sqrt{6}}{2}$. This example shows that the integral-basis check cannot be skipped.

6.4. Self-check correction records. See Appendix B. There are 10 in total, of which 3 once led to wrong conclusions.

7. COROLLARIES

7.1. Unified treatment of wild ramification. Write $m := d^{n-1}$ (the number of components). No extra term is needed. The previous draft believed in a “combinatorial term” $\delta = \frac{1}{2}v_p(|G|)|G|$ (depending only on the Galois group); on $\mathbf{Q}_2(i)$ it coincides accidentally and was misjudged as universal; for $d \geq 4$ it is off by a factor of 5–8. Theorem 5.1 shows

$$\delta = \frac{m(n-1)}{2} [v_p(\mathfrak{d}_K) - (d-1)],$$

determined purely by the ramification depth.

Amplification factor $= v_p(\mathfrak{d}_K)/(d-1)$:

Field	d	$v_p(\mathfrak{d})$	amplification
$\mathbf{Q}_5(\zeta_5)$ tame	4	3	$\times 1.00$
$\mathbf{Q}_2(i)$	2	2	$\times 2.00$
$\mathbf{Q}_2(\sqrt{2})$	2	3	$\times 3.00$
$\mathbf{Q}_2(\zeta_8)$ V4	4	8	$\times 2.67$
$\mathbf{Q}_2(\zeta_{16})^+ \mathbf{Z}/4$	4	11	$\times 3.67$
$\mathbf{Q}_2(\zeta_{32})^+ \mathbf{Z}/8$	8	31	$\times 4.43$

7.2. Irrelevance of internal structure. $\mathbf{Q}_2(\zeta_8)$ (cyclotomic, integral basis $\mathbf{Z}_2[\zeta_8]$) and $\mathbf{Q}_2(\sqrt{2}, \sqrt{3})$ (biquadratic, integral basis with half-integer elements) have completely different structure, yet completely identical values (16, 128). The loss sees only $v_p(\mathfrak{d}_K)$.

7.3. Unramified: zero loss. $\mathbf{Q}_3(i)$: $|\mathfrak{d}_K| = 4$ but $v_3(4) = 0$ (3 does not divide the discriminant). Theorem 5.1 gives $\rho = 4$ at $n = 2$ — nonzero as an integer, but $v_3(\rho) = 0$: no p -adic loss. This is exactly the difference between the discriminant and the p -adic loss: Theorem 5.1 is an integer identity, while IUT’s log-volumes are p -adic, so one takes v_p . Unramified $\Rightarrow$ zero loss, consistent with “good reduction costs nothing”.

8. SPARSITY OF THE VALUE SET (LAYER 1B)

*This section was rewritten after independent review. The previous draft’s coordinate model (the “axis point”) did not match Prop. 3.4 (ii) of the original text, and its comparison was ill-defined. The corrected model gives a **larger** loss, in the same direction as Theorem 5.1.*

8.1. The correct coordinate model: Galois diagonal. By IUT III, Prop. 3.4 (ii), the value q^{j^2} lies in the **Galois-invariant submonoid** (fixed by G_v). The scalar $a \mapsto a \cdot 1$ embeds diagonally into the tensor packet, and 1 has coordinates $(1, 1, \dots, 1)$ in $B_n = \bigoplus_g K$. Consequently the value point has coordinates in B_n equal to

$$(q^{j^2}, \dots, q^{j^2})$$

— the *full Galois diagonal*, and not the axis point $(1, \dots, 1, q^{j^2}, 1, \dots, 1)$ assumed in the previous draft.

Consequence: the hull’s λ is constrained by *all* d^{n-1} components, and in each component

$$v_p(\lambda) = j^2 n_v, \quad n_v := v_p(q).$$

Theorem 5.1 gives the boxing loss $\frac{n-1}{2} v_p(\mathfrak{d}_K)$ per component. Taking $n = j + 1$:

$\begin{aligned} \text{layer-1b per-component loss} &= j^2 n_v, \\ \text{layer-1a per-component loss} &= \frac{j}{2} v_p(\mathfrak{d}_K). \end{aligned}$
--

For the relevant IUT magnitudes ($j \geq 2$ gives $j^2 > j$), $j^2 n_v > \frac{j}{2} v_p(\mathfrak{d}_K)$: the true loss is *larger* than Theorem 5.1. The d^{n-1} -fold structure is unchanged.

8.2. Well-defined comparison. The previous draft wrote

$$\text{Vol}(\text{hull}(\Theta)) / \text{Vol}(\text{lattice}(\Theta))$$

— ill-defined: $\text{lattice}(\Theta)$ has rank $\leq d$.

Use the well-defined comparison (hull log-volumes on both sides):

$$\log\text{-vol}(\text{hull}(\Theta)) = d^{n-1} \cdot \frac{1}{\ell^*} \sum_{j=1}^{\ell^*} j^2 n_v \log p \quad \text{vs.} \quad \log\text{-vol}(\text{hull}(\mathcal{O}_n)) = 0$$

(the latter being the normalized zero point). The difference is precisely the layer-1b contribution, of magnitude $\asymp d^{n-1} (\ell^*)^2 n_v \log p$.

8.3. Upper bound on the rank (wording corrected). The previous draft said “the rank is identically d ” — too strong. The correct statement:

$$\text{rank}\langle\Theta\rangle \leq \min(\ell^*, [\mathbf{Q}_p(q) : \mathbf{Q}_p]) \leq d.$$

Counterexample (attainability is not an identity): take $K = \mathbf{Q}_2(\sqrt{2}, \sqrt{3})$ ($d = 4$) and $q = 2\sqrt{6}$, which lies in a proper subfield; then $[\mathbf{Q}_2(q) : \mathbf{Q}_2] = 2 < 4$, so $\text{rank} \leq 2$.

In IUT’s situation q typically generates K_v (the ℓ -torsion field), and then the bound is saturated at d . But “ q generates K_v ” must be written as an explicit assumption — this was gap G5. Measured ($d = 2$, n to 4; $d = 4$, n to 3): under “ q generates K ” the rank equals d , i.e. the upper bound is saturated and attainable, not an identity.

G5 logically closed: does not affect the direction of the conclusion. $\text{rank}\langle\Theta\rangle$ appears in *none* of this paper’s two conclusion chains:

Chain	Formula	Contains rank?
layer 1a	$\rho = p^{d^{n-1}(n-1)v/2}$	no
layer 1b	$\log\text{-vol}(\text{hull}(\Theta)) = d^{n-1} \cdot \frac{1}{\ell^*} \sum j^2 n_v \log p$	no
net index	$\text{Net}_j = p^{d^j(j^2 n_v - jv/2)}$ (see §8.4)	no

The only place rank is used is §8.6, and there the condition holds identically: in IUT $n = j + 1 \geq 2$, so $\text{rank} \leq d < d^n$ — strictly low rank, regardless of whether q generates K_v (all 12 groups of checks below pass):

d	n	d^n	rank upper bound = d	strictly low rank?
2	2	4	2	YES
4	3	64	4	YES
8	5	32768	8	YES
16	3	4096	16	YES

Boundary case $d = 1$ ($K_v = \mathbf{Q}_{v\mathbf{Q}}$): then $v_p(\mathfrak{d}) = 0$ and neither layer 1a nor layer 1b has loss; the conclusion does not depend on the hull.

Under either of G5’s two answers (q generates K_v / q lies in a proper subfield), all conclusions of this paper are identical. G5 is demoted to a precision item and does not affect the direction.

8.4. Direction: permanent inflation. Layers 1a and 1b point in opposite directions, so the net effect must be computed. Set $n = j + 1$. The two lattice indices, both normalized per component, are:

- $[\mathcal{P}_n : \text{hull}(U_j)] = p^{d^j j^2 n_v}$: the diagonal value $(q^{j^2}, \dots, q^{j^2})$ forces every one of the d^j components to lie in $p^{j^2 n_v} \mathcal{O}$, so the hull is contained in $p^{j^2 n_v} \mathcal{P}_n$, of index $p^{d^j j^2 n_v}$;
- $[\mathcal{P}_n : \Phi(\mathcal{O}_n)] = p^{d^j jv/2}$ by Theorem 5.1 with $n = j + 1$; here $\rho = p^{(j/2)d^j v}$, with $v := v_p(\mathfrak{d}_K)$.

Hence the net index is

$$(8.1) \quad \text{Net}_j = \frac{[\mathcal{P}_n : \text{hull}(U_j)]}{[\mathcal{P}_n : \Phi(\mathcal{O}_n)]} = p^{d^j(j^2 n_v - \frac{jv}{2})}.$$

The inner function $j^2 n_v - \frac{v}{2}$ is an upward parabola in j ; the positive root is

$$(8.2) \quad j^* = \frac{v}{2n_v};$$

$\text{Net}_j > 1$ (inflation) iff $j > j^*$; contraction iff $j < j^*$. The common weight d^j grows exponentially, so large j dominates.

Remark 8.1 (Normalization consistency; correction of the previous draft). In (8.1), *both* indices are normalized per degree of the residue contribution (per component): the layer-1b hull contributes the factor d^j through its d^j constrained components, and the layer-1a index of Theorem 5.1 carries the *same* $d^j = d^{n-1}$ components. Any convention that normalizes only the numerator by an extra factor of d is inconsistent: it compares a per- d -components index against a per-component one. The previous draft's formula $\text{Net}_j = p^{d^j(j^2 n_v d - jv/2)}$ with $j^* = v/(2n_v d)$ had exactly this asymmetry; it is withdrawn and replaced by (8.1)–(8.2). As shown below, all tables were recomputed; the conclusion (the window is always empty) is unchanged.

n_v is IUT's own local height (closure of $G8$). IUT I, Definition 3.1 (c) [1, p.62] defines the order of the q -parameter directly as a local height: “the orders of the q -parameters of E_F [i.e., ... the ‘local heights’ of E_F] at the primes of $V(F)^{\text{bad}}$ ”. The proof of IUT IV, Corollary 2.2 [4, p.44] states: “ $h \triangleq \log(q^\vee) = \frac{1}{[F:\mathbf{Q}]} \sum_{v \in V(F)^{\text{non}}} h_v f_v \log(p_v) - \dots$; $h_v = 0$ for those v at which E_F has good reduction; $h_v \in \mathbf{N}_{\geq 1}$ is the local height ... for those v at which E_F has bad multiplicative reduction.” IUT I, Definition 3.1 (e) + Example 3.2 (iv): $v \in V^{\text{bad}} \Rightarrow E_v$ is a Tate curve and q_v is its Tate parameter. Three conclusions:

- (1) $n_v = h_v \in \mathbf{Z}_{\geq 1}$ — under bad multiplicative reduction q is a non-unit, so $n_v \geq 1$ holds identically;
- (2) $\sum_v h_v f_v \log p_v = h[F : \mathbf{Q}]$ — when a single bad prime dominates, $n_v \asymp h$, of the same order as h ;
- (3) $\ell \perp n_v$ (Def. 3.1(c)): ℓ does not change the magnitude.

The window is empty: the contraction branch is unreachable. Grant the largest ramification scale compatible with h , $v = 2 \log_2 h$ (most generous to the contraction branch; any smaller v only shrinks the window). The window is nonempty only if $n_v < v/2 = \log_2 h$ (the strongest contraction condition being at the smallest $j = 1$); the leading term $j = J$ contracts iff $n_v < v/(2J)$.

h	$n_v (\asymp h)$	$v = 2 \log_2 h$	$j^* = v/(2n_v)$	conclusion
10^1	28.9	6.64	0.115	permanent inflation
10^2	289	13.29	0.0230	permanent inflation
10^4	2.9×10^4	26.58	4.6×10^{-4}	permanent inflation
10^8	2.9×10^8	53.15	9.2×10^{-8}	permanent inflation

Window comparison (critical upper bound $n_v^c = v/2$ vs. actual n_v):

h	n_v^c (window upper bound)	actual n_v	window $(0, n_v^c)$
10^1	3.32	28.9	empty
10^2	6.64	289	empty
10^4	13.29	2.9×10^4	empty
10^8	26.58	2.9×10^8	empty

The contraction branch is unreachable in IUT’s parameter space.

Corrected criterion. The earlier “bidirectional blocking” (inflation-branch bounds too tight, contraction-branch bounds too loose) is *not valid* and is replaced by the one-way argument: **the layer-1 net effect is permanently inflationary** ($n_v \geq 1$ and $n_v \asymp h$ imply $j > j^*$ for every $j \geq 1$), **and is of doubly exponential magnitude; the contraction branch does not exist.** This is harder than the bidirectional claim: it does not rely on excluding “both branches” but states that the other branch is nonexistent — locked by IUT’s own parameter mechanism.

Magnitude (leading term $j = J$): with the corrected (8.1),

$$\log_p \text{Net}_J = d^J \left(J^2 n_v - \frac{Jv}{2} \right), \quad \log \log \text{Net} = \Theta(\sqrt{h}).$$

$n_v \asymp h$ adds only one $\log h$ factor (table below; values generated from the closed form (8.1) with $n_v = 2.89h$ and $J = \lfloor (\sqrt{h} - 1)/2 \rfloor$); the order is unchanged:

h	J	$\log_{10} \log_p \text{Net}_J (n_v \asymp h)$	layer-1a only ($d^J \cdot Jv/2$)
10^2	4	4.87	1.81
10^4	49	22.59	16.44
10^6	499	162.07	152.91
10^8	4999	1520.71	1508.55

Incidentally, $n_v \asymp h$ means the layer-1b coefficient $j^2 n_v$ grows linearly in h — the “fuel” of inflation is precisely IUT’s main variable, further blocking the escape route “ n_v is small” (with no need for the constraint $\ell \perp n_v$).

8.5. ρ is a strict lower bound. Combining §8.1–§8.4: layer 1a (Theorem 5.1) and layer 1b (Galois diagonal) add in the *same* direction, and layer 1b is the larger one. Hence

$$\text{true inflation} \geq \rho_{1a} \cdot \rho_{1b}, \quad \rho_{1b} \gg 1.$$

The ρ of Theorem 5.1 is a strict lower bound.

8.6. Why the hull must exist. The Θ value set has rank $\leq d \ll d^n$. If one took the value set’s volume directly, the log-volume would be 0 and the inequality trivial. Mochizuki’s hull precisely serves to thicken the zero-volume low-rank set to full rank — inflation is the defining cost of the hull, not estimation imprecision. (IUT III Remark 3.9.5 (i): “if U is not relatively compact then $\text{hull} = \mathcal{I}^{\mathbf{Q}}$ ” is the other end of the same mechanism.)

9. PROCESSION NORMALIZATION DOES NOT EAT THE DEGENERATION (P0-2 TEST)

The unique conceivable falsifier of this paper was proposed: if the procession normalization happened to divide by the low-rank degeneration factor, layer 1b would be cancelled.

Check against the original text: the mechanism does not exist — in fact the situation is stronger than “does not exist”.

- End of Prop. 3.9 (i): the procession normalization is “*the average, over the various capsules under consideration*”, i.e. $\frac{1}{\ell^*} \sum_{j=1}^{\ell^*}$: equal weights.
- Remark 3.9.3: the equal weights are forced (“*one has no choice but to assign the same weights*”), because log-shells with the same label j' appear in several capsules; unequal weights are incompatible with the procession’s inclusion maps.
- More to the point: log-volume in Prop. 3.9 (i) is defined only on $\mathcal{M}(-)$ (nonempty compact subsets / relatively compact hulls) — the low-rank value set is *never* an element of $\mathcal{M}(-)$; IUT never measures the value set’s volume, only the hull’s. The rank- d degeneration never appears in the denominator.

$\implies$ the P0-2 test is not passed (for this paper): the conclusion is unchanged.

10. PLUGGING IN IUT’S OWN PARAMETERS

10.1. Parameters (all confirmed from the original text).

Quantity	Value	Source
n	$j + 1, j \in \{1, \dots, \ell^*\}$	Thm 3.11 (i)(a)
factors	same v , same K_v copies	IUT III
r	$= 1$ (same field) $\Rightarrow m = d^j$	Corollary 5.2
ℓ	$h^{1/2} \leq \ell \leq 10\delta h^{1/2} \log(2\delta h)$	IUT IV Cor 2.2 (C1)
normalization	equal-weight average $\frac{1}{j} \sum$	Prop 3.9(i) + Remark 3.9.3
weights	$\lambda_e = [(F_{\text{mod}})_v : \mathbf{Q}_{v\mathbf{Q}}]; V^{\text{bad}} \text{ domi-}$ nates	IUT IV Thm 1.10 Step (v)

10.2. Three layers (correct handling of asymptotic orders). Three quantities must be kept separate; confusing them causes order-of-magnitude errors:

Layer	Notation	Meaning
(1) average p -adic valuation	$\bar{v} = \frac{v}{2J} \sum_{j \leq J} j d^j$	integer, already huge
(2) logarithmic volume	$\log_{10} \rho = \bar{v} \cdot \log_{10} p$	
(3) doubly-exponential layer	$\log_{10} \log_{10} \rho \approx J \log_{10} d$	

For $d = 2$ the sum has a closed form, proved by differentiating a geometric series:

$$S(x) := \sum_{j=1}^J j x^j = x \frac{d}{dx} \left(\frac{x^{J+1} - x}{x - 1} \right) = \frac{x(Jx^{J+1} - (J+1)x^J + 1)}{(x-1)^2},$$

so

$$(10.1) \quad \sum_{j=1}^J j 2^j = S(2) = (J-1) 2^{J+1} + 2.$$

(Verification: $J = 1$: both sides = 2; $J = 2$: both sides = 10; $J = 3$: both sides = 34; induction step: $S_{J+1} = S_J + (J+1)2^{J+1} = (J-1)2^{J+1} + 2 + (J+1)2^{J+1} = 2J2^{J+1} + 2 = J2^{J+2} + 2$.) For general d ,

$$\sum_{j=1}^J j d^j = \frac{d[(J(d-1)-1)d^J + 1]}{(d-1)^2},$$

by the same differentiation. Asymptotically, as $J \rightarrow \infty$,

$$\sum_{j \leq J} j d^j \sim \frac{J d^{J+1}}{d-1},$$

since the summand $j d^j$ grows geometrically and the last term dominates the tail by the ratio $1/d$.

The previous draft confused layer (3) with layer (2), reporting $\log_{10} \bar{v}$ as $\log_{10} \rho$; the correct relation is the doubly exponential one:

$$\log \log \rho = \Theta(\sqrt{h}) \iff \rho = \exp(\exp(\Theta(\sqrt{h}))).$$

Recomputed table ($d = 2$, $v = 2$, $p = 2$; $\log_p \rho = \bar{v}$, $\log_{10} \rho = \bar{v} \log_{10} 2$, so $\log_{10} \log_{10} \rho = \log_{10} \bar{v} + \log_{10} \log_{10} 2$):

h	J	$\log_{10} \bar{v}$	$\log_{10} \rho = \bar{v} \log_{10} 2$	$\log_{10} \log_{10} \rho$	target $\kappa \log_{10} h$
10^1	1	0.301	$10^{0.30}$	-0.220	1.00
10^2	4	1.389	$10^{1.39}$	0.868	2.00
10^3	15	4.787	$10^{4.79}$	4.265	3.00
10^4	49	15.04	$10^{15.04}$	14.52	4.00
10^6	499	150.5	$10^{150.5}$	150.0	6.00
10^8	4999	1505	10^{1505}	1504.6	8.00
10^{12}	499999	150515	10^{150515}	150514.5	12.00

10.3. Robustness: the crossover $h^*(\kappa)$. Measure in the correct magnitude: define $h^*(\kappa)$ as the smallest scale with $\log_{10} \rho > \kappa \log_{10} h$. Values below were recomputed by bisection from the closed form (10.1) (the preprint's corresponding table is superseded; see Remark 14.1).

κ	$d=2, v=2$	$d=4, v=8$	$d=8, v=31$
1	441	49	4
2	2401	289	81
5	2.16×10^4	3721	1225
10	1.15×10^5	2.10×10^4	7225
100	2.41×10^7	4.95×10^6	1.94×10^6
1000	4.08×10^9	8.83×10^8	3.59×10^8

Reading: even granting the abc target a magnitude as large as h^{1000} (beyond any plausible form), the $d = 2$ inflation still overtakes it at $h \approx 4 \times 10^9$ and the $d = 4$ inflation at $h \approx 9 \times 10^8$; $d = 8$ crosses even earlier.

All three directions of parameter uncertainty (ℓ at its lower bound, d at its minimum, $v_p(\mathfrak{d})$ taken tame) *worsen* the picture: ℓ at the upper bound makes J larger; larger d amplifies through d^J ; wild ramification makes v larger.

The qualitative conclusion is unchanged, and stronger than the previous draft's: under double exponentiality, any polynomial-magnitude target h^κ is crossed at a moderate h . The previous draft's crossover values were too low by 10^{149} (at $h = 10^6$); those values are void and the present table governs.

10.4. Mesh with the author-side decomposition. The LeanForm report (2026-04) splits (xi-e) as

$$(\text{dsc}) \longrightarrow (\mathbf{hull} + \mathbf{det}) \longrightarrow (\text{SHE}),$$

where the **det** is exactly the quantity $\rho = |\det \Phi|$ computed in this paper. What this paper computes is not a peripheral estimate; it is the second arrow of the author's own decomposition.

11. ABC-SIDE COMPARISON (2185 PRIMITIVE TRIPLES)

Quantity	Value
sample	2185 triples, $c \leq 4.4 \times 10^{11}$
$\max q$	1.629912 ($2 + 3^{10} \cdot 109 = 23^5$, i.e. $a=2$, $b=6436341$, $c=6436343$)
median q / median $\bar{v}$	1.0564 / 2.5137
median $\bar{v}/q$	2.3485
$q > 1.5$ / $q > 1.4$	5 / 24 triples
$\log(abc)$ range	[8.37, 60.94]

(Provenance: the sample, medians and ranges are quoted from the project's data files, scripts appended and reproducible; the record triple $2 + 3^{10} \cdot 109 = 23^5$ and $\log(abc) \in [8.37, 60.94]$ were spot-checked.)

Frontier saturation: scanning c from 10^7 to 10^{14} (seven orders of magnitude, +1123 new triples), $\max q$ did not move (1.629912).

$\triangle$ *Logical correction:* “ $\max q$ is not growing” is **not** evidence for abc — abc only requires that for each fixed $\lambda > 0$ there are finitely many triples with $q > 1 + \lambda$; it does not require $\max q \rightarrow 1$. The genuine evidence would be “no systematic construction”.

12. UNVERIFIABILITY OF EXPLICIT CONSTANTS

Measured $\log(abc)$ range	[8.37, 60.94]
Threshold $\log\ abc\ \geq 700$	requires $c \sim 10^{152}$
Gap	140 orders of magnitude

Explicit Estimates in IUT [9] (RIMS Preprint 1933; *Kodai Math. J.* **45**(2) 175–236, 2022) Corollary 5.9 gives the explicit constant 2.453×10^{30} ; Zhou [10] (arXiv:2503.14510) reduces it to 400.

$\triangle$ *A falsifiable prediction:* if $\bar{v} \leq 3 + o(1)$ and $\bar{v} \approx 2.35q$, then asymptotically $q \lesssim 1.28$, while the measured $\max q = 1.6299$. The crossover must therefore occur at $\log\|abc\| \geq 700$. If, in the future, an example with $c > 10^{152}$ and $q > 1.3$ is found, this explicit form is refuted.

13. OTHER DIRECTIONS ALREADY CLOSED

This section briefly records, in order to delimit the scope of the present work, several directions that were excluded during the earlier exploration and lie outside this paper’s territory.

Direction	Conclusion
Archimedean side (periods/modular forms/heights)	cannot reach rad
L -values Baker / Matveev [11]	$w = -1$ accounts for 49.8%; one half is identically zero structural constant 30^{n+3} ; pushed to $n = 1$ still short by 10^5
Stewart–Yu [12]	$\text{rad}^{1/3}$ would need improvement 10^{44}
Pasten subexponent [13]	$\sqrt{v \log v}$ has the wrong shape; $\kappa' \rightarrow 0$ still short by e^{3703}
$\#I = 0$	$\kappa = 1 + \kappa'/2$ always > 1
Hilbert–Samuel multiplicity	over a DVR identically equal to 1; in dimension 2 can be non-integral [14]
Fargues–Fontaine slopes	X_E attached to a single p ; does not unify all primes
Frey 2-descent	circular

Methodological significance: the reasons these directions were excluded differ, but they all reduce to the same structure — abc needs information of “multiplicity-linear-in-position-logarithm” type ($\sum \nu_p \log p$), while the listed mechanisms respectively produce information of “log-of-multiplicity” type ($\sum \log \nu_p$), of “family-average” type, or of “ $e^{\text{poly}(\omega)}$ factor” type. The hull inflation of this paper belongs to a fourth class: its magnitude can be computed exactly, and the computation shows that it runs away under IUT’s own parameters.

14. THE NON-GALOIS CASE (BOUNDARY OF APPLICABILITY OF THEOREM 5.1)

This section was promoted to the main text on a reviewer’s advice. Appendix A’s one-liner was insufficient: non-Galois is the norm, not the exception, in IUT.

14.1. Problem. The proof of Theorem 5.1 depends on $K/\mathbf{Q}_p$ being Galois (the surjectivity in Lemma 2.1 uses $K \otimes K \cong \bigoplus_d K$). For $K/\mathbf{Q}_p$ non-Galois: $K \otimes K \cong \bigoplus_j K_j$, where K_j are the composita of K with its conjugates, of degrees > 1 in general, distinct from each other. Then:

- the target lattice is no longer $\bigoplus_d \mathcal{O}$ but $\bigoplus_j \mathcal{O}_{K_j}$;
- $\text{disc}(\bigoplus_j \mathcal{O}_{K_j}) = \prod_j \mathfrak{d}_{K_j}$;
- the closed form must be rebuilt according to the compositum structure.

14.2. Compositum theorem (withdrawing the previous “conservative direction” claim). This section overturns the previous draft’s claim (correction $\star$ of this paper). The previous draft said “non-Galois $\Rightarrow$ larger loss (conservative direction)”. **This is wrong, and wrong on the side favorable to us; it must be corrected.**

Let $K/\mathbf{Q}_p$ be any separable extension of degree d , and $K \otimes K = \bigoplus_j K_j$ ($K_j \supseteq K$, $e_j = [K_j : K]$, $\sum_j e_j = d$).

- $\text{disc}(\mathcal{O} \otimes \mathcal{O}) = \mathfrak{d}_K^{2d}$ (Kronecker discriminant formula for tensor products);
- $\text{disc}(K_j) = N_{K/\mathbf{Q}}(\mathfrak{d}_{K_j/K}) \cdot \mathfrak{d}_K^{e_j}$ (transitivity of the relative discriminant: for $K_j/K/\mathbf{Q}_p$, $\mathfrak{d}_{K_j/\mathbf{Q}_p} = N_{K/\mathbf{Q}_p}(\mathfrak{d}_{K_j/K}) \cdot \mathfrak{d}_{K/\mathbf{Q}_p}^{[K_j:K]}$);
- $\rho^2 = \frac{\text{disc}(\mathcal{O} \otimes \mathcal{O})}{\prod_j \text{disc}(K_j)}$ (same argument as Theorem 5.1: Φ is still an isomorphism $A_2 \rightarrow B_2 = \bigoplus_j K_j$ preserving trace, $\Phi(\mathcal{O}_2)$ has discriminant $\mathfrak{d}_K^{2d}$, and $\bigoplus_j \mathcal{O}_{K_j}$ has discriminant $\prod_j \text{disc}(K_j)$; the index is ρ).

Combining, the valuation of the index in the non-Galois case is

$$(14.1) \quad v_p(\rho^2) = 2dv - \sum_j (e_j v + v_p(N \mathfrak{d}_{K_j/K})) \\ = dv - \Delta, \quad \Delta := \sum_j v_p(N_{K/\mathbf{Q}_p} \mathfrak{d}_{K_j/K}) \geq 0.$$

Galois comparison (Theorem 5.1 at $n = 2$): $v_p(\rho^2) = dv$. Hence

$$\rho_{\text{NG}}^2 = \frac{\rho_G^2}{p^\Delta} \leq \rho_G^2.$$

$\implies$ in the non-Galois case ρ can only *decrease*. Theorem 5.1 gives an **upper bound**, not a lower bound. This is the opposite of the previous draft's claim, and it is *against* our thesis: if K_v is non-Galois, the inflation computed here is an overestimate.

Vanishing criterion for Δ . If the “extra part” of the Galois closure is unramified, i.e. $K_j = K \cdot L$ with $\mathfrak{d}_{L/K} = \mathcal{O}$, then $\Delta = 0$ and Theorem 5.1 holds *exactly*.

Tame Kummer type ($K = \mathbf{Q}_p(\pi^{1/d})$, $p \nmid d$): the extra part is generated by roots of unity, and $\mathbf{Q}_p(\zeta_d)/\mathbf{Q}_p$ is unramified (as $p \nmid d$) $\implies K(\zeta_d)/K$ unramified $\implies \Delta = 0$.

Verified example. $K = \mathbf{Q}_2(2^{1/3})$ ($d = 3$, S_3 -type, non-Galois):

Quantity	Value
$\mathfrak{d}_K$	$-108 = 2^2 \cdot 27$, $v = 2$
components	$K \oplus \tilde{K}$, $\tilde{K} = K(\omega)$ (degree 6)
$\mathfrak{d}_{\tilde{K}}$	$-34992 = -2^4 \cdot 3^7$, $v_2 = 4$
Δ	$4 - 2v = 4 - 4 = \mathbf{0}$
$v_2(\rho^2)$	$dv - \Delta = 6 - 0 = \mathbf{6}$
Theorem 5.1 (Galois formula)	$dv = 6 \checkmark$

(Steps: $2dv = 12$; $\sum_j (e_j v + v_p(N \mathfrak{d}_{K_j/K})) = 1 \cdot 2 + 2 \cdot 2 + 0 = 6$; $12 - 6 = 6$. The tower formula gives $v_2(N \mathfrak{d}_{\tilde{K}/K}) = v_2(\mathfrak{d}_{\tilde{K}}) - 2v_2(\mathfrak{d}_K) = 4 - 4 = 0$, i.e. $\tilde{K}/K$ unramified, since $\mathbf{Q}_2(\zeta_3) = \mathbf{Q}_2(\sqrt{-3})$ is unramified over $\mathbf{Q}_2$.)

Corollary for IUT: $K = F(E[\ell])$ localized; for ℓ prime, $\ell \neq p$, $\mathbf{Q}_p(\zeta_\ell)/\mathbf{Q}_p$ is unramified $\implies \Delta \rightarrow 0$ and Theorem 5.1 is exact.

14.3. Explicit bound for Δ (G4 demoted to a precision item).

Localization in the original text.

Location	Content
IUT I Remark 3.1.7(ii) [1, p. 68]	“Write $F(\mu_\ell) \subseteq K$ for the subextension of K generated by the ℓ -th roots of unity”
same page	$\text{Gal}(K/F(\mu_\ell)) \cong SL_2(\mathbf{F}_\ell)$
IUT I Def 3.1(c)	$\ell \neq p_v$
IUT I Prop 1.8(vii) [1, p. 19]	at bad multiplicative reduction, the ℓ -torsion gives tame ramification, degree dividing ℓ
IUT II §1 [2, p. 20]	locally contains $\mu_{4\ell}$ (compatible with $\sqrt{-1} \in F + \mu_\ell \subseteq K$)
IUT IV Cor 2.2 (C1)	$\ell \asymp h^{1/2}$
IUT I Def 3.1(a)	F contains $\sqrt{-1}$ but not μ_ℓ (hierarchy: F no, K yes)

Bound on Δ . $\text{Gal}(K/F) \subseteq GL_2(\mathbf{F}_\ell) \implies [K : F(\mu_\ell)] \mid |SL_2(\mathbf{F}_\ell)| = \ell(\ell^2 - 1)$. Conservatively,

$$N := [\tilde{K}_v : \mathbf{Q}_{v\mathbf{Q}}] \leq [F : \mathbf{Q}] \cdot |GL_2(\mathbf{F}_\ell)|, \quad |GL_2(\mathbf{F}_\ell)| = \ell(\ell - 1)^2(\ell + 1) \sim \ell^4;$$

$$\Delta \leq v_p(\mathfrak{d}_{\tilde{K}}) \leq N(1 + v_p(N))$$

(Serre’s bound $v_p(\mathfrak{d}_{L/\mathbf{Q}_p}) \leq [L : \mathbf{Q}_p](1 + v_p([L : \mathbf{Q}_p]))$, [8, III §6]). Δ is a polynomial in ℓ , hence in h ($\ell \asymp h^{1/2}$).

★ *Core comparison.*

h	ℓ	N	$\Delta_{\max}$	$\log_{10} \Delta_{\max}$	J	$\log_{10} \log_p \rho$
10^2	10	1.8×10^4	5.3×10^4	4.73	4	1.39
10^4	100	2.0×10^8	7.9×10^8	8.90	49	15.04
10^6	1000	2.0×10^{12}	1.0×10^{13}	13.00	499	150.51
10^8	10^4	2.0×10^{16}	1.2×10^{17}	17.08	4999	1505.15
10^{12}	10^6	2.0×10^{24}	1.6×10^{25}	25.20	499999	150515.00

Share $\log_{10} \Delta_{\max} / \log_{10} \log_p \rho$: 10^2 : 3.40; 10^6 : 0.086; 10^{12} : 0.000167.

$$\frac{\Delta_{\max}}{\log_p \rho} = \frac{\text{poly}(h)}{\exp(c\sqrt{h})} \longrightarrow 0.$$

Stress test: even blowing $\Delta_{\max}$ up to N^2 (far beyond any wild bound), at $h = 10^6$ one gets 4.0×10^{24} versus 10^{150} (more precisely $\log_p \rho = \bar{v} \approx 10^{150.51}$): still 126 orders of magnitude short.

Remark 14.1 (Recomputed values superseding the preprint’s tables). The crossover table of §10.3 and the comparison table of this subsection were recomputed from the closed forms of this paper. They supersede the corresponding tables of the Chinese preprint. In particular the last row here ($\Delta_{\max} = 1.6 \times 10^{25}$) follows from the preprint’s own formula $\Delta_{\max} = N(1 + v_2(N))$ at $\ell = 10^6$ ($N = 2.0 \times 10^{24}$, $v_2(N) = 7$); the preprint’s displayed 6.2×10^{25} does not satisfy that formula and is corrected. All conclusions are unaffected.

Verdict. **G4 is demoted to a precision item.** Three pieces assembled:

- (1) original text: $K_v/\mathbf{Q}_{v\mathbf{Q}}$ non-Galois is the norm;
- (2) §14.2 closed form $\rho_{\text{NG}}^2 = \rho_{\text{G}}^2/p^\Delta$: Theorem 5.1 is always an upper bound;
- (3) this round: $\Delta \leq \text{poly}(h) \ll \exp(\Theta(\sqrt{h}))$.

$\implies$ the discount by Δ is asymptotically negligible; the crossover shifts to the right boundedly; the layer-1 conclusion is unchanged.

Honest reservations.

- $p_v \mid 6$ may have wild contribution ($E_{\text{mod}}[2 \cdot 3 \cdot 5]$ can be present); the tame argument does not apply and cases must be checked individually;
- $\Delta = 0$ is sufficient but not necessary: IUT only guarantees the global picture, and localization does not preserve surjectivity. But this round's conclusion does not rely on $\Delta = 0$, only on $\Delta \leq \Delta_{\text{max}}$.

15. GAP LIST FROM THE ORIGINAL TEXT

Ordered by value. G1–G3 affect the precision of the conclusion; G4/G5/G8 are closed (see rows); the only substantial gaps are G2/G7 (layer 2).

#	Gap	Why needed	Impact
G1	exact relation of $h = \log(q^\vee)$ to $\log \text{rad}$ of abc	§10 uses “target $\sim h$ ” as the comparison	greatly mitigated by §10.3 robustness: conclusion holds for κ up to 50–100. The order of h still needs confirmation from the original text
G2	precise form of $\text{Ind1}/\text{Ind2}$ and how hull saturates on the Ind track	the quantifiable part of layer 2	not computed. Needs the full definition of Ind2 in IUT III + the indeterminacy $\Phi(\mathcal{P})/\Xi(\mathcal{P})$ of Remark 3.9.5(iii)
G3	explicit formula for C_Θ in IUT IV Thm 1.10	the chain from qualitative inequality to quantitative one	endpoints known ($2.453 \times 10^{30} \rightarrow 400$), the middle of the chain unverified
G4	whether K_v is Galois	—	demoted to a precision item (§14.3): closed form $\rho_{\text{NG}}^2 = \rho_G^2/p^\Delta + \Delta \leq \text{poly}(h) \Rightarrow$ the discount is asymptotically negligible
G5	whether q generates K_v	—	logically closed: rank enters none of the layer-1a/1b/net-index chains; the only use (§8.6) needs only $\text{rank} \leq d < d^n$, which holds since $n = j + 1 \geq 2$
G5'	full coordinates of Prop. 3.4(ii) (per-component exponent distribution)	item-by-item confirmation of the all-diagonal model	precision item only
G6	actual d and $v_p(\mathfrak{d})$ of V^{bad}	which value §10 plugs in	scanned with full coverage at $d = 2, 4, 8$
G7	exact statement of (9-1) + the λ_e weight distribution	layer 2	no literature-level solution; awaits LANA [6]
G8	order of magnitude of $n_v = v_p(q)$	—	closed: $n_v = h_v$ (IUT I Def 3.1(c) + IUT IV Cor 2.2), $h_v \in \mathbf{Z}_{\geq 1}$ and $\asymp h \Rightarrow$ permanent inflation (§8.4)

16. CONCLUSION

Within IUT’s own parameter range, the integral-structure inflation of the hull operation is doubly exponential:

$$\log \log \rho = \Theta(\sqrt{h}).$$

Measured by crossover: even granting the target a magnitude as large as h^{1000} , the $d = 4$ inflation still overtakes it at $h \approx 9 \times 10^8$. The conclusion is robust to parameter uncertainty in all three directions.

Accurate statement about the “lower bound”: the ρ of Theorem 5.1 is the *exact* layer-1a value; layer 1b adds in the same direction (§8.5), making it a lower bound on the true inflation; while the compositum theorem of §14.2 gives $\rho_{\text{NG}}^2 = \rho_G^2/p^\Delta$ — i.e. in the non-Galois case ρ can only *decrease* — but §14.3 proves $\Delta \leq \text{poly}(h) \ll \exp(\Theta(\sqrt{h}))$, so the discount is asymptotically negligible. The three pieces combined leave the layer-1 conclusion unchanged.

Direction is locked: $n_v = h_v \in \mathbf{Z}_{>1}$ with $n_v \asymp h$ (§8.4) $\implies$ the layer-1 net effect is permanently inflationary and the contraction branch is unreachable in IUT’s parameter space. Hence the conclusions above rely on no assumption of “branch luck”.

This paper does not falsify IUT. What it proves is the narrower statement: *even if (9-1) holds, layer-1 inflation alone already suffices to make Corollary 3.12 fail to yield abc.*

The three gaps (Scholze–Stix non-commutativity, LANA compatibility, and the inflation of this paper) occupy the same position, are of different types, and are mutually independent. The only agent that can decide (9-1) is the follow-up work of LANA.

APPENDIX A. APPLICABILITY CONDITIONS OF THEOREM 5.1

The proof of Theorem 5.1 uses:

- (1) $K/\mathbf{Q}_p$ Galois (surjectivity in Lemma 2.1 relies on $K \otimes K$ splitting completely into d copies of K);
- (2) the trace-preservation Lemma 3.1 (valid for every Galois K and every n);
- (3) the discriminant relation (0.1).

For the non-Galois case, see §14.

APPENDIX B. SELF-CHECK CORRECTION RECORDS (10 ITEMS)

#	Error	Consequence / correction
B1	Bareiss division silently truncated without an assertion	reported $n = 4, 5$ for $\mathbf{Q}_2(\zeta_{16})^+$ as $-3, -1$; the hard constraint $v_p(\det) \geq 0$ catches it immediately; switched to Fraction + p -adic tracking; true value 1056
B2	$v_2(\mathfrak{d}_{\zeta_{32}^+})$ misused as 65	inflation column wrong; actual discriminant 2^{31} , i.e. 31
B3	degree of $\mathbf{Q}_2(\zeta_{32})$ labeled 8	entire row void; $[\mathbf{Q}_2(\zeta_{32}) : \mathbf{Q}_2] = 16$
B4	automorphism of $\mathbf{Q}_5(\zeta_5)$ assumed to be the identity	“all” not full rank ($4 \neq 16$); corrected to $\sigma_a(\theta) = \theta^a$
B5	mixed tensor product component count taken as 2^{n-1}	all mixed-field values void; true count $m = 2^{n-r}$, r the $\mathbf{F}_2$ -rank of the square classes
B6	§8 coordinate model error: value point assumed to be the axis point $(1, \dots, 1, q^{j^2}, 1, \dots, 1)$	contradicts Prop. 3.4(ii); per the original text, the value lies in the Galois-invariant submonoid and its coordinates are the full diagonal $(q^{j^2}, \dots, q^{j^2})$; the corrected loss is larger
B7	§8.2 inequality ill-defined: $\text{Vol}(\text{lattice}(\Theta))$ undefined for a low-rank lattice	comparison meaningless; changed to hull log-volumes on both sides
B8	§8.3 wording too strong: “rank identically d ”	counterexample exists; corrected to $\text{rank} \leq \min(\ell^*, [\mathbf{Q}_p(q) : \mathbf{Q}_p]) \leq d$, counterexample $q = 2\sqrt{6}$
B9	§10 asymptotic-order error: treating $\log_{10} \log_{10} \rho$ as $\log_{10} \rho$	crossover table underestimated by 10^{149} ; the truth is doubly exponential, $\log \log \rho = \Theta(\sqrt{h})$; everything recomputed
B10	§14.2 directional claim wrong: “non-Galois $\Rightarrow$ larger loss”	wrong on the side favorable to us; the compositum theorem proves $\rho_{\text{NG}} \leq \rho_{\text{G}}$; Theorem 5.1 is an upper bound

Three methodological lessons. Lesson from B1. -3 was once written into a report as a “new phenomenon” (albeit marked “not used as a conclusion”). The correct move is to stop it on the spot with a theoretical constraint: $v_p \geq 0$ is a hard constraint; a negative value is necessarily a bug.

Lesson from B9. When the value of a formula is itself a huge integer (2^{500}), “taking $\log_{10}$ ” and “taking $\log_{10} \log_{10}$ ” differ by one symbol, but the conclusions differ by an exponential layer. *Whenever large numbers are involved, the definition of every layer of quantity must be written out explicitly.*

Lesson from B10. (Same kind as B1 but more insidious.) B1 is a numerical violation of a hard constraint, decidable on the spot; B10 is a *directional* claim in the reasoning, with no hard constraint to catch it — and it was wrong *on the side favorable to us*. Such errors are the hardest to self-detect, because they make the conclusion “look stronger”.

Rule. Every directional claim favorable to our own conclusion must be given a proof or explicitly labeled as a conjecture; it may not be exempted from proof on the grounds of “conservative direction”.

Lesson from B5. The automorphism of $\mathbf{Q}_2(\sqrt{2})$ is $\theta \mapsto -\theta$; it cannot be written in the form $\theta \mapsto \theta^a$ — the latter form is valid only for Kummer/cyclotomic types.

Reproducibility. All scripts, data, and excerpts of the original text are appended to the project files and reproduce every number in this paper.

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