

10/24/26: FACTORING A PYTHAGOREAN DATE

SCOTT DUKE KOMINERS

ABSTRACT. Written numerically, the date October 24, 2026 corresponds to a Pythagorean triple—the last one of the twenty-first century! The associated 10–24–26 right triangle has area twice its perimeter; a classical factorization shows that exactly three integer-sided right triangles have this property and, more generally, counts those with any prescribed integer inradius. We also observe that 60 is the smallest perimeter shared by two noncongruent integer-sided right triangles, one of which is the 10–24–26 triangle.

On October 24, 2026, the date records a Pythagorean triple:

$$10^2 + 24^2 = 26^2.$$

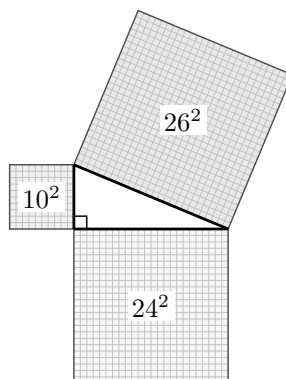


FIGURE 1. The date triangle and the three squares in its Pythagorean identity $10^2 + 24^2 = 26^2$.

Let m , d , and y be the month, day, and two-digit year. We call a date *Pythagorean* if $m^2 + d^2 = y^2$; thus the month and day are the legs and the year is the hypotenuse.

Our particular Pythagorean date gives the triangle $\Delta := (10, 24, 26) = 2(5, 12, 13)$. The 5–12–13 triangle is equable: its area and perimeter both equal 30. Dilation by 2 multiplies area by 4 and perimeter by 2, so our date triangle satisfies the identity

$$\text{area}(\Delta) = 120 = 2 \cdot 60 = 2 \text{ perimeter}(\Delta).$$

This corresponds to an inradius condition. Indeed, every triangle T of area K , semiperimeter s , and inradius r satisfies $K = rs$; since the perimeter is $2s$, it follows that

$$(1) \quad \frac{\text{area}(T)}{\text{perimeter}(T)} = \frac{r}{2}.$$

Thus the displayed area–perimeter identity says precisely that our date triangle Δ has inradius 4.

Date: October 24, 2026.

I used LLMs to assist in preparing this recreational math note, particularly GPT-6 Astra and Claude Opus 5.5 (both accessed in part via Poe with the support of Quora, where I am an advisor). The subject matter and $\sqrt{a^2 + b^2} = \text{celebration}$ are my own, and of course any errors remain my responsibility.

There are exactly two other integer-sided right triangles with the same area-to-perimeter ratio. Together, the three form something of a “Pythagorean trio:”

$$(9, 40, 41), \quad (10, 24, 26), \quad (12, 16, 20).$$

We derive this list as the $r = 4$ case of a general elementary classification below. The factorization approach we use is the classical argument for equable right triangles, run with an arbitrary inradius; it appears explicitly in the work of Omland [2]. Curry [1] instead derives the general classification by first counting the primitive triangles and then recovering all integer-sided right triangles by dilation; Robbins [3] gives a short independent count of the primitive ones. (Recall that an integer-sided right triangle is *primitive* if the greatest common divisor of its three side lengths is 1.)

Our purpose here is to use the factorization as an economical route to the particular geometry and calendar consequences of 10/24/26. As a final application, we observe that October 24, 2026 is the last Pythagorean date of the twenty-first century.

A FACTORIZATION

Let T be a right triangle with integer legs a and b , hypotenuse c , and inradius r . The incircle, together with the radii to its points of tangency on the two legs, cuts off an $r \times r$ square at the right angle, so both tangent segments from the right-angle vertex have length r . Writing x and y for the tangent lengths from the acute vertices at the far ends of the legs a and b , respectively, equality of tangent lengths gives $a = r + x$, $b = r + y$, and $c = x + y$. Hence

$$(2) \quad 2r = a + b - c.$$

Moreover, $a + b - c$ is even, because

$$0 = a^2 + b^2 - c^2 \equiv a + b - c \pmod{2}.$$

Thus the inradius of an integer-sided right triangle is an integer.

Theorem 1. *For every positive integer r , the integer-sided right triangles of inradius r are in bijection with the unordered factorizations*

$$\xi\eta = 2r^2$$

into positive integers. The factorization $2r^2 = \xi\eta$ corresponds to the triangle

$$(3) \quad (a, b, c) = (2r + \xi, 2r + \eta, 2r + \xi + \eta).$$

Consequently, the number $N(r)$ of such triangles is

$$(4) \quad N(r) = \frac{\tau(2r^2)}{2},$$

where $\tau(n)$ denotes the number of positive divisors of n .

Proof. By (2), we have $c = a + b - 2r$. Substitution into $a^2 + b^2 = c^2$ gives

$$a^2 + b^2 = (a + b - 2r)^2,$$

which rearranges to

$$(5) \quad (a - 2r)(b - 2r) = 2r^2.$$

Both factors in (5) are positive; for example, $2r = a + b - c < a$ because $b < c$. Thus $\xi = a - 2r$ and $\eta = b - 2r$ give a positive factorization of $2r^2$.

Conversely, if $\xi\eta = 2r^2$ and (a, b, c) is defined by (3), then

$$a^2 + b^2 - c^2 = 4r^2 - 2\xi\eta = 0$$

and $(a+b-c)/2 = r$. Hence (3) gives an integer-sided right triangle of inradius r . Interchanging ξ and η only interchanges the legs, so unordered factorizations give precisely the congruence classes of triangles.

Finally, $2r^2$ is not a square. Its divisors therefore pair off without a fixed point, giving (4). $\square$

If we have

$$r = 2^\alpha \prod_{i=1}^{\ell} p_i^{\beta_i},$$

where the p_i are distinct odd primes, then Theorem 1 immediately gives the more explicit formula

$$(6) \quad N(r) = (\alpha + 1) \prod_{i=1}^{\ell} (2\beta_i + 1).$$

POWER-OF-2 INRADIUS

Specializing Theorem 1 to $r = 2^k$ yields the family containing both the equable 5–12–13 triangle and the 10–24–26 date triangle.

Corollary 2. *For every integer $k \geq 0$, there are exactly $k + 1$ integer-sided right triangles T satisfying*

$$(7) \quad \frac{\text{area}(T)}{\text{perimeter}(T)} = 2^{k-1}.$$

They are indexed by $0 \leq j \leq k$ and have sides

$$\begin{aligned} a_{k,j} &= 2^{k+1} + 2^j, \\ b_{k,j} &= 2^{k+1} + 2^{2k+1-j}, \\ c_{k,j} &= 2^{k+1} + 2^j + 2^{2k+1-j}, \end{aligned}$$

with $\gcd(a_{k,j}, b_{k,j}, c_{k,j}) = 2^j$.

Proof. Equation (1) turns the condition (7) into the requirement that the inradius be 2^k . The unordered factorizations of

$$2(2^k)^2 = 2^{2k+1}$$

are

$$2^j \cdot 2^{2k+1-j} \quad (0 \leq j \leq k);$$

substitution into (3) gives the displayed sides. For the gcd assertion, it suffices to show that $\gcd(a_{k,j}, b_{k,j}) = 2^j$. (For any integer-sided right triangle, the greatest common divisor of the legs equals that of all three sides: if g divides both legs, then g^2 divides c^2 , so g divides c .) Put $h = k - j$. After division by 2^j , the first two sides are

$$2^{h+1} + 1 \quad \text{and} \quad 2^{h+1}(2^h + 1).$$

Their greatest common divisor is 1, since $2^{h+1} + 1$ is coprime to 2^{h+1} , because it is odd, and is coprime to $2^h + 1$, because $(2^{h+1} + 1) - 2(2^h + 1) = -1$. $\square$

By the gcd assertion, the $j = 0$ triangle is the only primitive member of each family, so for every $k \geq 0$ there is a unique primitive integer-sided right triangle of inradius 2^k . Since $(a_{k,j}, b_{k,j}, c_{k,j}) = 2^j(a_{k-j,0}, b_{k-j,0}, c_{k-j,0})$, the j -th triangle in Corollary 2 is 2^j times the unique primitive triangle of inradius 2^{k-j} . Thus the family for inradius 2^k is obtained by doubling every triangle of inradius 2^{k-1} and adjoining one new primitive triangle.

For $k = 1$, Corollary 2 recovers the classical fact that the only integer-sided right triangles with area equal to their perimeter are

$$(6, 8, 10) \quad \text{and} \quad (5, 12, 13).$$

For $k = 2$, the three factorizations

$$32 = 1 \cdot 32 = 2 \cdot 16 = 4 \cdot 8$$

give

$$(8) \quad (9, 40, 41), \quad (10, 24, 26), \quad (12, 16, 20).$$

Equivalently, the three triangles are

$$(9, 40, 41), \quad 2(5, 12, 13), \quad 4(3, 4, 5).$$

Our Pythagorean date triangle is the middle member; $(12, 16, 20)$ also corresponds to a Pythagorean date (December 16, 2020).

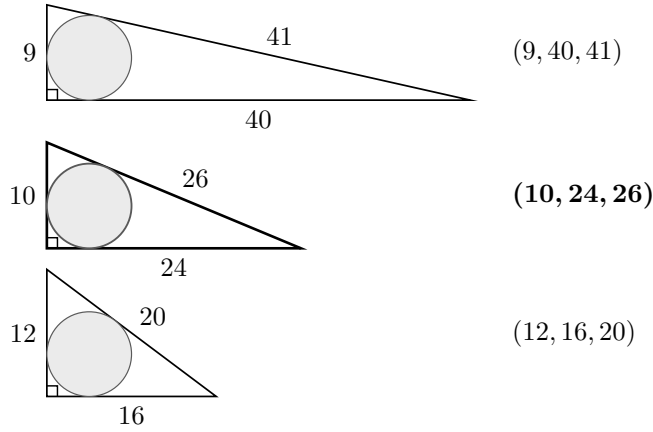


FIGURE 2. The Pythagorean trio: the three integer-sided right triangles of inradius 4, drawn to a common scale. Their incircles are congruent.

A SECOND COINCIDENCE

The perimeter 60 of the 10–24–26 triangle has a further distinguishing property.

Proposition 3. *The smallest positive integer that occurs as the perimeter of two noncongruent integer-sided right triangles is 60. The two triangles are*

$$(10, 24, 26) = 2(5, 12, 13) \quad \text{and} \quad (15, 20, 25) = 5(3, 4, 5).$$

Proof. Every integer-sided right triangle is an integer multiple of a primitive one. By Euclid's parametrization, after interchanging the legs if necessary, every primitive Pythagorean triangle has a unique pair of *generators* $u > v > 0$ with $\gcd(u, v) = 1$ and $u \not\equiv v \pmod{2}$, and its sides are

$$u^2 - v^2, \quad 2uv, \quad u^2 + v^2,$$

giving perimeter $2u(u + v)$. The primitive perimeters at most 60 are thus

$$12, \quad 30, \quad 40, \quad 56,$$

arising respectively from $(u, v) = (2, 1), (3, 2), (4, 1), (4, 3)$. Indeed, when $u \geq 5$, we have $2u(u + v) \geq 2 \cdot 5 \cdot 6 = 60$, with equality only for $(u, v) = (5, 1)$. That pair is excluded because

5 and 1 are both odd—and fittingly so, since Euclid’s formula applied to $(5, 1)$ produces the non-primitive triangle $(24, 10, 26)$, which is precisely our date triangle. Hence every admissible pair with $u \geq 5$ gives perimeter greater than 60. In particular, no primitive triangle has perimeter 60, and each primitive perimeter in the displayed list comes from a unique generator pair (u, v) .

If two noncongruent integer-sided right triangles have the same perimeter $P \leq 60$, then their primitive reductions must therefore be two distinct triangles from the displayed list: a fixed primitive triangle has only one dilation with perimeter P . Hence P is divisible by two distinct members of that list. The smallest least common multiple of a pair is

$$\text{lcm}(12, 30) = 60.$$

Thus no $P < 60$ can occur twice. At $P = 60$, the primitive perimeters 12 and 30 divide P , whereas 40 and 56 do not; hence they yield exactly the two triangles in the statement. $\square$

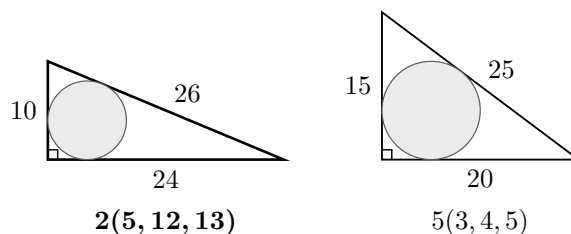


FIGURE 3. The smallest pair of noncongruent integer-sided right triangles with a common perimeter. Both perimeters are 60; their inradii are 4 and 5, respectively.

CODA: A CALENDAR COROLLARY

Corollary 4. *There are exactly twelve Pythagorean dates in each century. Using the $m/d/y$ convention, they are:*

$$\begin{array}{cccc}
 3/4/05, & 4/3/05, & 6/8/10, & 8/6/10, \\
 5/12/13, & 12/5/13, & 9/12/15, & 12/9/15, \\
 8/15/17, & 12/16/20, & 7/24/25, & \mathbf{10/24/26}.
 \end{array}$$

Moreover, October 24, 2026 is the last Pythagorean date of the twenty-first century.

Proof. Let r be the inradius of the triangle corresponding to a Pythagorean date (m, d, y) . Equation (2) gives

$$2r = m + d - y < m \leq 12,$$

because $d < y$. Hence $r \leq 5$. Theorem 1 gives the following complete lists for these five inradii:

r	integer-sided right triangles of inradius r
1	(3, 4, 5)
2	(5, 12, 13), (6, 8, 10)
3	(7, 24, 25), (8, 15, 17), (9, 12, 15)
4	(9, 40, 41), (10, 24, 26), (12, 16, 20)
5	(11, 60, 61), (12, 35, 37), (15, 20, 25).

Imposing $m \leq 12$ and $d \leq 31$, and retaining both orders of the legs when both give valid dates, yields exactly the twelve dates displayed above. Their largest year entry is 26, and it occurs only for 10/24/26. $\square$

Because the defining equation $m^2 + d^2 = y^2$ is symmetric in m and d , being Pythagorean is a property of the calendar day itself rather than of the way it is written: in day/month/year notation, our date is 24/10/26, and $24^2 + 10^2 = 26^2$ still holds.

The table also shows that no Pythagorean date has inradius 5, so our date triangle has the largest inradius possible. All three triangles of inradius 3 occur as dates (in 2015, 2017, and 2025), and two members of the Pythagorean trio supply the century's last two Pythagorean dates, 12/16/20 and 10/24/26.

Allowing the two-digit year to be a leg gives sixteen further dates per century, the last two being 7/25/24 and 10/26/24. Thus October 24, 2026 is the last date of the century whose three entries form a Pythagorean triple in *any* order. After that, the calendar is silent until April 5, 2103 (4/5/03), and there are no further Pythagorean dates in our strict sense until March 4, 2105—barring some truly substantial calendar reform, that is: the third member of the inradius-4 trio, (9, 40, 41), is still waiting for September 40, 2041.

REFERENCES

- [1] S. B. Curry, *Right triangles with inradius r* , Alabama J. Math., Spring/Fall 2005, 1–8.
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HARVARD BUSINESS SCHOOL; DEPARTMENT OF ECONOMICS AND CENTER OF MATHEMATICAL SCIENCES AND APPLICATIONS, HARVARD UNIVERSITY; AND A16Z FRONTIER RESEARCH

Email address: `kominers@fas.harvard.edu`