

A COUNTEREXAMPLE TO LIU'S CONJECTURE ON FANO INDICES IN DIMENSION FOUR

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ABSTRACT. We prove that Liu's conjecture on Fano indices fails in dimension four: 1389 is the index of a terminal Calabi–Yau fourfold and of a four-dimensional log canonical singularity, but it is not the Fano index of any canonical weighted projective space of dimension four.

1. INTRODUCTION

We work over $\mathbb{C}$. For a well-formed weighted projective space $\mathbb{P}(a_0, \dots, a_n)$ with canonical singularities, the Fano index is $q = a_0 + \dots + a_n$. The *index* of a Calabi–Yau variety S (a normal projective variety with $K_S \sim_{\mathbb{Q}} 0$) is the least $m \geq 1$ with $mK_S \sim 0$, and the index of a log canonical singularity $P \in Y$ is the least $m \geq 1$ such that mK_Y is Cartier at P . Following [7, Definition 4.1], let

- $I_{n,\text{wps}}$ be the set of Fano indices of n -dimensional well-formed weighted projective spaces with canonical singularities, together with $\{1, \dots, n\}$;
- $I_{n,\text{term}}$ be the set of indices of n -dimensional terminal Calabi–Yau varieties;
- $I_{n,\text{loc}}$ be the set of indices of n -dimensional log canonical singularities.

Conjecture 1.1 ([7, Conjecture 4.2]). $I_{n,\text{wps}} = I_{n,\text{term}} = I_{n,\text{loc}}$ for every $n \geq 2$.

Liu proves the conjecture for $n \leq 3$ [7, Proposition 4.3] and computes $I_{4,\text{wps}}$ by an exhaustive search [7, Theorem 4.4], [8].

Theorem 1.2. $1389 \in I_{4,\text{term}} \cap I_{4,\text{loc}}$ and $1389 \notin I_{4,\text{wps}}$. In particular, Conjecture 1.1 fails for $n = 4$.

The two memberships come from a symmetry of order 1389 of one Calabi–Yau threefold, the hypersurface X_{78} of $\mathbb{P}(1, 1, 11, 26, 39)$ (Sections 2–4). The non-membership is a finite statement about the 1,301,769,806 partitions of 1389 into five parts (Section 5). It is proved in the Lean 4 kernel from a certificate of witnesses, together with the polynomial identities of Section 2 (Section 6). Figure 1 shows the plan of the proof.

Remark 1.3. Read literally, $I_{4,\text{loc}}$ contains every positive integer: for $m \geq 2$ the group μ_m acting on $\mathbb{C}^4$ with weights $(1, 1, 1, m-2)$ is free in codimension one, so $\mathbb{C}^4/\mu_m$ is klt, and $dx_1 \wedge \dots \wedge dx_4$ has weight $m+1 \equiv 1 \pmod{m}$, so its index is m (by the argument in the proof of Proposition 4.2). Since $3487 \notin I_{4,\text{wps}}$ by [7, Theorem 4.4], the literal reading fails at once, and the intended reading of $I_{n,\text{loc}}$ concerns

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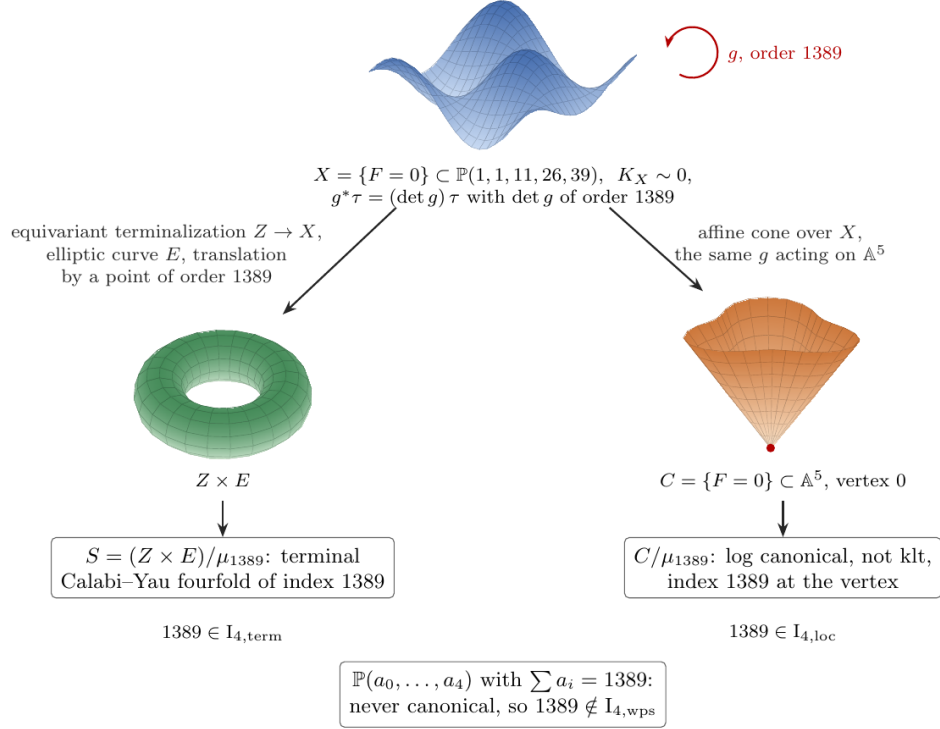


FIGURE 1. The plan of the proof. One symmetry g of order 1389 of the threefold X gives a terminal Calabi–Yau fourfold of index 1389 (Section 3) and a log canonical singularity of index 1389 (Section 4); no canonical weighted projective fourspace has Fano index 1389 (Section 5).

log canonical singularities that are not klt. The singularity of Proposition 4.2 is log canonical and not klt, so Theorem 1.2 holds for both readings.

2. THE THREEFOLD

Let $w = (w_0, \dots, w_4) = (1, 1, 11, 26, 39)$ and

$$F = x_0^{77}x_1 + x_1^{67}x_2 + x_2^7x_0 + x_3^3 + x_4^2 \in \mathbb{C}[x_0, \dots, x_4].$$

Lemma 2.1. (1) F is quasi-homogeneous of degree $78 = w_0 + \dots + w_4$ for the weights w .

(2) Any three of the w_i are coprime.

(3) The partial derivatives of F have no common zero in $\mathbb{C}^5$ other than 0.

Proof. (1) and (2) are direct. For (3), let x be a common zero; then $x_3 = x_4 = 0$. With $\partial_i = \partial F / \partial x_i$,

$$x_0\partial_0 - 77x_1\partial_1 + 5159x_2\partial_2 = 36114x_2^7x_0,$$

since $x_0\partial_0 = 77x_0^{77}x_1 + x_2^7x_0$, $x_1\partial_1 = x_0^{77}x_1 + 67x_1^{67}x_2$ and $x_2\partial_2 = x_1^{67}x_2 + 7x_2^7x_0$. Hence $x_2^7x_0 = 0$. If $x_2 = 0$, then $\partial_1 = x_0^{77}$ and $\partial_2 = x_1^{67} + 7x_2^6x_0$ give $x_0 = x_1 = 0$. If $x_0 = 0$, then $\partial_0 = x_2^7$ gives $x_2 = 0$ and ∂_2 gives $x_1 = 0$. $\square$

Let $C = \{F = 0\} \subset \mathbb{A}^5$ and $X = \{F = 0\} \subset \mathbb{P} = \mathbb{P}(1, 1, 11, 26, 39)$. By Lemma 2.1, X is a quasi-smooth hypersurface of degree 78; $\mathbb{P}$ and X are well formed [4, 6.10]. Hence X has cyclic quotient singularities and $\omega_X \cong \mathcal{O}_X(78 - \sum w_i) = \mathcal{O}_X$ [4, 6.14]. Quotient singularities are klt [6, 5.20], and the discrepancies of a klt variety with Cartier canonical divisor are integers, so X is a canonical Calabi–Yau threefold with $K_X \sim 0$.

Fix a primitive 1389th root of unity ζ and put $g = \text{diag}(\zeta, \zeta^{1312}, \zeta^{992}, 1, 1)$.

Lemma 2.2. (1) $F(gx) = F(x)$.

(2) $\det g = \zeta^{2305}$ is a primitive 1389th root of unity.

(3) For $0 < k < 1389$, every fixed point $x \in \mathbb{C}^5$ of g^k has $x_0 = x_1 = x_2 = 0$.

Proof. (1) The monomials $x_0^{77}x_1$, $x_1^{67}x_2$ and $x_2^7x_0$ acquire the factors

$$\zeta^{77+1312} = \zeta^{1389}, \quad \zeta^{67 \cdot 1312 + 992} = \zeta^{64 \cdot 1389}, \quad \zeta^{7 \cdot 992 + 1} = \zeta^{5 \cdot 1389}.$$

(2) $1389 = 3 \cdot 463$ and $2305 = 5 \cdot 461$ are coprime. (3) The integers 1, 1312 = $2^5 \cdot 41$ and 992 = $2^5 \cdot 31$ are prime to 1389, so ζ^k , ζ^{1312k} and ζ^{992k} differ from 1. $\square$

The matrix g commutes with the $\mathbb{C}^*$ -action $t \cdot x = (t^{w_0}x_0, \dots, t^{w_4}x_4)$, so $G = \langle g \rangle \cong \mu_{1389}$ acts on C , on $\mathbb{P}$ and on X . Let σ be the Poincaré residue of $dx_0 \wedge \dots \wedge dx_4 / F$ along C ; it generates ω_C , and $g^*\sigma = \det(g)\sigma$ by Lemma 2.2(1).

Lemma 2.3. $H^0(X, \omega_X)$ is spanned by a form τ with $g^*\tau = \det(g)\tau$.

Proof. Let $\xi = \sum w_i x_i \partial / \partial x_i$ and let $\rho: C \setminus \{0\} \rightarrow X$ be the quotient map. The 3-form $\iota_\xi \sigma$ is $\mathbb{C}^*$ -invariant, since σ has weight $\sum w_i - 78 = 0$, and $\iota_\xi \iota_\xi \sigma = 0$. Let $X^\circ \subset X$ be the open set over which ρ is a principal $\mathbb{C}^*$ -bundle; its complement has codimension at least 2 because X is well formed. On X° there is a nowhere vanishing 3-form τ with $\rho^*\tau = \iota_\xi \sigma$. As ω_X is reflexive, τ is a nowhere vanishing section of $\omega_X \cong \mathcal{O}_X$, so it spans $H^0(X, \omega_X)$. Since g commutes with ξ and with ρ , $g^*\tau = \det(g)\tau$. $\square$

3. A TERMINAL CALABI–YAU FOURFOLD OF INDEX 1389

Proposition 3.1. $1389 \in I_{4, \text{term}}$.

Proof. By [3, Proposition 2.2] there is a G -equivariant projective birational morphism $\pi: Z \rightarrow X$ with Z terminal and $K_Z = \pi^*K_X$. Then $\pi^*\tau$ is a nowhere vanishing section of ω_Z , and $g^*\pi^*\tau = \det(g)\pi^*\tau$ by Lemma 2.3. Let E be an elliptic curve with a point p of order 1389, and let G act on $Z \times E$ by $g(z, e) = (gz, e + p)$. The action is free, so $S = (Z \times E)/G$ is a terminal projective fourfold and $Z \times E \rightarrow S$ is étale. With η a translation-invariant 1-form on E , the form $\pi^*\tau \wedge \eta$ spans $H^0(Z \times E, \omega_{Z \times E})$ and g multiplies it by $\det(g)$. For $m \geq 1$, the sections of $\mathcal{O}_S(mK_S)$ are the G -invariant sections of $\omega_{Z \times E}^{\otimes m}$, so $H^0(S, \mathcal{O}_S(mK_S)) \neq 0$ if and only if $\det(g)^m = 1$, that is, $1389 \mid m$. As K_S is numerically trivial, a nonzero section of $\mathcal{O}_S(mK_S)$ has no zeros, so $mK_S \sim 0$ exactly when $1389 \mid m$. This is the argument of [3, Corollary 4.1]. Hence S is a terminal Calabi–Yau fourfold of index 1389. $\square$

4. A LOG CANONICAL SINGULARITY OF INDEX 1389

Lemma 4.1. C is normal and Gorenstein, log canonical, and not klt.

Proof. By Lemma 2.1(3), C is a hypersurface whose only singular point is 0, so C is Gorenstein and, by Serre's criterion, normal; its local ring at 0 is a domain, so C is irreducible and no coordinate x_j divides F .

Let $B \rightarrow \mathbb{A}^5$ be the weighted blow-up of 0 with weights w . It is covered by the charts $U_i = \mathbb{A}^5 / \mu_{w_i}$, where $\varphi_i: \mathbb{A}_y^5 \rightarrow \mathbb{A}_x^5$ is $x_i = y_i^{w_i}$, $x_j = y_j y_i^{w_j}$ ($j \neq i$), and $\varepsilon \in \mu_{w_i}$ acts by $y_i \mapsto \varepsilon y_i$, $y_j \mapsto \varepsilon^{-w_j} y_j$. Quasi-homogeneity gives $F \circ \varphi_i = y_i^{78} F_i$, where F_i is F with x_i replaced by 1 and x_j by y_j ; it does not involve y_i . Let $S_i = \{F_i = 0\}$ in the coordinates y_j , $j \neq i$. If F_i and its derivatives vanish at u (with $u_i = 1$), the Euler identity $78F = \sum w_j x_j \partial_j F$ gives $\partial_i F(u) = 0$, contradicting Lemma 2.1(3); so S_i is smooth. The strict transform W of C is covered by $W_i = V_i / \mu_{w_i}$ with $V_i = \mathbb{A}_{y_i}^1 \times S_i$, and the exceptional divisor $E_W = W \cap \pi^{-1}(0)$ is covered by D_i / μ_{w_i} with $D_i = \{0\} \times S_i$.

The action of μ_{w_i} on V_i is free in codimension one. A non-identity ε fixes no point with $y_i \neq 0$. Its fixed points in D_i satisfy $y_j = 0$ for some $j \neq i$ with $\varepsilon^{w_j} \neq 1$, which exists because the w_j with $j \neq i$ are coprime; and $\{y_j = 0\}$ contains no component of S_i because $x_j \nmid F$. Hence $q_i: V_i \rightarrow W_i$ is étale in codimension one, $q_i^* K_{W_i} = K_{V_i}$ and $q_i^* E_W = D_i$. The pair (V_i, D_i) is a smooth variety with a smooth divisor, hence log canonical, so (W, E_W) is log canonical by [6, 5.20].

The Jacobian of φ_i is $w_i y_i^{\sum w_j - 1}$, so

$$\varphi_i^*(dx/F) = w_i y_i^{\sum w_j - 1 - 78} dy / F_i = w_i y_i^{-1} dy / F_i,$$

and the pullback of σ to V_i is $w_i y_i^{-1}$ times a generator of ω_{V_i} . Thus the pullback of σ to W is a rational section of ω_W with divisor $-E_W$, that is, $K_W + E_W = \psi^* K_C$ for $\psi: W \rightarrow C$. By [6, Lemma 2.30], every divisor over C has the same discrepancy for C as for (W, E_W) . So C is log canonical, and E_W has discrepancy -1 , so C is not klt. $\square$

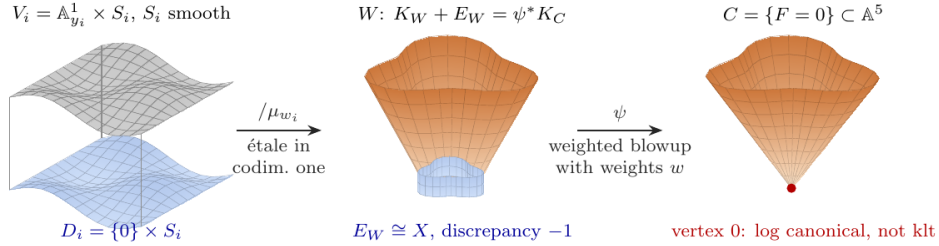


FIGURE 2. The proof of Lemma 4.1. The weighted blowup ψ replaces the vertex of C by $E_W \cong X$, and $K_W + E_W = \psi^* K_C$. On the chart V_i the pair (V_i, D_i) is a smooth variety with a smooth divisor, and $V_i \rightarrow W_i$ is a quotient by μ_{w_i} that is étale in codimension one.

Proposition 4.2. *Let $Y = C/G$ and let $P \in Y$ be the image of 0. Then $P \in Y$ is a four-dimensional log canonical singularity that is not klt and has index 1389. In particular, $1389 \in \mathbf{I}_{4,\text{loc}}$.*

Proof. By Lemma 2.2(3), the fixed locus of g^k on C for $0 < k < 1389$ lies in the curve $\{x_0 = x_1 = x_2 = 0, x_3^3 + x_4^2 = 0\}$. So $p: C \rightarrow Y$ is étale in codimension one, $K_C = p^*K_Y$, and Y is log canonical and not klt by Lemma 4.1 and [6, 5.20].

For $m \geq 1$, the sections of $\mathcal{O}_Y(mK_Y)$ near P are the G -invariant sections of $\omega_C^{\otimes m}$, that is, $f\sigma^m$ with $g^*f = \det(g)^{-m}f$. If $1389 \mid m$, then σ^m is invariant and generates. Conversely, if mK_Y is Cartier at P with generator $f\sigma^m$, then $f(0) \neq 0$, and $g(0) = 0$ gives $f(0) = \det(g)^{-m}f(0)$, so $1389 \mid m$ by Lemma 2.2(2). $\square$

5. WEIGHTED PROJECTIVE SPACES OF FANO INDEX 1389

For $a = (a_0, \dots, a_4) \in \mathbb{Z}_{>0}^5$ with $q = \sum a_i$ and $k \in \mathbb{Z}$, put

$$\text{age}_a(k) = \sum_{i=0}^4 \left\{ \frac{ka_i}{q} \right\} = k - \sum_{i=0}^4 \left\lfloor \frac{ka_i}{q} \right\rfloor.$$

By Kasprzyk's criterion [5, Proposition 2.5], a well-formed $\mathbb{P}(a)$ has canonical singularities if and only if $\text{age}_a(k) \in \{1, 2, 3\}$ for every $k \in \{2, \dots, q-2\}$.

Lemma 5.1. *Let $a \in \mathbb{Z}_{>0}^5$, $q = \sum a_i$, $k \in \mathbb{Z}$, and let $r_i \in \{0, \dots, q-1\}$ be the residue of ka_i modulo q . If $r_i \geq 1$ for $i = 1, 2, 3, 4$ and $r_1 + r_2 + r_3 + r_4 \leq q-1$, then $\text{age}_a(q-k) = 4$.*

Proof. Since $\sum ka_i = kq$, we have $r_0 \equiv -(r_1 + \dots + r_4) \pmod{q}$, and $1 \leq r_1 + \dots + r_4 \leq q-1$ gives $r_0 = q - (r_1 + \dots + r_4) \geq 1$. All five residues are positive, so $\{(q-k)a_i/q\} = 1 - r_i/q$ and $\text{age}_a(q-k) = 5 - \sum r_i/q = 4$. $\square$

Proposition 5.2. *Let $q = 1389$. For all integers $1 \leq a_4 \leq a_3 \leq a_2 \leq a_1$ with $2a_1 + a_2 + a_3 + a_4 \leq q$ there is $k \in \{2, \dots, q-2\}$ such that the residues r_i of ka_i modulo q satisfy $r_1, \dots, r_4 \geq 1$ and $r_1 + r_2 + r_3 + r_4 \leq q-1$.*

Proof. The proof is a finite verification, carried out in the Lean kernel (Section 6). For $t = (t_1, \dots, t_4)$ with $t_4 \leq t_3 \leq t_2 \leq t_1$ let $x_i = 1 + 8t_i$ and let Q_t be the box $\prod_i \{x_i, \dots, x_i + 7\}$. Every admissible $(a_1, \dots, a_4)$ lies in Q_t for $t_i = \lfloor (a_i - 1)/8 \rfloor$, and then $2x_1 + x_2 + x_3 + x_4 \leq q$. There are 368,253 such t . For each of them the certificate lists values of k , 2,624,293 in all and at most 70 per box; one value suffices for 231,139 boxes. The condition on k is equivalent to

$$(1) \quad \sum_{i=1}^4 (((ka_i + q - 1) \bmod q) + 1) \leq q - 1,$$

because $((ka + q - 1) \bmod q) + 1$ equals the residue of ka when that residue is positive and equals q otherwise. Write $a_i = x_i + \delta_i$ with $0 \leq \delta_i \leq 7$ and $c_i = (kx_i + q - 1) \bmod q$. Then $(ka_i + q - 1) \bmod q = c_i + k\delta_i - q \lfloor (c_i + k\delta_i)/q \rfloor$ and

$$\left\lfloor \frac{c_i + k\delta_i}{q} \right\rfloor = \# \left\{ j \geq 1 : \delta_i \geq \left\lceil \frac{jq - c_i}{k} \right\rceil \right\},$$

a sum of at most $\lfloor (c_i + 7k)/q \rfloor$ indicator functions of δ_i . The checker stores the 8^4 points of a box in 17-bit fields of one natural number and evaluates (1), the order conditions and the linear condition for all of them by additions, multiplications by integers, shifts and bitwise conjunctions; a box passes when every admissible point satisfies (1) for some listed k . The soundness of the checker, namely that a passing box has this property, is proved in Lean, and the kernel evaluates the checker on all boxes. $\square$

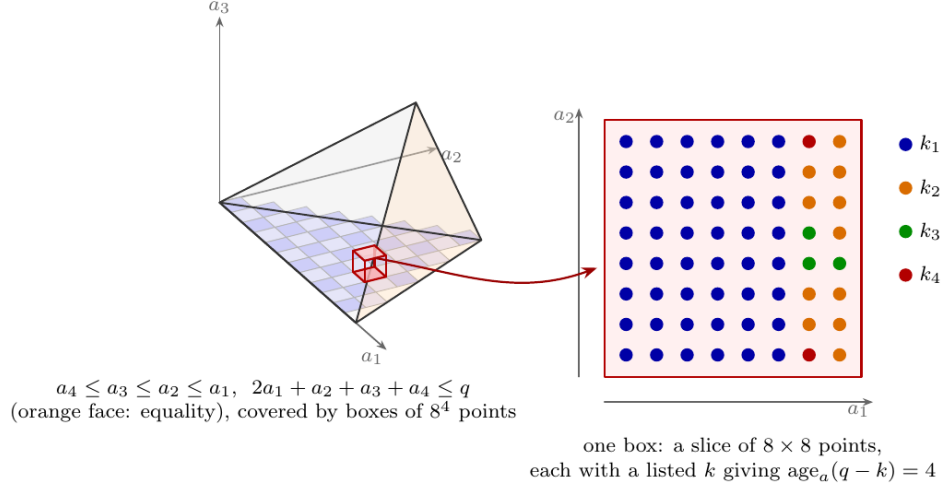


FIGURE 3. The certificate of Proposition 5.2. The sorted weights with $2a_1 + a_2 + a_3 + a_4 \leq q$ are covered by boxes of 8^4 points (drawn in the coordinates a_1, a_2, a_3). For each box the certificate lists values of k ; every point of the box is served by one of them. The right panel shows the points of one slice of one box, coloured by the listed value that serves them.

Theorem 5.3. *Every $a \in \mathbb{Z}_{>0}^5$ with $\sum a_i = 1389$ has $\text{age}_a(k) = 4$ for some $k \in \{2, \dots, 1387\}$. Hence no well-formed weighted projective space $\mathbb{P}(a_0, \dots, a_4)$ of Fano index 1389 has canonical singularities, and $1389 \notin I_{4, \text{wps}}$.*

Proof. Order the weights so that $a_0 \geq a_1 \geq \dots \geq a_4$. Then $2a_1 + a_2 + a_3 + a_4 \leq a_0 + \dots + a_4 = 1389$. Proposition 5.2 gives $k \in \{2, \dots, 1387\}$, and Lemma 5.1 gives $\text{age}_a(1389 - k) = 4$ with $1389 - k \in \{2, \dots, 1387\}$. Kasprzyk's criterion excludes canonical singularities, and $1389 > 4$. $\square$

Proof of Theorem 1.2. Combine Propositions 3.1 and 4.2 with Theorem 5.3. $\square$

Remark 5.4. Since $\varphi(1389) = 924 \leq 984$, the number 1389 lies in the set $\{m : \varphi(m) \leq 984\}$ that contains $I_{4, \text{wps}}$ by [7, Theorem 4.4]; it is one of the elements of the set J_4 of [7, Section 4]. Theorem 5.3 agrees with the list [8]. The neighbouring index 1383 lies in $I_{4, \text{wps}}$: $\mathbb{P}(674, 461, 195, 35, 18)$ is well formed and passes Kasprzyk's criterion.

6. FORMAL VERIFICATION

The finite and algebraic parts of the proof are formalized in Lean 4 with its mathematical library [9, 2]. In the formalization the set $I_{4, \text{wps}}$ is defined as the union of $\{1, 2, 3, 4\}$ with the sums $\sum a_i$ over the well-formed $a \in \mathbb{Z}_{>0}^5$ that satisfy Kasprzyk's criterion. The formal statements are Lemma 2.1, Lemma 2.2, Proposition 5.2, the first part of Theorem 5.3, and the statement $1389 \notin I_{4, \text{wps}}$. The kernel checks every proof, and each of these theorems depends only on the axioms of propositional extensionality, choice and quotient soundness.

The certificate of Proposition 5.2 is split into 4,022 statements, each asserting that the checker accepts one list of boxes. Each statement is closed by evaluating the checker in the kernel, so no compiled code is trusted; checking all of them takes about 15 minutes on four cores. The steps that rest on the literature, namely Kasprzyk's criterion, the results of [4] and [3], and the singularity theory of Sections 2–4, are proved in the text and not formalized, since the library does not yet contain weighted projective spaces or discrepancies.

Independently of the formalization, programs in C, C++, Julia and Python enumerate all 1,301,769,806 partitions of 1389 into five parts, test Kasprzyk's criterion on the 874,763,737 well-formed ones, and find no canonical one. The same enumeration for $q = 1383$ finds $\mathbb{P}(674, 461, 195, 35, 18)$.

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DATA AVAILABILITY

Deep Bhattacharjee computed the Lean formalization and the Julia, C, C++, shell and Python computations with the help of AI assistance (Claude, Anthropic). The Lean formalization of the finite and algebraic parts of the proof, together with the certificate generator and the search programs, is archived at Zenodo [1], doi:10.5281/zenodo.23387505. The paper is available at <https://github.com/creelie/fuzzy-guacamole/blob/main/preprintLiuFanoIndexConjecture/LiuFanoIndexConjecture.pdf>.

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