

THE BRIDGE INDEX AND THE NONORIENTABLE FOUR-GENUS OF KNOTS WITH AT MOST THIRTEEN CROSSINGS

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ABSTRACT. We determine two invariants for knots that the KnotInfo database leaves open. KnotInfo lists the bridge index of 536 prime knots with thirteen crossings as either 3 or 4; we prove that 243 of them have bridge index and meridional rank 4, so the meridional rank conjecture holds for them. The lower bound comes from representations of the knot group in $GL_4(\mathbb{F}_q)$ that send a meridian to a transvection or a reflection and fix no nonzero vector. KnotInfo leaves the smooth nonorientable four-genus open for 3525 prime knots with eleven to thirteen crossings; we determine it for 2537 of them and narrow the interval for 376 more. Upper bounds come from pinch moves, lower bounds from the Goeritz-lattice obstructions of Jabuka and Kelly evaluated by an exhaustive search for lattice embeddings. Every bound is a certificate checked by a program independent of the search that found it.

1. INTRODUCTION

The KnotInfo database [17] records, for each prime knot with at most thirteen crossings, the values of several dozen invariants, and an interval where the value is not known. This paper closes many of these intervals for two invariants, the bridge index and the smooth nonorientable four-genus, both in the version 2026.10.5 of the database. The two parts are independent and use different methods; they share the source of the diagrams, the form of the results, a list of certificates, one per knot, each checked by a short program, and the treatment of the knots that remain open.

The bridge index. Let $K \subset S^3$ be a knot, $G = \pi_1(S^3 \setminus K)$ its group and $m \in G$ a meridian. The *bridge index* $b(K)$ is the least number of local maxima of a height function on a knot isotopic to K . The *meridional rank* $\mu(K)$ is the least number of meridians, that is conjugates of $m^{\pm 1}$, that generate G . The overpasses of a b -bridge presentation give b meridians generating G , so $\mu(K) \leq b(K)$. Cappell and Shaneson asked whether equality always holds [15, Problem 1.11]; this is the meridional rank conjecture.

Blair, Kjurukova, Velazquez and Villanueva defined the Wirtinger number $\omega(D)$ of a diagram D and proved that its minimum over the diagrams of K is $b(K)$ [3]. In particular

$$(1.1) \quad b(K) \leq \omega(D) \quad \text{for every diagram } D \text{ of } K.$$

Blair, Kjurukova and Morrison bounded $\mu(K)$ from below by surjections of G onto Coxeter groups sending meridians to reflections, and so computed $b(K) = \mu(K)$ for 595,515 knots up to sixteen crossings [1]. KnotInfo records the bridge index of thirteen-crossing knots from these bounds and from the plain sphere number [2]. In the version 2026.10.5, 536 knots with thirteen crossings have the entry [3, 4].

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Theorem 1.1. *For each of the 243 knots K listed in Appendix A,*

$$b(K) = \mu(K) = 4.$$

In particular the meridional rank conjecture holds for these knots.

The upper bound is (1.1) applied to the KnotInfo diagram of K , whose Wirtinger number is 4. The lower bound is Lemma 2.1, applied to a representation $\rho: G \rightarrow \mathrm{GL}_4(\mathbb{F}_q)$ with $q \in \{3, 5\}$ that sends the meridian to a transvection of $\mathrm{Sp}_4(\mathbb{F}_q)$ or to a reflection of an orthogonal group $\mathrm{O}_4(\mathbb{F}_5)$. Reflections in Coxeter groups are pseudo-reflections too; the transvections, of order 3 and 5, are what settles most of these knots. Table 1 gives the counts.

group	image of a meridian	knots
$\mathrm{O}_4(\mathbb{F}_5)$	reflections	15
$\mathrm{Sp}_4(\mathbb{F}_3)$	transvections	67
$\mathrm{Sp}_4(\mathbb{F}_5)$	transvections	161

TABLE 1. The certificates of Theorem 1.1, by the group used.

The nonorientable four-genus. Every knot $K \subset S^3 = \partial B^4$ bounds a smooth surface in B^4 ; if K is not slice, it bounds nonorientable ones as well. The *nonorientable four-genus* $\gamma_4(K)$ is the least first Betti number of a smooth compact nonorientable surface $F \subset B^4$ with $\partial F = K$; for slice knots one sets $\gamma_4(K) = 1$. It was studied by Murakami and Yasuhara [18] and by Gilmer and Livingston [10]. Its values on small knots were computed by Jabuka and Kelly for eight and nine crossings [14], by Ghanbarian for ten [9] and by Fairchild for nonalternating knots with eleven [8]. Lee and Sabloff [16] extended the computation to fourteen crossings with an algorithmic search for Möbius bands in the style of Dunfield and Gong [7], together with the Arf invariant obstruction of Yasuhara [20] and a linking form obstruction of Murakami and Yasuhara [18]. KnotInfo incorporates these data. In its version 2026.10.5, 3525 knots with eleven to thirteen crossings have an interval $[a, b]$ with $a < b$ as the entry for γ_4 .

Theorem 1.2. *The value $\gamma_4(K)$ is determined for 2537 of these knots: 520 have $\gamma_4(K) = 1$, 1995 have $\gamma_4(K) = 2$ and 22 have $\gamma_4(K) = 3$. For 2177 of them a pinch move brings the upper end of the KnotInfo interval down to its lower end; for 163 the lower bound of Theorem 7.1 raises the lower end to the upper end; for 197 the two meet in between. For 376 further knots the interval is narrowed.*

The knots are listed in Appendix C, and Table 3 gives the counts by crossing number and by KnotInfo interval. Part I (Sections 2 to 4) proves Theorem 1.1, Part II (Sections 5 to 8) proves Theorem 1.2. Both parts end with what the methods do not reach.

Part I. The bridge index

2. A LOWER BOUND FROM PSEUDO-REFLECTIONS

A *pseudo-reflection* of a vector space V is an element $g \in \mathrm{GL}(V)$ with $\mathrm{rank}(g - 1) = 1$. It fixes the hyperplane $\ker(g - 1)$ pointwise.

Lemma 2.1. *Let V be a vector space of dimension n over a field and let $\rho: G \rightarrow \mathrm{GL}(V)$ be a homomorphism such that $\rho(m)$ is a pseudo-reflection. Let V^G be the subspace of vectors fixed by $\rho(G)$. Then*

$$\mu(K) \geq n - \dim V^G.$$

Proof. Let $m_1, \dots, m_k$ be meridians generating G , with $m_i = g_i m^{\epsilon_i} g_i^{-1}$ and $\epsilon_i = \pm 1$. Since $\rho(m)^{-1} - 1 = -\rho(m)^{-1}(\rho(m) - 1)$, each $\rho(m_i)$ is a pseudo-reflection, and $H_i = \ker(\rho(m_i) - 1)$ is a hyperplane. The intersection $H_1 \cap \dots \cap H_k$ has dimension at least $n - k$ and is fixed by every $\rho(m_i)$, hence by $\rho(G)$. So $n - k \leq \dim V^G$. $\square$

Corollary 2.2. *If $\dim V = 4$, $\rho(m)$ is a pseudo-reflection and $V^G = 0$, then $b(K) \geq \mu(K) \geq 4$.*

In a diagram with arcs $x_1, \dots, x_r$, the Wirtinger presentation makes such a ρ an assignment of matrices $X_i = \rho(x_i)$ to the arcs that satisfies $X_c = X_o^{\pm 1} X_a X_o^{\mp 1}$ at each crossing, where x_o is the overarc and x_a, x_c are the underarcs. Since the X_i generate $\rho(G)$,

$$V^G = \bigcap_{i=1}^r \ker(X_i - 1).$$

Both conditions are finite computations once the X_i are written down.

Transvections. Let ω be a nondegenerate alternating form on $V = \mathbb{F}_q^4$. For $v \neq 0$ the transvection $t_v(x) = x + \omega(x, v)v$ lies in $\mathrm{Sp}(V, \omega)$, $t_v - 1$ has rank one, $t_{-v} = t_v$, and $g t_v g^{-1} = t_{gv}$ for $g \in \mathrm{Sp}(V, \omega)$. A representation with $X_i = t_{v_i}$ is therefore the same thing as a coloring of the arcs by vectors with

$$v_c = \pm t_{v_o}^{\pm 1}(v_a)$$

at every crossing, and $\ker(X_i - 1) = v_i^\perp$. So $V^G = (\mathrm{span}\{v_i\})^\perp$, which is 0 exactly when the v_i span V .

Reflections. Let B be a nondegenerate symmetric bilinear form on $\mathbb{F}_q^4$, q odd. For $B(v, v) \neq 0$ the reflection $r_v(x) = x - 2\frac{B(x, v)}{B(v, v)}v$ satisfies $g r_v g^{-1} = r_{gv}$ for $g \in \mathrm{O}(B)$, and again $V^G = (\mathrm{span}\{v_i\})^\perp$.

3. THE CERTIFICATES FOR THE BRIDGE INDEX

For each knot of Appendix A the file `data/certificates.json` contains:

- the PD code of K from KnotInfo, with edges numbered $1, \dots, 2n$ along the orientation and $[a, b, c, d]$ the edges at a crossing counterclockwise from the incoming underedge a ;
- a prime q and a 4×4 matrix over $\mathbb{F}_q$ for every edge;
- four edges on distinct arcs, the seeds of a Wirtinger coloring.

The program `verify/verify.py` uses only the Python standard library. For every knot it checks:

- (1) the matrices are constant along each arc, where an arc is a maximal run of edges between two undercrossings;
- (2) at each crossing $[a, b, c, d]$ the relation $X_c = X_b^{-1} X_a X_b$ holds if the overstrand runs from b to d , and $X_c = X_b X_a X_b^{-1}$ if it runs from d to b (changing this convention at every crossing at once replaces K by its mirror image, which has the same b and μ);
- (3) each $X_i - 1$ has rank 1, and the matrices $X_i - 1$ stacked together have rank 4, so that $V^G = 0$;
- (4) starting from the four seed arcs, the coloring moves of [3] (if the overarc and one underarc at a crossing are colored, color the other underarc) color every arc, so $\omega(D) \leq 4$.

By Corollary 2.2 and (1.1), items (1)–(4) prove $b(K) = \mu(K) = 4$. With the option `--knotinfo` the program also checks that each PD code is the one in KnotInfo and that KnotInfo lists the bridge index as [3, 4]. All 243 certificates are checked in a few seconds. The program rejects altered certificates: a changed matrix entry, a seed set on three arcs, one crossing with its overstrand reversed, or identity matrices.

Remark 3.1. The certificates were found by a search, which plays no part in the proof. If the four seed vectors do not span V , every arc vector lies in their span, so a certificate must have seeds forming a basis. A coloring is then determined by the Gram matrix of the seeds: six entries of an alternating matrix, or six off-diagonal entries of a symmetric matrix with a fixed diagonal. The search runs over all such matrices and propagates the colors from the seeds through the crossings. The Wirtinger relations are then checked at the remaining crossings.

4. THE KNOTS WHOSE BRIDGE INDEX REMAINS OPEN

For the other 293 knots with entry [3, 4] (Appendix B) we found neither bound. The exhaustive searches described above found no representation satisfying Corollary 2.2 with meridians sent to transvections of $\mathrm{Sp}_4(\mathbb{F}_q)$ for $q \in \{3, 4, 5, 7, 8, 9, 11\}$, to reflections of $\mathrm{O}_4(\mathbb{F}_q)$ for $q \in \{3, 5, 7\}$ (both isometry types), or to pseudo-reflections of $\mathrm{GL}_4(\mathbb{F}_2)$ and $\mathrm{GL}_4(\mathbb{F}_3)$ with eigenvalue 1 or -1 . Random Reidemeister moves (SnapPy [5]) produced no diagram of Wirtinger number 3; 300 diagrams were tried for each knot. Abelian colorings give nothing either: for every prime $p < 110$ and every irreducible factor g of the Alexander polynomial modulo p , the colorings of the diagram by the Alexander quandle $\mathbb{F}_p[t]/(g)$, $a \triangleright b = ta + (1-t)b$, form a space of dimension at most 3, while dimension 4 would give $\mu(K) \geq 4$. Nor is any of these knots the closure of a 3-braid $\Delta^{2d}w$ in Murasugi's normal form with $|d| \leq 4$ and w either $\sigma_1^{-p_1}\sigma_2^{q_1} \cdots \sigma_1^{-p_s}\sigma_2^{q_s}$ of length at most $17 - 4 \max(|d| - 1, 0)$ or σ_2^m with $|m| \leq 20$ (53,309 braids, identified by SnapPy), which would give $b(K) \leq 3$. A diagram of Wirtinger number 3 would give $b(K) = \mu(K) = 3$, and a representation as in Corollary 2.2 over another field, or a Coxeter quotient of rank 4, would give $b(K) = \mu(K) = 4$.

Part II. The nonorientable four-genus

5. PINCH MOVES

Definition 5.1. Let D be a diagram of a knot K and c a crossing of D . Of the two smoothings of c , one respects the orientation of K and gives a two-component link; the other gives a knot K' . We say K' is obtained from K by a *pinch move* at c .

Lemma 5.2. *If K' is obtained from K by a pinch move, then $\gamma_4(K) \leq 1$ when K' is slice, and $\gamma_4(K) \leq \gamma_4(K') + 1$ in general.*

Proof. The pinch move is a band move, so there is a cobordism $\Sigma \subset S^3 \times [0, 1]$ from K to K' formed by $K \times [0, 1]$ and one band. Since K' is connected, the band does not respect the orientation of K , and Σ is a Möbius band with an open disk removed. If K' bounds a disk Δ in B^4 , then $\Sigma \cup \Delta$ is a Möbius band bounded by K , so $\gamma_4(K) \leq 1$. If K' bounds a surface F with $b_1(F) = \gamma_4(K')$, then $\Sigma \cup F$ is a nonorientable surface bounded by K with $b_1 = b_1(F) + 1$. This is the bound used by Jabuka and Kelly [14]. $\square$

A knot that is not slice has $\gamma_4 \geq 1$. For the knots whose KnotInfo interval starts at 2, the lower bound is the obstruction to bounding a Möbius band recorded by KnotInfo from Lee and Sabloff [16]: either Yasuhara's condition $4 \operatorname{Arf}(K) - \sigma(K) \in \{0, \pm 2\} \pmod{8}$ fails [20], or the linking form obstruction of Murakami and Yasuhara [18] applies. The further lower bounds are those of Section 7. Theorem 1.2 therefore rests on these obstructions, on the slice status and four-genus values recorded in KnotInfo for the targets, on the certificates of Section 6 and on the lattice computations of Section 7.

6. THE PINCH CERTIFICATES AND THEIR VERIFICATION

A certificate for a knot K consists of a planar diagram code D , a crossing c of D , a choice of smoothing, and a target: either the unknot or a knot T of KnotInfo. It asserts that

- (1) D is a diagram of K ;
- (2) the chosen smoothing of c gives a one-component diagram D' ;
- (3) D' represents T or its mirror image;
- (4) the bound $\beta = 1$ if T is slice or the unknot, and $\beta = \gamma_4(T) + 1$ otherwise, is smaller than the upper end of the KnotInfo interval of K .

Since γ_4 and sliceness are invariant under mirror image, (1) to (4) and Lemma 5.2 give $\gamma_4(K) \leq \beta$.

The certificates were found with SnapPy [5]: for every open knot we applied every pinch move to the KnotInfo diagram and to up to twenty-five diagrams obtained from it by random Reidemeister moves, and identified the resulting knots by hyperbolic structure. A candidate enters the file only if the verifier below accepts it; when it did not, the search went on with other crossings and other diagrams of the same knot. A second pass used the knots settled in the first pass as targets; the number of certificates with such a target is 2, and the verifier checks them after the certificates of their targets. A last pass applied every pinch move to twenty diagrams of each knot left open.

The verifier `crosscap4d/verify/verify.py` does not use SnapPy. It reads the KnotInfo database and uses Regina [4] for the topology. For (1) and (3) it proves that two diagrams represent the same knot up to mirror image in one of two ways.

- *Isotopy.* Both diagrams are simplified by Reidemeister moves to the same number of crossings, and the closures of the two minimal diagrams under Reidemeister moves that add at most one crossing share a diagram. Diagrams are compared by Regina's canonical signatures, taken up to reflection, rotation and reversal.
- *Complements.* The triangulated complements of the two diagrams have combinatorially isomorphic triangulations, possibly after Pachner moves that add at most one tetrahedron, or one of the triangulations obtained from forty simplifications of each complement under fixed seeds of Regina's random engine, together with the triangulations one Pachner move away from the smallest of them, occurs for both diagrams. Each of these triangulations is a triangulation of the complement it came from, so a common one shows that the complements are homeomorphic, and by the theorem of Gordon and Luecke [12] the knots agree up to mirror image.

Both tests are exact: a positive answer is a proof. Regina's simplification is randomised, so the verifier makes up to four attempts, each from a fixed seed, and gives each certificate two minutes. For the unknot the verifier uses Regina's recognition of the solid torus. Item (2) is checked by following the strands of D' , and (4) by reading KnotInfo.

The verifier accepts all 2728 certificates. It rejects certificates corrupted in four ways, on a sample of twenty: the other smoothing of the crossing, the next crossing, a wrong target, and a bound lowered by one (Table 2). The seven modified certificates with the next crossing that it accepts are valid certificates: that crossing gives the same target.

7. LOWER BOUNDS FROM DEFINITE LATTICES

Let D be a diagram of a knot K with a checkerboard colouring. Following Gordon and Litherland [11], give each crossing c the weight $\eta(c) = \pm 1$ according to the position of the shaded regions at c , let $X_0, \dots, X_n$ be the unshaded regions, and let G be the $n \times n$ matrix with $g_{ij} = -\sum \eta(c)$ over the crossings meeting X_i and X_j for $i \neq j$, and $g_{ii} = -\sum_{k \neq i} g_{ik}$ with k running from 0 to n . This is the Goeritz matrix; $\det G = \pm \det K$, and with $\mu = \sum \eta(c)$ over

corruption	rejected
other smoothing of the crossing	20 of 20
next crossing of the diagram	13 of 20
wrong target ($12a_1$)	20 of 20
bound lowered by one	20 of 20

TABLE 2. Corrupted pinch certificates rejected by the verifier.

the crossings of type II for the shaded surface,

$$(7.1) \quad \sigma(K) = \text{sign}(G) - \mu.$$

The form G is the intersection form of the double cover of B^4 branched along the shaded surface pushed into B^4 [11]. If D is reduced and alternating, the forms G_+ and G_- of the two colourings are positive and negative definite.

Theorem 7.1 (Jabuka and Kelly [14, Theorems 2.10–2.12]). *Let K be a knot with a reduced alternating diagram, let $P_+ = G_+$ and $P_- = -G_-$, of ranks $r_{\pm}$, and let $s \equiv \sigma(K) + 4 \text{Arf}(K) \pmod{8}$. Call $\ell \in \mathbb{N}$ admissible if $\ell \mid \det K$ and $\det K/\ell$ is a square.*

- (1) *If $s = 2$ and $P_- \oplus [\ell]$ embeds in $\mathbb{Z}^{r-+1}$ for no admissible ℓ , or $s = 6$ and $P_+ \oplus [\ell]$ embeds in $\mathbb{Z}^{r++1}$ for no admissible ℓ , then $\gamma_4(K) \geq 2$.*
- (2) *If $s = 0$ and neither $P_+ \oplus [\ell]$ embeds in $\mathbb{Z}^{r++1}$ nor $P_- \oplus [\ell]$ in $\mathbb{Z}^{r-+1}$, for any admissible ℓ , then $\gamma_4(K) \geq 2$.*
- (3) *If $s = 4$ and neither P_+ embeds in $\mathbb{Z}^{r++2}$ nor P_- in $\mathbb{Z}^{r-+2}$, then $\gamma_4(K) \geq 3$.*

Here $\mathbb{Z}^N$ carries the standard positive definite form.

We recall the argument. If K bounds a Möbius band $\Sigma \subset B^4$, the double branched cover $W(\Sigma)$ has $b_1 = 0$ [14] and $b_2 = 1$, its form is $[\pm\ell]$ with ℓ admissible by a theorem of Gilmer and of Owens and Strle [14, Corollary 2.8], and the congruence of Gilmer and Livingston [10] fixes the sign of ℓ by s . Gluing $W(\Sigma)$ with reversed orientation to the double cover bounded by the shaded surface whose Goeritz form has the opposite sign gives a closed definite smooth four-manifold whose form contains $G \oplus [\mp\ell]$ in full rank. By Donaldson's theorem [6] that form is diagonal; changing the sign of the whole form gives (1) and (2). In (3) a surface with $b_1 = 2$ would give a definite $W(\Sigma)$ of rank two, of either sign, and the same gluing embeds P_+ or P_- in a diagonal lattice of rank two more. In (1) the statement of [14] names the target lattice as ε -definite; the argument above, which is theirs, produces the lattice of the sign of G , and it is in this form that we apply it.

Since γ_4 is invariant under mirror image, and mirroring changes the sign of every weight $\eta(c)$, of G , of μ and of σ , it does not matter which of the two weight conventions is called positive, as long as σ is computed from (7.1) with the same weights.

Lemma 7.2. *Let P be a positive definite lattice of rank r and $\ell \in \mathbb{N}$. Then $P \oplus [\ell]$ embeds in $\mathbb{Z}^{r+1}$ if and only if there are an embedding $\phi: P \rightarrow \mathbb{Z}^{r+1}$ and an integer $k \geq 1$ with $\ell = k^2|w|^2$, where w is a primitive vector spanning the orthogonal complement of $\phi(P)$.*

Proof. The orthogonal complement of a sublattice of rank r in $\mathbb{Z}^{r+1}$ is $\mathbb{Z}w$ for a primitive w , and a vector of norm ℓ orthogonal to $\phi(P)$ is kw with $k^2|w|^2 = \ell$. $\square$

Lemma 7.3. *Let P be a positive definite Gram matrix of rank r and $N \geq r$. Every embedding of P in $\mathbb{Z}^N$ is carried by a signed permutation of the coordinates to one found by the following search. The basis vectors $e_1, \dots, e_r$ are placed in turn; if the vectors already placed are supported on the first t coordinates, the next vector is $(x, y, 0)$ with $x \in \mathbb{Z}^t$ arbitrary, subject to the norm*

and the inner products with the vectors already placed, and y a nonincreasing sequence of positive integers.

Proof. By induction on the number of vectors placed. Given an embedding $v_1, \dots, v_r$ whose first $i-1$ vectors are of the stated form, supported on the first t coordinates, the signed permutations of the coordinates beyond t fix $v_1, \dots, v_{i-1}$; one of them makes the entries of v_i beyond t nonnegative and nonincreasing, with its nonzero entries first, and it does not change the shape of the vectors already placed. $\square$

We take the KnotInfo diagram of each open alternating knot, compute both Goeritz forms and σ from (7.1), check that the two colourings give the same σ , that $|\sigma|$ agrees with KnotInfo and that the forms are definite, and run the search of Lemma 7.3 to the end: for (1) and (2) it lists the norms $|w|^2$ of Lemma 7.2 over all embeddings, for (3) it decides whether an embedding exists. Of the 3525 knots, the hypotheses of Theorem 7.1 hold and raise the KnotInfo lower end for 382: 360 reach $\gamma_4 \geq 2$ (314 by (1) and 46 by (2)) and 22 reach $\gamma_4 \geq 3$ by (3). The data are in `crosscap4d/data/lower_bounds.json`.

The program `crosscap4d/lower/verify_lower.py` recomputes every entry with the basis order of the search shuffled under a fixed seed, and runs three tests of the conventions. On the alternating knots with at most eleven crossings whose γ_4 KnotInfo records, the bound never exceeds the recorded value, while the same program with the sign of σ reversed contradicts the recorded value for more than two hundred of them. On the alternating knots that carry a pinch certificate, the bound never exceeds the upper bound of that certificate.

crossings	KnotInfo	knots	settled	narrowed	open
11	[1, 2]	14	8	0	6
11	[1, 3]	9	8	1	0
11	[2, 3]	36	33	0	3
12	[1, 2]	87	43	0	44
12	[1, 3]	87	49	35	3
12	[2, 3]	278	265	0	13
12	[2, 4]	3	3	0	0
13	[1, 2]	890	431	0	459
13	[1, 3]	677	318	338	21
13	[1, 4]	3	1	2	0
13	[1, 5]	1	0	0	1
13	[2, 3]	1437	1375	0	62
13	[2, 4]	3	3	0	0
all		3525	2537	376	612

TABLE 3. Open entries of γ_4 in KnotInfo 2026.10.5 and what the certificates give. “Settled”: the value is now known; “lowered”: the upper bound is lower.

8. THE KNOTS WHOSE NONORIENTABLE FOUR-GENUS REMAINS OPEN

The remaining 988 knots, with their narrowed intervals, are listed in the file `still_open.json` of the folder `crosscap4d/data`. For a knot with interval $[1, 2]$ that is not settled, no pinch move we tried gives a slice knot, and Theorem 7.1 does not apply or does not obstruct. Theorem 7.1 needs a definite diagram, so it says nothing about the nonalternating knots.

The bound $|\nu(K) - \sigma(K)/2| \leq \gamma_4(K)$ of Ozsváth, Stipsicz and Szabó [19], with $\nu = \Upsilon_K(1)$, does not raise any of these lower ends where we could evaluate it. For the open knots with Hom's invariant $\varepsilon(K) = 0$ the complex $CFK^\infty(K)$ is ε -equivalent to that of the unknot [13], so $\nu(K) = 0$ and the bound is $|\sigma(K)|/2$, which never exceeds the lower end of the KnotInfo interval. For the others, the knot Floer complexes we could compute are complexes over $\mathbb{F}[U, V]/(UV)$, which do not determine $\Upsilon_K(1)$.

Remark 8.1. The 520 Möbius bands found here are not in the data of Lee and Sabloff [16], whose search follows Dunfield and Gong [7] and runs over a finite family of bands chosen in advance. The knots settled at $\gamma_4 = 2$ were outside their aim, which was Möbius bands; for them the target of the pinch move is a knot known to bound a Möbius band but not a disk.

DATA AVAILABILITY

The certificates, the verifiers and the search programs for both parts are in the repository <https://github.com/creelie/conjecture-to-choose>: Part I in the folders **data**, **verify** and **search**, Part II in the folder **crosscap4d**. Release 1.3.0 of the repository is archived on Zenodo under doi:10.5281/zenodo.23561157; on GitHub it is <https://github.com/creelie/conjecture-to-choose/releases/tag/v1.3.0>.

USE OF AI TOOLS

The searches, the verification programs and drafts of the text were produced with Claude Code (Anthropic) under the direction of the author, who is responsible for the content.

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APPENDIX A. THE KNOTS OF THEOREM 1.1

13a₆₉, 13a₇₀, 13a₇₂, 13a₇₃, 13a₇₄, 13a₇₆, 13a₇₉, 13a₉₆, 13a₁₀₄, 13a₁₀₆, 13a₁₀₇, 13a₁₀₈, 13a₁₅₆, 13a₁₅₇, 13a₁₆₀, 13a₁₇₁, 13a₂₃₅, 13a₃₄₃, 13a₃₅₃, 13a₃₅₄, 13a₃₅₅, 13a₃₅₈, 13a₃₅₉, 13a₃₆₀, 13a₃₈₈, 13a₃₉₀, 13a₃₉₆, 13a₃₉₉, 13a₄₀₀, 13a₄₀₂, 13a₄₀₃, 13a₄₀₈, 13a₄₁₃, 13a₄₁₄, 13a₅₀₅, 13a₅₂₄, 13a₅₂₅, 13a₅₂₆, 13a₅₂₇, 13a₅₂₈, 13a₅₂₉, 13a₅₃₅, 13a₅₃₆, 13a₅₄₁, 13a₅₄₂, 13a₅₄₃, 13a₅₄₄, 13a₅₄₅, 13a₅₅₄, 13a₅₅₅, 13a₅₅₇, 13a₅₅₈, 13a₅₉₆, 13a₅₉₇, 13a₆₀₁, 13a₆₂₁, 13a₆₃₀, 13a₆₃₁, 13a₆₃₂, 13a₆₃₆, 13a₆₃₇, 13a₆₅₇, 13a₆₆₄, 13a₆₆₅, 13a₈₁₅, 13a₈₆₇, 13a₁₃₉₂, 13a₁₅₃₈, 13a₁₉₉₅, 13a₂₀₀₆, 13a₂₀₀₈, 13a₂₆₁₃, 13a₂₆₁₄, 13a₂₆₁₅, 13a₂₆₁₆, 13a₂₆₁₇, 13a₂₆₁₉, 13a₂₆₂₀, 13a₂₈₀₀, 13a₂₈₀₁, 13a₃₀₃₈, 13a₃₀₄₀, 13a₃₁₀₇, 13a₃₁₄₄, 13a₃₁₄₆, 13a₃₁₇₉, 13a₃₂₅₇, 13a₃₃₃₃, 13a₃₃₃₄, 13a₃₃₄₃, 13a₃₃₄₄, 13a₃₃₇₆, 13a₃₃₇₇, 13a₃₃₇₈, 13a₃₃₈₀, 13a₃₃₈₁, 13n₁₆₀, 13n₁₆₁, 13n₁₆₂, 13n₁₆₃, 13n₁₆₄, 13n₁₆₅, 13n₁₆₆, 13n₁₆₇, 13n₁₆₈, 13n₁₆₉, 13n₁₇₃, 13n₁₇₄, 13n₁₇₅, 13n₁₇₆, 13n₁₇₇, 13n₁₇₈, 13n₁₇₉, 13n₁₈₀, 13n₁₈₁, 13n₁₈₂, 13n₂₁₁, 13n₄₂₉, 13n₄₃₀, 13n₄₃₁, 13n₄₃₂, 13n₄₃₃, 13n₄₃₄, 13n₄₃₅, 13n₄₄₀, 13n₄₄₁, 13n₄₄₂, 13n₄₄₉, 13n₄₆₇, 13n₆₀₁, 13n₆₀₉, 13n₆₁₀, 13n₉₂₉, 13n₉₃₀, 13n₉₃₁, 13n₉₃₂, 13n₉₃₃, 13n₉₅₇, 13n₉₅₈, 13n₉₅₉, 13n₉₆₀, 13n₉₆₁, 13n₉₆₂, 13n₉₆₃, 13n₉₆₄, 13n₉₆₅, 13n₉₆₆, 13n₉₆₇, 13n₉₆₈, 13n₉₇₇, 13n₉₇₈, 13n₉₇₉, 13n₉₈₀, 13n₉₈₁, 13n₉₈₂, 13n₉₈₃, 13n₁₀₄₂, 13n₁₀₅₈, 13n₁₀₅₉, 13n₁₀₆₀, 13n₁₀₆₁, 13n₁₀₆₂, 13n₁₀₆₃, 13n₁₀₆₄, 13n₁₀₇₂, 13n₁₀₇₃, 13n₁₀₇₄, 13n₁₀₇₅, 13n₁₀₇₆, 13n₁₀₇₇, 13n₁₀₇₈, 13n₁₀₈₂, 13n₁₀₈₃, 13n₁₀₈₄, 13n₁₁₀₃, 13n₁₁₀₄, 13n₁₁₀₅, 13n₁₁₀₆, 13n₁₂₃₉, 13n₁₂₈₀, 13n₁₂₈₁, 13n₁₂₈₂, 13n₁₂₈₃, 13n₁₂₈₄, 13n₁₂₈₅, 13n₁₂₈₆, 13n₁₂₈₇, 13n₁₂₈₈, 13n₁₂₈₉, 13n₁₂₉₀, 13n₁₂₉₁, 13n₁₂₉₂, 13n₁₂₉₃, 13n₁₂₉₄, 13n₁₂₉₅, 13n₁₃₀₅, 13n₁₃₀₆, 13n₁₃₀₇, 13n₁₃₀₈, 13n₁₃₀₉, 13n₁₃₁₀, 13n₁₃₁₁, 13n₁₃₁₂, 13n₁₃₁₃, 13n₁₃₁₄, 13n₁₃₁₅, 13n₁₃₁₆, 13n₁₃₁₇, 13n₁₃₁₈, 13n₁₃₁₉, 13n₁₃₂₀, 13n₁₃₂₁, 13n₁₃₂₂, 13n₁₃₂₃, 13n₁₃₂₄, 13n₁₃₃₂, 13n₁₃₃₃, 13n₁₃₃₄, 13n₁₃₆₃, 13n₁₃₇₇, 13n₁₃₇₈, 13n₁₃₇₉, 13n₁₃₈₀, 13n₁₃₈₁, 13n₁₃₈₅, 13n₁₃₈₆, 13n₂₉₃₃, 13n₂₉₃₄, 13n₂₉₃₅, 13n₂₉₃₆, 13n₂₉₃₇, 13n₂₉₅₄, 13n₂₉₅₅, 13n₂₉₅₆, 13n₃₃₉₀, 13n₃₅₀₅, 13n₃₇₂₃, 13n₃₇₂₅, 13n₃₇₂₆, 13n₃₇₂₇, 13n₃₇₆₃, 13n₃₈₃₈, 13n₃₉₁₅.

APPENDIX B. THE KNOTS WITH BRIDGE INDEX ENTRY [3, 4] THAT REMAIN OPEN

13a₁₂₉, 13a₁₃₅, 13a₁₃₆, 13a₁₃₇, 13a₁₉₄, 13a₁₉₆, 13a₁₉₇, 13a₁₉₈, 13a₂₃₃, 13a₂₃₄, 13a₂₃₉, 13a₂₄₂, 13a₂₄₃, 13a₂₄₄, 13a₂₅₃, 13a₂₆₅, 13a₂₆₈, 13a₂₆₉, 13a₂₇₀, 13a₂₇₁, 13a₂₇₂, 13a₃₀₄, 13a₃₀₉, 13a₃₃₈, 13a₃₆₂, 13a₃₆₅, 13a₃₆₆, 13a₃₆₉, 13a₃₇₃, 13a₃₇₅, 13a₃₈₇, 13a₃₉₁, 13a₃₉₂, 13a₄₇₁, 13a₄₈₃, 13a₄₈₇, 13a₅₁₆, 13a₅₁₈, 13a₅₃₀, 13a₆₂₃, 13a₆₂₉, 13a₃₁₄₅, 13a₃₁₄₇, 13a₃₁₄₈, 13a₃₁₄₉, 13a₃₁₆₇, 13a₃₁₆₈, 13a₃₁₆₉, 13a₃₁₇₀, 13a₃₁₈₈, 13a₃₁₈₉, 13a₃₁₉₀, 13a₃₁₉₁, 13a₃₂₀₄, 13a₃₂₀₅, 13a₃₂₀₇, 13a₃₂₀₈, 13a₃₂₀₉, 13a₃₂₁₀, 13a₃₂₁₃, 13a₃₂₁₆, 13a₃₂₁₉, 13a₃₂₂₀, 13a₃₂₂₁, 13a₃₂₂₂, 13a₃₂₂₃, 13a₃₂₃₂, 13a₃₂₃₄, 13a₃₂₄₈, 13a₃₂₅₉, 13a₃₂₆₁, 13a₃₂₆₂, 13a₃₂₆₃, 13a₃₂₆₄, 13a₃₂₆₆, 13a₃₂₇₄, 13a₃₂₇₅, 13a₃₂₇₆, 13a₃₂₉₁, 13a₃₂₉₂, 13a₃₂₉₃, 13a₃₂₉₄, 13a₃₂₉₅, 13a₃₂₉₈, 13a₃₃₀₁, 13a₃₃₀₂, 13a₃₃₀₃, 13a₃₃₁₇, 13a₃₃₂₀, 13a₃₃₂₃, 13a₃₃₃₀, 13a₃₃₃₁, 13a₃₃₃₅, 13a₃₃₄₅, 13a₃₃₄₆, 13a₃₃₄₇, 13a₃₃₄₈, 13a₃₃₅₁, 13a₃₃₅₄, 13a₃₃₅₅, 13a₃₃₅₆, 13a₃₃₆₃, 13a₃₃₆₅, 13a₃₃₆₈, 13a₃₃₇₄, 13a₃₃₇₅, 13a₃₃₈₈, 13a₃₃₉₀, 13a₃₃₉₃, 13a₃₃₉₄, 13a₃₃₉₅, 13a₃₃₉₆, 13a₃₃₉₉, 13a₃₄₀₀, 13a₃₄₀₁, 13a₃₄₀₂, 13a₃₄₀₃, 13a₃₄₀₄, 13a₃₄₀₅, 13a₃₄₀₇, 13a₃₄₀₉, 13a₃₄₁₀, 13a₃₄₁₁, 13a₃₄₁₂, 13a₃₄₁₃, 13a₃₄₁₅, 13a₃₄₁₇, 13a₃₄₁₈, 13a₃₈₁₁, 13a₃₈₁₂, 13n₃₂₆, 13n₃₂₉, 13n₃₃₀, 13n₃₄₁, 13n₃₄₄, 13n₃₄₅, 13n₃₄₈, 13n₃₅₁, 13n₃₅₂, 13n₃₅₅, 13n₃₅₇, 13n₃₅₈, 13n₅₁₀, 13n₅₁₃, 13n₅₁₄, 13n₅₁₈, 13n₅₂₀, 13n₅₂₁, 13n₅₂₄, 13n₅₂₅, 13n₅₂₆, 13n₅₂₉, 13n₅₃₀, 13n₅₃₁, 13n₅₈₂, 13n₅₉₀, 13n₅₉₁, 13n₅₉₈, 13n₅₉₉, 13n₆₀₀, 13n₆₂₇, 13n₆₃₂, 13n₆₃₃, 13n₆₄₄, 13n₆₄₇, 13n₆₄₈, 13n₆₅₁, 13n₆₅₂, 13n₆₆₁, 13n₆₆₂, 13n₆₆₃, 13n₆₉₄, 13n₆₉₅, 13n₆₉₆, 13n₇₀₄, 13n₇₀₅, 13n₇₀₆, 13n₇₀₇, 13n₇₀₈, 13n₇₀₉, 13n₇₁₂, 13n₇₁₃, 13n₇₁₆, 13n₇₁₇, 13n₇₁₈, 13n₈₃₄, 13n₈₃₈, 13n₈₃₉, 13n₈₅₇, 13n₉₂₆, 13n₉₉₁, 13n₉₉₂, 13n₉₉₃, 13n₁₀₀₀, 13n₁₀₀₁, 13n₁₀₀₄, 13n₁₀₀₅, 13n₁₀₀₆, 13n₁₀₀₇,

13n₁₀₁₁, 13n₁₀₁₂, 13n₁₀₁₃, 13n₁₀₃₉, 13n₁₀₄₀, 13n₁₀₄₁, 13n₁₀₄₃, 13n₁₀₄₄, 13n₁₀₄₅, 13n₁₀₄₆, 13n₁₀₄₇, 13n₁₀₄₈,
 13n₁₁₇₅, 13n₁₁₇₈, 13n₁₁₇₉, 13n₁₂₀₀, 13n₁₂₀₁, 13n₁₂₀₂, 13n₁₂₁₀, 13n₁₂₁₁, 13n₁₂₁₂, 13n₁₂₇₀, 13n₁₂₇₆, 13n₁₂₉₆,
 13n₁₃₆₇, 13n₁₃₇₀, 13n₁₃₇₁, 13n₁₃₇₄, 13n₁₃₇₅, 13n₁₃₇₆, 13n₃₇₃₉, 13n₃₇₄₂, 13n₃₇₄₅, 13n₃₇₄₈, 13n₃₇₇₀, 13n₃₇₇₃,
 13n₃₇₇₆, 13n₃₇₇₉, 13n₃₈₁₄, 13n₃₈₄₀, 13n₃₈₄₂, 13n₃₈₄₄, 13n₃₈₅₄, 13n₃₈₅₅, 13n₃₈₅₆, 13n₃₈₇₀, 13n₃₈₇₁, 13n₃₈₇₂,
 13n₃₈₇₃, 13n₃₈₇₄, 13n₃₈₉₇, 13n₃₈₉₈, 13n₃₈₉₉, 13n₃₉₀₀, 13n₃₉₀₁, 13n₃₉₀₃, 13n₃₉₀₄, 13n₃₉₀₅, 13n₃₉₁₁, 13n₃₉₁₄,
 13n₃₉₁₆, 13n₃₉₁₈, 13n₃₉₁₉, 13n₃₉₂₀, 13n₃₉₂₁, 13n₃₉₂₂, 13n₃₉₂₄, 13n₃₉₂₅, 13n₃₉₂₈, 13n₃₉₂₉, 13n₃₉₃₁, 13n₃₉₃₂,
 13n₃₉₃₃, 13n₃₉₃₄, 13n₃₉₃₅, 13n₃₉₃₆, 13n₃₉₃₇, 13n₃₉₃₈, 13n₃₉₃₉, 13n₃₉₄₀, 13n₃₉₄₁, 13n₄₅₈₇, 13n₄₅₈₉, 13n₄₆₃₃,
 13n₄₆₃₄, 13n₄₆₃₇, 13n₄₆₃₈, 13n₄₆₃₉, 13n₄₆₄₀, 13n₄₆₄₁, 13n₄₆₄₂, 13n₄₆₄₃, 13n₄₆₄₄, 13n₄₆₄₅.

APPENDIX C. THE KNOTS OF THEOREM 1.2

$\gamma_4 = 1$ (**520 knots**).

12a_133	12n_843	13a_619	13a_1231	13a_1907	13a_3266
12a_200	13a_5	13a_680	13a_1237	13a_2059	13a_3274
12a_212	13a_6	13a_687	13a_1250	13a_2094	13a_3275
12a_218	13a_12	13a_699	13a_1277	13a_2166	13a_3279
12a_313	13a_45	13a_712	13a_1316	13a_2218	13a_3294
12a_335	13a_79	13a_714	13a_1320	13a_2308	13a_3298
12a_373	13a_129	13a_721	13a_1324	13a_2311	13a_3302
12a_390	13a_131	13a_726	13a_1362	13a_2323	13a_3306
12a_457	13a_136	13a_730	13a_1380	13a_2328	13a_3328
12a_474	13a_138	13a_732	13a_1412	13a_2339	13a_3333
12a_611	13a_156	13a_734	13a_1417	13a_2389	13a_3343
12a_639	13a_158	13a_776	13a_1421	13a_2402	13a_3354
12a_698	13a_166	13a_777	13a_1436	13a_2411	13a_3355
12a_709	13a_180	13a_786	13a_1438	13a_2414	13a_3356
12a_750	13a_197	13a_815	13a_1454	13a_2502	13a_3374
12a_818	13a_224	13a_823	13a_1477	13a_2505	13a_3399
12a_878	13a_232	13a_824	13a_1506	13a_2506	13a_3403
12a_882	13a_244	13a_827	13a_1511	13a_2511	13a_3408
12a_895	13a_247	13a_845	13a_1514	13a_2560	13a_3413
12a_916	13a_253	13a_861	13a_1523	13a_2595	13a_3415
12a_932	13a_265	13a_906	13a_1532	13a_2632	13a_3417
12a_1069	13a_307	13a_916	13a_1598	13a_2645	13a_3418
12a_1070	13a_318	13a_925	13a_1608	13a_2676	13a_3462
12a_1092	13a_338	13a_988	13a_1609	13a_2788	13a_3463
12a_1094	13a_342	13a_990	13a_1624	13a_2828	13a_3475
12a_1102	13a_343	13a_995	13a_1627	13a_2864	13a_3477
12a_1115	13a_353	13a_1000	13a_1648	13a_2870	13a_3491
12a_1206	13a_365	13a_1027	13a_1656	13a_2893	13a_3494
12a_1208	13a_369	13a_1068	13a_1658	13a_2925	13a_3497
12n_32	13a_375	13a_1072	13a_1665	13a_3012	13a_3505
12n_70	13a_381	13a_1077	13a_1666	13a_3026	13a_3515
12n_147	13a_392	13a_1087	13a_1671	13a_3028	13a_3522
12n_382	13a_396	13a_1103	13a_1673	13a_3039	13a_3556
12n_390	13a_410	13a_1106	13a_1712	13a_3046	13a_3579
12n_444	13a_476	13a_1122	13a_1719	13a_3057	13a_3598
12n_495	13a_505	13a_1131	13a_1722	13a_3058	13a_3609
12n_554	13a_518	13a_1134	13a_1727	13a_3076	13a_3611
12n_555	13a_519	13a_1140	13a_1731	13a_3167	13a_3618
12n_597	13a_530	13a_1163	13a_1756	13a_3181	13a_3653
12n_632	13a_537	13a_1169	13a_1758	13a_3188	13a_3657
12n_642	13a_538	13a_1176	13a_1767	13a_3190	13a_3674
12n_769	13a_550	13a_1186	13a_1851	13a_3198	13a_3689
12n_784	13a_591	13a_1193	13a_1856	13a_3242	13a_3692
12n_831	13a_610	13a_1230	13a_1862	13a_3258	13a_3741

13a_3758	13a_4443	13n_370	13n_1628	13n_2949	13n_4193
13a_3764	13a_4449	13n_406	13n_1638	13n_2966	13n_4202
13a_3772	13a_4475	13n_418	13n_1670	13n_2977	13n_4247
13a_3776	13a_4479	13n_428	13n_1677	13n_2985	13n_4272
13a_3786	13a_4486	13n_520	13n_1678	13n_3108	13n_4300
13a_3787	13a_4487	13n_529	13n_1688	13n_3190	13n_4305
13a_3799	13a_4493	13n_530	13n_1693	13n_3198	13n_4306
13a_3805	13a_4511	13n_561	13n_1726	13n_3236	13n_4313
13a_3807	13a_4513	13n_563	13n_1789	13n_3311	13n_4333
13a_3811	13a_4524	13n_620	13n_1932	13n_3334	13n_4334
13a_3866	13a_4544	13n_635	13n_1935	13n_3338	13n_4351
13a_3905	13a_4577	13n_671	13n_1976	13n_3344	13n_4354
13a_3928	13a_4591	13n_813	13n_1992	13n_3351	13n_4380
13a_3930	13a_4603	13n_816	13n_2021	13n_3402	13n_4457
13a_3955	13a_4620	13n_839	13n_2022	13n_3452	13n_4477
13a_3993	13a_4629	13n_852	13n_2080	13n_3501	13n_4485
13a_4054	13a_4647	13n_855	13n_2088	13n_3526	13n_4509
13a_4077	13a_4648	13n_892	13n_2115	13n_3602	13n_4515
13a_4103	13a_4654	13n_932	13n_2150	13n_3641	13n_4517
13a_4119	13a_4696	13n_988	13n_2193	13n_3667	13n_4521
13a_4141	13a_4697	13n_994	13n_2245	13n_3695	13n_4543
13a_4152	13a_4705	13n_999	13n_2265	13n_3723	13n_4614
13a_4178	13a_4743	13n_1005	13n_2383	13n_3739	13n_4632
13a_4195	13a_4744	13n_1057	13n_2404	13n_3740	13n_4642
13a_4198	13a_4746	13n_1061	13n_2406	13n_3745	13n_4657
13a_4211	13a_4794	13n_1062	13n_2409	13n_3752	13n_4666
13a_4220	13a_4848	13n_1067	13n_2453	13n_3771	13n_4709
13a_4247	13a_4874	13n_1111	13n_2503	13n_3776	13n_4746
13a_4249	13n_7	13n_1114	13n_2610	13n_3899	13n_4870
13a_4268	13n_30	13n_1201	13n_2629	13n_3902	13n_4896
13a_4289	13n_50	13n_1243	13n_2690	13n_3916	13n_4940
13a_4338	13n_54	13n_1275	13n_2694	13n_3919	13n_5006
13a_4342	13n_78	13n_1277	13n_2706	13n_3921	13n_5009
13a_4365	13n_149	13n_1302	13n_2741	13n_3923	13n_5012
13a_4367	13n_153	13n_1328	13n_2765	13n_3933	13n_5015
13a_4394	13n_157	13n_1419	13n_2773	13n_3968	13n_5041
13a_4398	13n_231	13n_1490	13n_2812	13n_4023	13n_5067
13a_4403	13n_290	13n_1492	13n_2834	13n_4030	13n_5089
13a_4412	13n_330	13n_1494	13n_2837	13n_4107	13n_5095
13a_4419	13n_341	13n_1500	13n_2851	13n_4115	13n_5104
13a_4426	13n_344	13n_1582	13n_2857	13n_4152	13n_5108
13a_4430	13n_347	13n_1585	13n_2926	13n_4155	
13a_4437	13n_360	13n_1609	13n_2942	13n_4162	

$\gamma_4 = 2$ (1995 knots).

11a_6	11a_144	11a_241	11a_299	11a_354	12a_44
11a_17	11a_161	11a_248	11a_305	11a_355	12a_47
11a_20	11a_163	11a_259	11a_308	11a_360	12a_49
11a_29	11a_173	11a_264	11a_314	11a_362	12a_52
11a_49	11a_178	11a_274	11a_318	11a_366	12a_55
11a_51	11a_188	11a_277	11a_319	12a_9	12a_64
11a_53	11a_192	11a_278	11a_329	12a_29	12a_66
11a_59	11a_203	11a_280	11a_337	12a_35	12a_67
11a_64	11a_206	11a_292	11a_338	12a_36	12a_76
11a_66	11a_214	11a_293	11a_339	12a_39	12a_85
11a_89	11a_240	11a_297	11a_352	12a_43	12a_88

12a_96	12a_551	12a_816	12a_1079	12n_153	12n_830
12a_97	12a_553	12a_817	12a_1080	12n_164	12n_863
12a_103	12a_556	12a_820	12a_1084	12n_169	12n_866
12a_113	12a_570	12a_822	12a_1088	12n_173	12n_874
12a_114	12a_575	12a_827	12a_1091	12n_174	12n_880
12a_117	12a_587	12a_828	12a_1097	12n_185	12n_881
12a_134	12a_590	12a_829	12a_1099	12n_186	13a_4
12a_136	12a_592	12a_832	12a_1103	12n_190	13a_8
12a_137	12a_595	12a_840	12a_1106	12n_211	13a_16
12a_143	12a_597	12a_841	12a_1107	12n_229	13a_25
12a_146	12a_610	12a_844	12a_1112	12n_243	13a_26
12a_148	12a_616	12a_846	12a_1114	12n_247	13a_29
12a_152	12a_628	12a_853	12a_1116	12n_260	13a_32
12a_156	12a_633	12a_855	12a_1120	12n_269	13a_33
12a_167	12a_636	12a_859	12a_1122	12n_289	13a_35
12a_176	12a_641	12a_869	12a_1137	12n_305	13a_40
12a_186	12a_647	12a_873	12a_1142	12n_317	13a_51
12a_188	12a_650	12a_876	12a_1143	12n_330	13a_52
12a_193	12a_651	12a_880	12a_1152	12n_361	13a_56
12a_195	12a_653	12a_881	12a_1159	12n_402	13a_58
12a_224	12a_655	12a_885	12a_1164	12n_432	13a_59
12a_239	12a_657	12a_890	12a_1171	12n_472	13a_65
12a_254	12a_661	12a_899	12a_1172	12n_503	13a_71
12a_264	12a_665	12a_905	12a_1177	12n_509	13a_82
12a_269	12a_669	12a_912	12a_1179	12n_517	13a_84
12a_276	12a_674	12a_923	12a_1180	12n_542	13a_85
12a_277	12a_681	12a_927	12a_1184	12n_557	13a_88
12a_289	12a_686	12a_937	12a_1189	12n_585	13a_89
12a_296	12a_689	12a_946	12a_1193	12n_603	13a_96
12a_297	12a_692	12a_950	12a_1197	12n_627	13a_97
12a_304	12a_693	12a_969	12a_1200	12n_638	13a_98
12a_331	12a_694	12a_974	12a_1222	12n_639	13a_103
12a_332	12a_702	12a_981	12a_1223	12n_651	13a_112
12a_338	12a_703	12a_984	12a_1229	12n_671	13a_122
12a_347	12a_706	12a_986	12a_1232	12n_672	13a_123
12a_370	12a_712	12a_994	12a_1240	12n_675	13a_125
12a_371	12a_714	12a_996	12a_1242	12n_682	13a_132
12a_389	12a_726	12a_1001	12a_1244	12n_688	13a_133
12a_392	12a_730	12a_1002	12a_1248	12n_691	13a_134
12a_426	12a_738	12a_1004	12a_1252	12n_692	13a_140
12a_428	12a_742	12a_1006	12a_1255	12n_694	13a_150
12a_437	12a_753	12a_1009	12a_1256	12n_707	13a_151
12a_446	12a_762	12a_1012	12a_1260	12n_712	13a_154
12a_449	12a_763	12a_1013	12a_1265	12n_717	13a_157
12a_460	12a_767	12a_1015	12a_1266	12n_722	13a_160
12a_461	12a_783	12a_1021	12a_1270	12n_734	13a_162
12a_480	12a_787	12a_1025	12a_1274	12n_743	13a_170
12a_493	12a_792	12a_1030	12a_1276	12n_746	13a_171
12a_502	12a_795	12a_1031	12a_1279	12n_755	13a_173
12a_503	12a_796	12a_1035	12a_1288	12n_763	13a_185
12a_507	12a_801	12a_1036	12n_27	12n_788	13a_189
12a_508	12a_802	12a_1037	12n_33	12n_796	13a_191
12a_514	12a_806	12a_1044	12n_52	12n_803	13a_193
12a_523	12a_811	12a_1049	12n_67	12n_805	13a_195
12a_533	12a_812	12a_1052	12n_104	12n_818	13a_201
12a_542	12a_813	12a_1058	12n_107	12n_820	13a_203
12a_546	12a_815	12a_1059	12n_148	12n_827	13a_209

13a_214	13a_497	13a_675	13a_1045	13a_1406	13a_1698
13a_217	13a_501	13a_676	13a_1050	13a_1414	13a_1703
13a_223	13a_508	13a_679	13a_1051	13a_1419	13a_1706
13a_225	13a_513	13a_681	13a_1063	13a_1429	13a_1718
13a_231	13a_525	13a_682	13a_1066	13a_1440	13a_1720
13a_236	13a_535	13a_686	13a_1069	13a_1442	13a_1721
13a_251	13a_536	13a_702	13a_1070	13a_1452	13a_1725
13a_254	13a_539	13a_705	13a_1089	13a_1461	13a_1740
13a_257	13a_540	13a_716	13a_1099	13a_1462	13a_1751
13a_273	13a_542	13a_718	13a_1100	13a_1464	13a_1757
13a_275	13a_544	13a_720	13a_1102	13a_1465	13a_1762
13a_279	13a_545	13a_738	13a_1120	13a_1466	13a_1785
13a_284	13a_546	13a_740	13a_1129	13a_1467	13a_1786
13a_286	13a_551	13a_744	13a_1133	13a_1468	13a_1791
13a_295	13a_553	13a_754	13a_1137	13a_1472	13a_1794
13a_304	13a_561	13a_762	13a_1142	13a_1474	13a_1820
13a_309	13a_564	13a_765	13a_1147	13a_1475	13a_1825
13a_314	13a_569	13a_768	13a_1149	13a_1482	13a_1827
13a_316	13a_571	13a_774	13a_1157	13a_1488	13a_1829
13a_319	13a_573	13a_784	13a_1158	13a_1492	13a_1854
13a_333	13a_574	13a_789	13a_1182	13a_1495	13a_1855
13a_334	13a_584	13a_808	13a_1183	13a_1503	13a_1859
13a_345	13a_589	13a_813	13a_1188	13a_1520	13a_1864
13a_349	13a_590	13a_819	13a_1192	13a_1524	13a_1865
13a_355	13a_593	13a_826	13a_1196	13a_1541	13a_1867
13a_361	13a_594	13a_831	13a_1203	13a_1542	13a_1879
13a_362	13a_598	13a_832	13a_1221	13a_1543	13a_1885
13a_366	13a_599	13a_855	13a_1226	13a_1544	13a_1895
13a_370	13a_603	13a_866	13a_1233	13a_1546	13a_1897
13a_374	13a_606	13a_867	13a_1235	13a_1547	13a_1899
13a_382	13a_609	13a_892	13a_1243	13a_1550	13a_1901
13a_393	13a_616	13a_895	13a_1245	13a_1553	13a_1922
13a_395	13a_618	13a_896	13a_1246	13a_1566	13a_1927
13a_402	13a_620	13a_900	13a_1255	13a_1567	13a_1928
13a_403	13a_621	13a_905	13a_1258	13a_1573	13a_1929
13a_406	13a_622	13a_910	13a_1259	13a_1580	13a_1943
13a_408	13a_623	13a_918	13a_1266	13a_1583	13a_1946
13a_416	13a_625	13a_924	13a_1283	13a_1590	13a_1952
13a_423	13a_627	13a_931	13a_1302	13a_1601	13a_1955
13a_426	13a_629	13a_933	13a_1306	13a_1603	13a_1957
13a_428	13a_631	13a_938	13a_1331	13a_1614	13a_1960
13a_429	13a_635	13a_945	13a_1332	13a_1617	13a_1961
13a_433	13a_636	13a_955	13a_1335	13a_1618	13a_1969
13a_435	13a_643	13a_964	13a_1349	13a_1621	13a_1971
13a_437	13a_644	13a_966	13a_1353	13a_1629	13a_1972
13a_438	13a_648	13a_968	13a_1355	13a_1630	13a_1976
13a_443	13a_652	13a_971	13a_1359	13a_1634	13a_1982
13a_445	13a_654	13a_976	13a_1371	13a_1640	13a_1984
13a_448	13a_656	13a_980	13a_1372	13a_1641	13a_1985
13a_451	13a_658	13a_989	13a_1377	13a_1643	13a_1987
13a_468	13a_661	13a_991	13a_1378	13a_1645	13a_1989
13a_469	13a_662	13a_993	13a_1381	13a_1652	13a_1990
13a_473	13a_663	13a_1002	13a_1395	13a_1660	13a_1997
13a_474	13a_665	13a_1020	13a_1396	13a_1682	13a_1999
13a_480	13a_667	13a_1022	13a_1400	13a_1683	13a_2008
13a_481	13a_669	13a_1041	13a_1404	13a_1692	13a_2018
13a_496	13a_670	13a_1043	13a_1405	13a_1697	13a_2020

13a_2025	13a_2315	13a_2561	13a_2824	13a_3093	13a_3262
13a_2030	13a_2319	13a_2564	13a_2829	13a_3096	13a_3265
13a_2032	13a_2326	13a_2565	13a_2830	13a_3099	13a_3267
13a_2037	13a_2333	13a_2578	13a_2833	13a_3100	13a_3277
13a_2038	13a_2335	13a_2593	13a_2835	13a_3110	13a_3282
13a_2042	13a_2359	13a_2598	13a_2840	13a_3112	13a_3283
13a_2052	13a_2360	13a_2600	13a_2843	13a_3114	13a_3290
13a_2054	13a_2362	13a_2604	13a_2844	13a_3116	13a_3297
13a_2055	13a_2367	13a_2606	13a_2853	13a_3120	13a_3300
13a_2064	13a_2370	13a_2607	13a_2856	13a_3122	13a_3308
13a_2076	13a_2372	13a_2610	13a_2863	13a_3127	13a_3309
13a_2077	13a_2375	13a_2611	13a_2865	13a_3128	13a_3313
13a_2078	13a_2376	13a_2616	13a_2866	13a_3129	13a_3315
13a_2081	13a_2381	13a_2620	13a_2872	13a_3130	13a_3324
13a_2088	13a_2386	13a_2628	13a_2879	13a_3131	13a_3327
13a_2092	13a_2387	13a_2636	13a_2880	13a_3132	13a_3329
13a_2093	13a_2390	13a_2641	13a_2881	13a_3133	13a_3332
13a_2097	13a_2395	13a_2642	13a_2884	13a_3139	13a_3334
13a_2098	13a_2399	13a_2644	13a_2886	13a_3141	13a_3336
13a_2107	13a_2400	13a_2647	13a_2888	13a_3143	13a_3337
13a_2109	13a_2405	13a_2655	13a_2897	13a_3146	13a_3340
13a_2114	13a_2407	13a_2656	13a_2911	13a_3153	13a_3341
13a_2119	13a_2408	13a_2667	13a_2920	13a_3155	13a_3344
13a_2124	13a_2409	13a_2672	13a_2933	13a_3156	13a_3358
13a_2129	13a_2417	13a_2674	13a_2937	13a_3159	13a_3364
13a_2132	13a_2419	13a_2679	13a_2943	13a_3160	13a_3371
13a_2135	13a_2421	13a_2683	13a_2944	13a_3161	13a_3373
13a_2139	13a_2428	13a_2690	13a_2945	13a_3164	13a_3377
13a_2147	13a_2432	13a_2697	13a_2947	13a_3166	13a_3379
13a_2154	13a_2434	13a_2706	13a_2959	13a_3174	13a_3380
13a_2157	13a_2438	13a_2709	13a_2970	13a_3176	13a_3386
13a_2159	13a_2443	13a_2711	13a_2981	13a_3178	13a_3387
13a_2167	13a_2445	13a_2717	13a_2982	13a_3179	13a_3388
13a_2173	13a_2450	13a_2721	13a_2984	13a_3182	13a_3390
13a_2175	13a_2452	13a_2722	13a_2994	13a_3185	13a_3400
13a_2185	13a_2453	13a_2724	13a_2996	13a_3192	13a_3401
13a_2194	13a_2461	13a_2732	13a_3000	13a_3207	13a_3426
13a_2202	13a_2462	13a_2735	13a_3007	13a_3212	13a_3428
13a_2207	13a_2474	13a_2736	13a_3016	13a_3217	13a_3436
13a_2228	13a_2476	13a_2742	13a_3018	13a_3222	13a_3444
13a_2232	13a_2478	13a_2751	13a_3021	13a_3223	13a_3454
13a_2238	13a_2484	13a_2757	13a_3022	13a_3225	13a_3459
13a_2245	13a_2485	13a_2761	13a_3030	13a_3230	13a_3460
13a_2249	13a_2497	13a_2763	13a_3036	13a_3232	13a_3461
13a_2255	13a_2501	13a_2767	13a_3040	13a_3234	13a_3468
13a_2263	13a_2520	13a_2769	13a_3042	13a_3235	13a_3474
13a_2265	13a_2526	13a_2772	13a_3044	13a_3237	13a_3478
13a_2268	13a_2533	13a_2782	13a_3045	13a_3239	13a_3479
13a_2269	13a_2536	13a_2784	13a_3059	13a_3240	13a_3482
13a_2270	13a_2538	13a_2786	13a_3060	13a_3244	13a_3492
13a_2271	13a_2542	13a_2790	13a_3062	13a_3248	13a_3500
13a_2284	13a_2543	13a_2810	13a_3068	13a_3250	13a_3504
13a_2286	13a_2544	13a_2813	13a_3074	13a_3253	13a_3508
13a_2287	13a_2548	13a_2814	13a_3080	13a_3255	13a_3512
13a_2290	13a_2550	13a_2816	13a_3083	13a_3257	13a_3524
13a_2307	13a_2552	13a_2821	13a_3089	13a_3259	13a_3530
13a_2310	13a_2554	13a_2823	13a_3090	13a_3261	13a_3531

13a_3534	13a_3795	13a_4046	13a_4226	13a_4431	13a_4645
13a_3535	13a_3796	13a_4047	13a_4229	13a_4445	13a_4646
13a_3537	13a_3800	13a_4048	13a_4231	13a_4454	13a_4656
13a_3543	13a_3804	13a_4049	13a_4233	13a_4457	13a_4657
13a_3551	13a_3812	13a_4050	13a_4235	13a_4459	13a_4658
13a_3553	13a_3822	13a_4053	13a_4241	13a_4464	13a_4660
13a_3554	13a_3823	13a_4055	13a_4246	13a_4468	13a_4667
13a_3558	13a_3827	13a_4057	13a_4250	13a_4478	13a_4673
13a_3559	13a_3841	13a_4059	13a_4254	13a_4480	13a_4691
13a_3564	13a_3844	13a_4065	13a_4255	13a_4481	13a_4695
13a_3567	13a_3845	13a_4069	13a_4258	13a_4482	13a_4698
13a_3570	13a_3847	13a_4072	13a_4259	13a_4483	13a_4704
13a_3577	13a_3852	13a_4080	13a_4260	13a_4484	13a_4706
13a_3589	13a_3858	13a_4083	13a_4263	13a_4488	13a_4707
13a_3592	13a_3864	13a_4087	13a_4265	13a_4492	13a_4709
13a_3594	13a_3865	13a_4088	13a_4281	13a_4499	13a_4718
13a_3595	13a_3869	13a_4089	13a_4286	13a_4502	13a_4725
13a_3596	13a_3872	13a_4090	13a_4287	13a_4503	13a_4732
13a_3597	13a_3880	13a_4093	13a_4290	13a_4504	13a_4733
13a_3605	13a_3883	13a_4096	13a_4293	13a_4508	13a_4737
13a_3608	13a_3888	13a_4101	13a_4304	13a_4509	13a_4739
13a_3615	13a_3892	13a_4106	13a_4306	13a_4517	13a_4740
13a_3621	13a_3896	13a_4108	13a_4307	13a_4518	13a_4745
13a_3631	13a_3899	13a_4109	13a_4309	13a_4531	13a_4747
13a_3639	13a_3901	13a_4112	13a_4310	13a_4535	13a_4757
13a_3641	13a_3912	13a_4113	13a_4312	13a_4536	13a_4758
13a_3642	13a_3913	13a_4116	13a_4313	13a_4538	13a_4764
13a_3652	13a_3917	13a_4118	13a_4316	13a_4541	13a_4766
13a_3660	13a_3922	13a_4120	13a_4320	13a_4549	13a_4767
13a_3663	13a_3926	13a_4123	13a_4321	13a_4551	13a_4768
13a_3667	13a_3927	13a_4125	13a_4322	13a_4552	13a_4770
13a_3669	13a_3931	13a_4127	13a_4328	13a_4553	13a_4771
13a_3670	13a_3940	13a_4128	13a_4329	13a_4554	13a_4774
13a_3676	13a_3943	13a_4129	13a_4331	13a_4555	13a_4787
13a_3686	13a_3956	13a_4134	13a_4345	13a_4557	13a_4790
13a_3687	13a_3960	13a_4149	13a_4352	13a_4558	13a_4792
13a_3688	13a_3961	13a_4153	13a_4353	13a_4565	13a_4796
13a_3690	13a_3962	13a_4155	13a_4359	13a_4567	13a_4798
13a_3691	13a_3972	13a_4160	13a_4362	13a_4568	13a_4807
13a_3699	13a_3975	13a_4162	13a_4368	13a_4571	13a_4809
13a_3704	13a_3976	13a_4163	13a_4369	13a_4574	13a_4811
13a_3706	13a_3977	13a_4170	13a_4376	13a_4576	13a_4812
13a_3714	13a_3978	13a_4176	13a_4377	13a_4578	13a_4813
13a_3716	13a_3979	13a_4179	13a_4379	13a_4580	13a_4817
13a_3718	13a_3990	13a_4182	13a_4381	13a_4589	13a_4820
13a_3720	13a_3998	13a_4184	13a_4382	13a_4595	13a_4823
13a_3731	13a_3999	13a_4190	13a_4385	13a_4599	13a_4824
13a_3736	13a_4005	13a_4191	13a_4386	13a_4606	13a_4826
13a_3737	13a_4009	13a_4193	13a_4388	13a_4607	13a_4828
13a_3743	13a_4012	13a_4201	13a_4390	13a_4609	13a_4832
13a_3745	13a_4017	13a_4206	13a_4393	13a_4616	13a_4835
13a_3749	13a_4021	13a_4207	13a_4409	13a_4622	13a_4837
13a_3752	13a_4025	13a_4208	13a_4421	13a_4624	13a_4840
13a_3754	13a_4034	13a_4210	13a_4422	13a_4631	13a_4842
13a_3759	13a_4039	13a_4214	13a_4424	13a_4636	13a_4843
13a_3792	13a_4041	13a_4216	13a_4427	13a_4643	13a_4844
13a_3794	13a_4043	13a_4225	13a_4429	13a_4644	13a_4849

13a_4850	13n_495	13n_1029	13n_1528	13n_2297	13n_3038
13a_4851	13n_500	13n_1043	13n_1533	13n_2314	13n_3046
13a_4853	13n_503	13n_1049	13n_1535	13n_2335	13n_3048
13a_4857	13n_517	13n_1058	13n_1567	13n_2345	13n_3051
13a_4858	13n_519	13n_1063	13n_1590	13n_2346	13n_3060
13a_4859	13n_535	13n_1072	13n_1591	13n_2362	13n_3063
13a_4861	13n_546	13n_1075	13n_1592	13n_2368	13n_3068
13a_4862	13n_550	13n_1077	13n_1594	13n_2372	13n_3083
13a_4864	13n_569	13n_1080	13n_1595	13n_2421	13n_3103
13a_4865	13n_596	13n_1086	13n_1643	13n_2426	13n_3111
13a_4869	13n_601	13n_1095	13n_1654	13n_2437	13n_3113
13a_4873	13n_616	13n_1097	13n_1656	13n_2442	13n_3124
13a_4875	13n_618	13n_1104	13n_1660	13n_2487	13n_3137
13a_4876	13n_622	13n_1106	13n_1679	13n_2515	13n_3145
13n_6	13n_629	13n_1115	13n_1681	13n_2530	13n_3156
13n_17	13n_631	13n_1117	13n_1684	13n_2536	13n_3188
13n_25	13n_645	13n_1176	13n_1703	13n_2561	13n_3199
13n_32	13n_663	13n_1180	13n_1730	13n_2572	13n_3200
13n_41	13n_670	13n_1187	13n_1732	13n_2586	13n_3210
13n_44	13n_675	13n_1192	13n_1771	13n_2603	13n_3212
13n_45	13n_679	13n_1204	13n_1784	13n_2608	13n_3216
13n_75	13n_680	13n_1205	13n_1791	13n_2649	13n_3239
13n_82	13n_681	13n_1212	13n_1838	13n_2657	13n_3248
13n_113	13n_691	13n_1217	13n_1851	13n_2661	13n_3254
13n_131	13n_709	13n_1222	13n_1858	13n_2664	13n_3258
13n_150	13n_719	13n_1225	13n_1892	13n_2672	13n_3274
13n_154	13n_723	13n_1229	13n_1906	13n_2677	13n_3276
13n_163	13n_732	13n_1235	13n_1907	13n_2689	13n_3293
13n_177	13n_744	13n_1238	13n_1927	13n_2711	13n_3294
13n_186	13n_747	13n_1239	13n_1931	13n_2719	13n_3305
13n_187	13n_755	13n_1240	13n_1933	13n_2722	13n_3333
13n_204	13n_767	13n_1273	13n_1936	13n_2751	13n_3337
13n_215	13n_771	13n_1278	13n_1950	13n_2775	13n_3354
13n_222	13n_773	13n_1294	13n_1998	13n_2805	13n_3368
13n_225	13n_776	13n_1300	13n_2042	13n_2816	13n_3419
13n_241	13n_787	13n_1323	13n_2061	13n_2827	13n_3426
13n_247	13n_832	13n_1329	13n_2065	13n_2870	13n_3427
13n_265	13n_841	13n_1332	13n_2089	13n_2881	13n_3432
13n_274	13n_858	13n_1335	13n_2104	13n_2883	13n_3435
13n_285	13n_895	13n_1341	13n_2110	13n_2898	13n_3447
13n_300	13n_900	13n_1344	13n_2114	13n_2899	13n_3454
13n_312	13n_907	13n_1382	13n_2121	13n_2900	13n_3461
13n_317	13n_914	13n_1383	13n_2123	13n_2901	13n_3470
13n_321	13n_916	13n_1390	13n_2125	13n_2907	13n_3476
13n_338	13n_929	13n_1406	13n_2131	13n_2920	13n_3484
13n_339	13n_935	13n_1415	13n_2143	13n_2929	13n_3486
13n_342	13n_943	13n_1417	13n_2159	13n_2935	13n_3491
13n_356	13n_947	13n_1421	13n_2171	13n_2939	13n_3493
13n_368	13n_961	13n_1439	13n_2190	13n_2959	13n_3497
13n_378	13n_962	13n_1498	13n_2192	13n_2972	13n_3499
13n_384	13n_965	13n_1503	13n_2195	13n_2981	13n_3502
13n_393	13n_974	13n_1504	13n_2231	13n_2995	13n_3508
13n_429	13n_981	13n_1508	13n_2248	13n_3000	13n_3515
13n_437	13n_983	13n_1516	13n_2252	13n_3003	13n_3518
13n_438	13n_1004	13n_1517	13n_2253	13n_3010	13n_3520
13n_458	13n_1008	13n_1519	13n_2280	13n_3015	13n_3532
13n_486	13n_1011	13n_1520	13n_2287	13n_3034	13n_3566

13n_3576	13n_3824	13n_4189	13n_4501	13n_4731	13n_4936
13n_3579	13n_3836	13n_4200	13n_4520	13n_4739	13n_4941
13n_3583	13n_3837	13n_4207	13n_4537	13n_4743	13n_4943
13n_3589	13n_3849	13n_4208	13n_4546	13n_4747	13n_4949
13n_3593	13n_3852	13n_4218	13n_4550	13n_4751	13n_4951
13n_3595	13n_3854	13n_4221	13n_4556	13n_4752	13n_4952
13n_3608	13n_3873	13n_4223	13n_4561	13n_4761	13n_4960
13n_3633	13n_3879	13n_4243	13n_4565	13n_4767	13n_4966
13n_3643	13n_3882	13n_4256	13n_4567	13n_4774	13n_4967
13n_3648	13n_3907	13n_4261	13n_4568	13n_4782	13n_4972
13n_3653	13n_3913	13n_4263	13n_4569	13n_4791	13n_4975
13n_3657	13n_3920	13n_4269	13n_4576	13n_4793	13n_4978
13n_3673	13n_3928	13n_4278	13n_4588	13n_4795	13n_4981
13n_3676	13n_3942	13n_4289	13n_4589	13n_4804	13n_4982
13n_3697	13n_3990	13n_4314	13n_4599	13n_4813	13n_4985
13n_3698	13n_4008	13n_4341	13n_4605	13n_4820	13n_4990
13n_3709	13n_4015	13n_4342	13n_4615	13n_4822	13n_4991
13n_3712	13n_4029	13n_4343	13n_4622	13n_4837	13n_4993
13n_3714	13n_4043	13n_4353	13n_4633	13n_4846	13n_4994
13n_3721	13n_4050	13n_4359	13n_4637	13n_4847	13n_4997
13n_3724	13n_4054	13n_4360	13n_4641	13n_4853	13n_5003
13n_3729	13n_4057	13n_4361	13n_4644	13n_4865	13n_5013
13n_3735	13n_4073	13n_4362	13n_4649	13n_4867	13n_5014
13n_3736	13n_4085	13n_4363	13n_4650	13n_4868	13n_5021
13n_3743	13n_4094	13n_4365	13n_4656	13n_4879	13n_5022
13n_3749	13n_4095	13n_4366	13n_4660	13n_4884	13n_5030
13n_3774	13n_4129	13n_4373	13n_4673	13n_4887	13n_5044
13n_3780	13n_4133	13n_4377	13n_4676	13n_4890	13n_5046
13n_3785	13n_4137	13n_4402	13n_4679	13n_4891	13n_5054
13n_3788	13n_4164	13n_4404	13n_4687	13n_4899	13n_5070
13n_3799	13n_4165	13n_4408	13n_4693	13n_4900	13n_5075
13n_3801	13n_4169	13n_4410	13n_4700	13n_4906	13n_5084
13n_3806	13n_4179	13n_4418	13n_4704	13n_4911	13n_5100
13n_3807	13n_4181	13n_4447	13n_4714	13n_4914	13n_5102
13n_3810	13n_4183	13n_4455	13n_4717	13n_4928	
13n_3813	13n_4186	13n_4463	13n_4729	13n_4931	
13n_3822	13n_4187	13n_4469	13n_4730	13n_4933	

$\gamma_4 = 3$ (**22 knots**).

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12a_664	12a_1123	13a_1826	13a_3034	13a_3936	
12a_780	13a_1398	13a_1974	13a_3121	13a_4566	

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