

Low-rank Grothendieck rings: an AI-assisted classification project, constructions, and open problems

Sébastien Palcoux*

26 October 2026

Abstract

We present a public AI-generated research project on fusion rings and their categorifications over $\mathbb{C}$, with data extending through rank twelve. The project reports unrestricted Grothendieck-ring classifications in ranks two, three, and four, a complete multiplicity-free classification through rank twelve, and classifications or partial classifications at higher multiplicities. The released census contains 1,434,042 rings in ranks three through twelve; ten categorification questions remain open within its stated bounds. The resources include research manuscripts, categorical models, exclusion arguments, verification code, and an interactive atlas. One categorical model per ring suffices for our classification problem; classifying all realizing categories up to equivalence is a further task. We summarize the current scope, compare it with AnyonWiki, and link to the online constructions catalogue and unresolved cases. The project's original research content was generated by AI under human direction; independent human verification remains pending.

1 The classification question and the current scope

For a fusion category $\mathcal{C}$ over $\mathbb{C}$, write $b_0 = 1, b_1, \dots, b_{r-1}$ for its simple-object classes. Its Grothendieck ring has multiplication and maximum fusion multiplicity

$$b_i b_j = \sum_{k=0}^{r-1} N_{ij}^k b_k, \quad N_{ij}^k \in \mathbb{Z}_{\geq 0}, \quad M(R) = \max_{i,j,k} N_{ij}^k.$$

The rank is r , and multiplicity-free means $M(R) = 1$. We count fusion rings up to based-ring isomorphism. All duality types are included, and commutativity is not assumed. The census enumerates fusion rings without filtering for categorifiability. The companion classification asks which of these rings are $K_0(\mathcal{C})$ for at least one complex fusion category.

The rank-one ring is $K_0(\text{Vec})$. In rank two the family is $X^2 = 1 + nX$, $n \geq 0$, and only $n = 0, 1$ are categorifiable [1]. Both have $M = 1$, since the maximum includes the unit coefficients. The higher-rank scope is summarized in Table 1.

The released census contains 1,434,042 based-isomorphism classes in ranks three through twelve: 1,433,502 recorded general exclusions, 530 realizations, and ten unresolved cases. The project reports a complete Grothendieck-ring classification at multiplicity one through rank twelve.

The new rank-twelve band contains 632 rings: 480 general exclusions and 152 realizations. Of the realized rings, 151 have known unitary models; R12_1_623 has a pseudo-unitary realization whose unitarity is unproved. This is an open question about unitarity, while its general complex categorifiability is decided.

At higher multiplicities, classification is complete within the displayed ranges wherever the unresolved count is zero. The complete contiguous ranges at ranks six, eight, and nine end at $M = 3, 2, 1$, respectively. There remain two open rings at rank six (one at each of $M = 4, 8$), four at rank eight ($M = 3$), and four at rank nine ($M = 2$).

*Contributor and project coordinator. Beijing Institute of Mathematical Sciences and Applications (BIMSA), Beijing, China. The project's AI provenance and verification status are stated explicitly below.

Rank	Census bound	Census rings	Excluded	Realized	Unresolved
2	$\text{all } n \geq 0$	symbolic	$n \geq 2$	2	0
3	$M \leq 1,000$	460,353	460,348	5	0
4	$M \leq 128$	889,530	889,515	15	0
5	$M \leq 20$	40,999	40,982	17	0
6	$M \leq 9$	25,421	25,362	57	2
7	$M \leq 5$	10,006	9,976	30	0
8	$M \leq 3$	4,500	4,403	93	4
9	$M \leq 2$	2,225	2,145	76	4
10	$M = 1$	194	136	58	0
11	$M = 1$	182	155	27	0
12	$M = 1$	632	480	152	0

Table 1: Reported project status at the pinned snapshot checked on 26 October 2026. Counts concern the displayed census ranges; they are based-ring counts, not numbers of tensor-equivalence classes. The project reports **unrestricted classification in ranks two, three, and four**; their finite census bounds are not hypotheses of those claims. All project claims remain subject to independent human verification.

Here “realized” means that a categorical model is recorded, “excluded” means that a general obstruction is recorded, and “complete” means that every ring in the stated census range has one of these two decisions. These labels describe the project’s reported mathematical conclusions. Neither a surviving necessary condition nor a failed construction attempt decides categorifiability.

2 Mathematical methods and verification

The project combines structural arguments with exact finite calculations. Its criteria include arithmetic and cyclotomic restrictions, formal codegrees, induction to the Drinfeld center, Frobenius–Schur indicators, fusion subcategories, module categories, and coherence obstructions. A positive decision requires a categorical construction together with an identification of its full fusion tensor with the census representative. An exclusion must apply to every possible complex categorification; an obstruction restricted to pivotal, spherical, pseudo-unitary, unitary, or braided categories retains that restriction.

For example, the [unrestricted rank-four manuscript](#) reports exactly fifteen Grothendieck rings, automatic sphericity of every categorification, and the sharp bounds

$$\text{FPdim}(\mathcal{C}) \leq 24 + 12\sqrt{3}, \quad M(K_0(\mathcal{C})) \leq 6.$$

Its argument reduces the problem theoretically to $\text{FPdim}(\mathcal{C}) < 1000$ and $M \leq 31$, then applies exact checks to 1,692 remaining rings. The pivotal classification of Dong–Palcoux–Plessis [2] is a separate result with its stated hypothesis. In higher ranks the project treats general complex categorification, not only unitary models: the rank-six list includes a reported spherical realization whose ring admits no pseudo-unitary categorification.

Within the reported classification through rank twelve, the [simple-ring audit](#) finds exactly sixteen nontrivial simple multiplicity-free Grothendieck rings: $\mathbb{Z}[C_p]$ for primes $p \leq 12$, and $K_0(\text{PSU}(2)_{2r-1})$ for $2 \leq r \leq 12$. Here simplicity means the absence of a proper nontrivial fusion subring, and $\text{PSU}(2)_k$ is the even-label fusion subcategory of $\text{SU}(2)_k$. This extends the rank-six pattern in [3, Corollary 5.9]; an analogous question in arbitrary rank was posed in [3, Question 5.10].

Three verification tasks must be distinguished: checking a released fusion tensor, checking a categorical argument or certificate, and proving that enumeration has missed no ring. For the recent rank-seven $M = 5$, rank-eight $M = 3$, rank-nine $M \leq 2$, and rank-ten through rank-twelve $M = 1$ releases, enumeration completeness relies on the supplied programs and producer completion records;

the full searches were not independently rerun. Exact tensor-validity and based-isomorphism checks have a narrower scope. The [verification guide](#) and [rank-twelve proof archive](#) record the executed checks and their mathematical dependencies.

The [correctness revision](#) repairs several model-identification arguments and computational verification procedures without changing the earlier classifications. Its records distinguish current proofs, superseded arguments, completed replays, and external computations not rerun. Automated checks, separate AI reviews, and source checksums do not replace independent human verification of categorical arguments or enumeration completeness.

3 Comparison with AnyonWiki

Our fusion-ring census strictly extends the rank–multiplicity ranges reported in AnyonWiki [4, Table 1], including multiplicity-free rings through rank twelve and larger ranges at higher multiplicities. Our census is entirely AI-generated under human direction, with independent human verification pending; AnyonWiki’s mathematical data were produced using software developed without AI assistance [4]. The categorification objectives also differ: for our Grothendieck-ring classification, one categorical model per ring suffices, whereas AnyonWiki provides a census of multiplicity-free pivotal fusion categories through rank seven up to equivalence, including compatible pivotal and braided structures [5].¹ Our Grothendieck-ring analysis also treats higher multiplicities.

AnyonWiki’s ring enumeration also includes higher multiplicities [4, 6]. Our categorification work uses established human mathematics and, for some positive cases, explicit AnyonWiki data; the [source and exact-comparison records](#) document those inputs.

4 Constructions and unresolved cases online

The [constructions catalogue](#) records specific realizations not, to our knowledge, explicitly described in the literature, together with their known ingredients and the project’s checks of the required hypotheses and fusion rules. These proposed realizations await independent verification; *new* is used only in this limited sense, without claiming novelty up to tensor equivalence or exoticness.

The [unresolved-cases page](#) maintains the current list of open rings, with exact fusion tensors, invariants, proof references, and the status of general, pivotal, spherical, pseudo-unitary, unitary, and braided categorification.

The [model catalogue](#) and [case directory](#) now include rank twelve in the same responsive interface, with filters for ring properties. The [theory and definitions page](#) explains these properties and the categorical structures under consideration. Individual records distinguish obstructions from implications between structures, and statements about a category itself from statements about a Galois conjugate. The live catalogues may evolve; the counts in this announcement refer to the fixed snapshot below.

5 Resources, provenance, and independent review

The project has three complementary public entry points:

Resource	Contents
Fusion Ring Census	Enumeration programs, canonical fusion tensors, count tables, and census manuscripts.
Grothendieck Ring Classification	Classification manuscripts, models, exclusions, decision records, and exact replay materials.
Interactive atlas	Unified ring exploration, constructions and case records, unresolved questions, definitions, manuscripts, and source evidence.

¹We retain the term “census” and the pivotal qualification used in [5], whose authors discuss correctness, completeness, and equivalence explicitly.

The [manuscript index](#) identifies the current mathematical accounts, and the [research-status page](#) explains their verification scope. The two repositories and the atlas are also listed in references [7, 8, 9].

Fixed snapshot. This revision describes classification commit [9891a7df8678](#) and census commit [a35155aae3b1](#), both checked on 26 October 2026. The [current publication manifest](#) and [all-rank decision table](#) record the current mathematical scope, including rank twelve. Rank-specific source manifests retain the exact census inputs used for each argument; these bindings are not replaced by the overall snapshot.

AI provenance. The project’s original mathematical arguments, research exposition, enumeration and verification code, and computational outputs were generated by AI under the contributor’s direction. This announcement was likewise prepared with ChatGPT. Cited theorems, known categories, and imported data retain their external authorship. Independent human verification and Lean formalization of the project remain pending.

Independent review is invited of the classification reductions, the scope of the exact certificates, categorical constructions and their coherence, and comparisons with the existing literature. Resolving a listed open case requires either a categorical model or an obstruction covering all possible categorifications.

References

- [1] V. Ostrik, *Fusion categories of rank 2*, Math. Res. Lett. **10** (2003), 177–183. [arXiv:math/0203255](#).
- [2] J. Dong, S. Palcoux, and A. Plessis, *Classifying Grothendieck rings of pivotal fusion categories of rank four*, [arXiv:2609.33272](#) (2026).
- [3] Z. Liu, S. Palcoux, and Y. Ren, *Classification of Grothendieck rings of complex fusion categories of multiplicity one up to rank six*, Lett. Math. Phys. **112** (2022), article 54. [doi:10.1007/s11005-022-01542-1](#).
- [4] C. Delaney, R. Razzouk, J. Slingerland, and G. Vercleyen, *AnyonWiki.org*, [arXiv:2610.30081](#) (2026). <https://anyonwiki.org/>.
- [5] G. Vercleyen and J. Slingerland, *On low-rank multiplicity-free complex fusion categories*, Commun. Math. Phys. **407** (2026), article 231. [doi:10.1007/s00220-026-05705-2](#).
- [6] G. Vercleyen and J. K. Slingerland, *On low rank fusion rings*, J. Math. Phys. **64** (2023), no. 9, 091703. [doi:10.1063/5.0148848](#).
- [7] S. Palcoux (project coordinator), *Fusion Ring Census*, AI-generated research repository (2026). <https://github.com/sebastienpalcoux/fusion-ring-census>.
- [8] S. Palcoux (project coordinator), *Grothendieck Ring Classification*, AI-generated research repository (2026). <https://github.com/sebastienpalcoux/grothendieck-ring-classification>.
- [9] S. Palcoux (project coordinator), *Fusion & Grothendieck Ring Atlas*, public project website (2026). <https://sebastienpalcoux.github.io/grothendieck-ring-classification/>.