

From Snapshots to Processes: Dynamic Contact Algebras as a Foundation for Process-Oriented GIS

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"There is no such entity as a self-subsistent point."

— A. N. Whitehead, "Space, Time, and Relativity" (1915)

For Waldo Tobler (1930–2018)

Abstract

Geographic information science has argued for three decades that representing change requires moving from time-stamped snapshots to events and processes. This ontological shift has not been matched by a formal foundation with the properties a GIS needs: explicit axioms, representation theorems, and decidable reasoning. We propose dynamic mereotopology, in the form of Vakarelov's dynamic contact algebras (DCAs), as a candidate foundation.

DCAs are point-free with respect to both space and time. Regions are histories, primitive relations include space contact, time contact and precedence, moments are recovered as clusters of the time-contact relation, and properties of time correspond to axioms about regions alone.

Using Grenon and Smith's distinction between SNAP (snapshot) and SPAN (process) ontologies, we argue that DCAs originate on the SNAP side, since they are abstracted from a snapshot model, yet their primitives already carry much of what a SPAN ontology requires. We then identify precisely what is still missing for a process-oriented GIS: process and participation primitives, continuity of change, freedom from global simultaneity, identity under splitting and merging, and granularity. We develop a SPAN-first extension. For two of its components we give full proofs: a point-free continuity axiom, shown to express exactly the intended condition in discrete time and in finitely variable dense time, and a set of correspondences showing which axioms of continuous space hold at every finite resolution of discrete data and which emerge only in the limit of refinement. The remaining proposals are stated as conjectures, and the framework is tested against three case studies: a flood, a wildfire, and a pandemic.

Keywords: dynamic mereotopology; contact algebra; point-free time; SNAP/SPAN; process ontology; temporal GIS; qualitative spatial reasoning

1 Introduction

1.1 The problem

Most of what geographers care about happens rather than sits. Floods rise and recede, fires spread, storm cells form, split and merge, coastlines erode, cities grow into farmland. Yet the dominant data model of GIS remains a stack of layers, each describing the world at an instant or averaged over an interval. Time enters as an attribute: a timestamp on an object, a date on a raster, a version number on a feature.

This snapshot view has well-known limits. It cannot say directly that one event caused or preceded another, that a region is the same region it was yesterday, or that a flood *passed through* a town rather than simply appearing in two frames. Change becomes a difference between snapshots, recovered by comparison after the fact, instead of a thing represented in its own right.

Since [Peuquet \(1994\)](#), GIScience has argued for event- and process-based representation. [Worboys \(2005\)](#) proposed treating events as first-class entities; [Galton \(2000, 2004\)](#) developed qualitative theories of spatial change; [Grenon and Smith \(2004\)](#) set out a two-sided ontology in which enduring things (SNAP) and unfolding processes (SPAN) are both basic. These proposals are ontologically rich. What they largely lack is a *region-based algebra* with the properties that made static qualitative spatial reasoning successful: a small set of primitive relations, explicit axioms, representation theorems guaranteeing that the axioms capture the intended models, and decidable reasoning fragments.

1.2 The thesis

A candidate foundation already exists, though it has received little attention in GIScience. Starting in 2010, Dimitar Vakarelov developed *dynamic mereotopology*: an algebraic theory of regions that change in time, formulated as *dynamic contact algebras* (DCAs) ([Vakarelov, 2010, 2012, 2014–2016, 2020](#)). It extends the contact algebras of static mereotopology, which underlie calculi like RCC-8, with relations such as time contact and precedence. Its most striking feature is that it is point-free in time as well as in space: moments are not assumed but recovered from relations among regions.

Our thesis has two parts. First, although DCAs are abstracted from a snapshot model and so begin on the SNAP side, their primitives already realize much of what a SPAN ontology asks for. Second, the remaining gap can be stated precisely, and closing it gives a concrete research agenda for a formally grounded, process-oriented GIS.

1.3 Contributions

1. An accessible exposition of DCAs for a GIScience audience, with the key constructions explained conceptually (Sections [3–4](#)).
2. An analysis of DCAs in SNAP/SPAN terms, separating what is already process-like from what remains snapshot-bound (Section [5](#)).
3. A list of requirements for process-oriented GIS, each assessed against DCAs (Section [6](#)).
4. A SPAN-first dynamic mereotopology (Section [7](#)): proposed primitives for processes and genidentity; a point-free continuity axiom with proofs of its meaning in discrete and finitely variable time (Theorems [7.4](#), [7.8](#) and [7.9](#)); and full proofs that continuous region structures are limits of refinement systems of discrete data, with extensionality and normality emerging only in the limit (Theorems [7.17](#), [7.22](#) and [7.26](#)).

5. Three case studies classifying typical queries by the formalism needed to express them (Section 8).

We aim throughout to explain concepts rather than assume them, since the paper sits between two communities, algebraic logic and GIScience, that rarely read each other.

2 Ontologies of geographic change

2.1 From snapshots to events

The earliest temporal GIS models attached time to data rather than to phenomena. In the *snapshot model* the world is a sequence of complete states, each a layer valid at a time. In *time-stamped object* models each feature carries a validity interval. Both treat time as an index on space.

Peuquet (1994) argued that a temporal GIS must support three kinds of question, about *where*, *what* and *when*, and that the snapshot model privileges the first at the expense of the others. Worboys (2005) went further: events such as a storm's passage, a merger of two parcels or the onset of a drought should be first-class entities with their own properties and relations, not reconstructions from differences between states. Hornsby and Egenhofer (2000) showed that even the notion of "the same object" through change needs explicit treatment, since objects can be created, destroyed, split, merged and reincarnated.

2.2 SNAP and SPAN

Grenon and Smith (2004) gave these intuitions a general ontological form. They distinguish two families of entity.

Continuants (SNAP entities) exist wholly at each moment they exist. A lake, a parcel or a city is fully present at any instant of its life; it has spatial parts but no temporal parts. A SNAP ontology describes the world one instant at a time, each description a snapshot.

Occurrents (SPAN entities) unfold over time and have temporal parts. A flood, a migration or a fire is never wholly present at an instant: at any moment only a phase of it exists. A SPAN ontology describes the world as extended in time, with processes as the basic entities.

The central claim is that neither view is complete. A full dynamic ontology needs both families and, crucially, the *trans-ontological* relations between them: a continuant *participates* in a process, a process *has* a continuant as its bearer, a process *begins* or *ends* at a boundary of a continuant's life.

Figure 1 shows the contrast for a growing flood. On the left the flood exists only as three extents at three times; its growth is inferred by comparing frames. On the right the flood is one entity, extended in space and time, and its growth is simply the shape of that entity. The question for this paper is which of these pictures dynamic mereotopology supports, and at what cost.

2.3 What is missing: a region-based algebra

Galton (2004) showed that fields, objects and processes can each be given space-time treatments, and Galton (2000) developed a detailed logic of continuous change for spatial relations. Yet GIScience has no counterpart, for change, of what RCC-8 and the intersection models provide for static relations: a small, well-understood set of region-based primitives with axioms, models and decision procedures. Dynamic mereotopology is a candidate for that role.

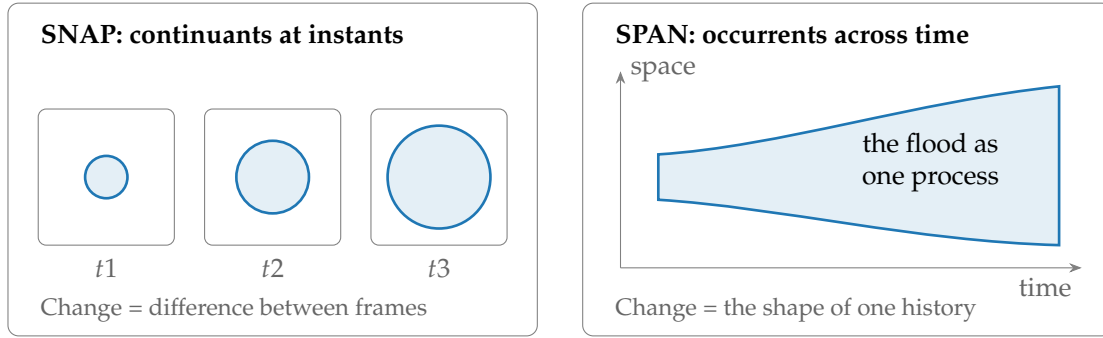


Figure 1: The same growing flood under a SNAP and a SPAN view. On the left the flood exists only as three extents at three times; on the right it is one entity extended in space and time.

3 Static mereotopology

3.1 Regions without points

Mereology is the theory of parts and wholes; *mereotopology* adds topological notions such as touching, boundary and interior. The tradition goes back to Whitehead, De Laguna and Tarski, whose shared motivation was that points are idealizations with no counterpart in experience: we observe extended things, never extensionless locations. A point-free theory takes regions as primitive and defines points, if at all, as derived constructions.

The modern algebraic form is the *contact algebra*. Tarski observed that the parts of a region behave like a Boolean algebra: regions can be joined ($a + b$), intersected ($a \cdot b$) and complemented (a^*), with 0 as the empty region and 1 as the whole of space. A contact algebra adds one relation, *contact*, written $a C b$ and read “ a touches b ”. In the formulation of [Dimov and Vakarelov \(2006\)](#) it satisfies:

- (C1) $a C b \Rightarrow a \neq 0 \text{ and } b \neq 0$
- (C2) $a C b, a \leq a', b \leq b' \Rightarrow a' C b'$
- (C3) $a C (b + c) \Rightarrow a C b \text{ or } a C c$
- (C4) $a C b \Rightarrow b C a$
- (C5) $a \cdot b \neq 0 \Rightarrow a C b$

In words: only nonempty regions touch; enlarging regions preserves contact; touching a union means touching one of its parts; contact is symmetric; and overlapping regions touch.

The intended model is the algebra of *regular closed* sets of a topological space: sets equal to the closure of their interior, which excludes dangling lines and isolated points. Two such regions are in contact when they share at least one point. Balls, polygons and ordinary map regions are all regular closed.

3.2 Derived relations and RCC-8

From contact and the Boolean operations one defines the relations familiar in GIS:

- **External contact:** a and b touch but do not overlap, so they meet only at their boundaries.
- **Non-tangential inclusion** $a \ll b$, defined as $\neg(a C b^*)$: a lies inside b away from its boundary.
- **Tangential inclusion:** a is part of b and touches b 's boundary.

These generate the eight relations of the Region Connection Calculus, RCC-8 ([Randell et al., 1992](#)): disconnected (DC), externally connected (EC), partial overlap (PO), equal (EQ), tangential and non-tangential proper part (TPP, NTPP), and their inverses (TPPi, NTPPi). The 4- and

9-intersection models of Egenhofer and Franzosa (1991) yield essentially the same classification for simple regions, and this is the vocabulary of qualitative spatial queries in GIS.

3.3 Representation: getting points back

The mathematical heart of the theory is the *representation theorem*. The approach goes back to Roeper (1997), who axiomatized regions with contact and a notion of bounded (“limited”) region, and proved a bijective correspondence, up to isomorphism and homeomorphism, between *complete* models of his axioms and locally compact Hausdorff spaces: each complete model is isomorphic to the regular closed sets of such a space, with the limited regions as those with compact closure and the points reconstructed from the regions. Models that are not complete embed into complete ones. Dimov and Vakarelov (2006) extended this line of work, proving that every contact algebra can be embedded in the regular closed sets of some topological space, with contact becoming “sharing a point”. The points of that space are constructed from the algebra itself, as *clans*: collections of regions that pairwise touch and are closed under enlargement and splitting. Intuitively a clan is a place, identified with all the regions that contain it.

The theorem guarantees that the axioms are complete for their intended models: anything true of all region systems in all such spaces follows from (C1)–(C5). It also means points are not lost, merely demoted from primitives to constructions.

One further axiom matters later. The **Efremovič axiom** says that separated regions can be separated by a buffer:

$$\neg(a \text{ C } b) \Rightarrow \exists c : \neg(a \text{ C } c) \text{ and } \neg(c^* \text{ C } b).$$

Under this axiom the maximal clans, called *clusters*, behave as well-separated points. In discrete models it corresponds to transitivity of the underlying adjacency relation (Düntsch and Vakarelov, 2007), which will be important in Section 4.

3.4 Discrete models and raster data

Contact algebras also have discrete models. Take a set of cells with a reflexive, symmetric adjacency relation, what Galton (1999) calls an *adjacency space*, and let two sets of cells touch when some cell of one is adjacent to some cell of the other. A raster with 4- or 8-neighbour adjacency is exactly such a model, so the same axioms govern vector and raster representations.

3.5 Continuity and conceptual neighbourhoods

Static relations constrain change. If two regions deform continuously, their RCC-8 relation cannot jump arbitrarily: disjoint regions must touch before they overlap. The *conceptual neighbourhood graph* (Freksa, 1992; Egenhofer and Al-Taha, 1992) records which relations can follow one another directly.

Figure 2 shows the graph. Under suitable continuity and finite-variability assumptions, made precise in Section 7.4, the history of a pair of regions traces a path through it; which edges are needed depends on the kind of motion allowed. This is the simplest formal content of the phrase “continuous change”, and static mereotopology can state the graph but cannot state that a history follows it, because a contact algebra has no time. Viewed as a dynamic algebra it has exactly one moment: its clock is stopped.

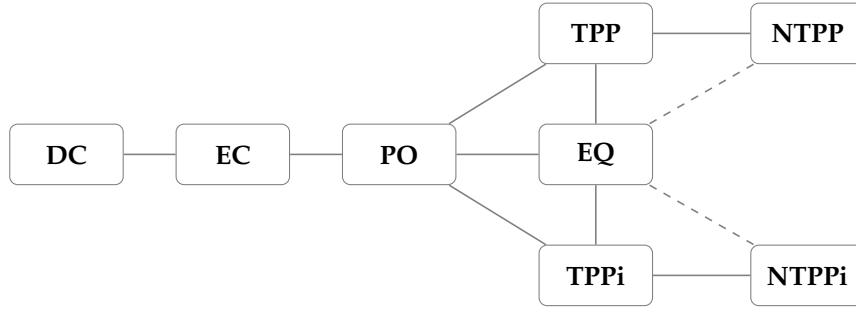


Figure 2: Conceptual neighbourhood graph of the eight RCC-8 relations. Solid edges form the standard graph Γ_0 : relations that a small continuous change can pass between directly. The dashed edges EQ–NTTP and EQ–NTPPi are also needed once regions may grow or shrink (Remark 7.11).

4 Dynamic contact algebras

4.1 Whitehead’s unfinished half

Whitehead’s case against points applied equally to instants. He argued that relativity requires both to be defined from relations between material things, and called this construction “an unwritten chapter of mathematics” (see Vakarelov, 2020, Section 1.2). Static mereotopology wrote the spatial half. Vakarelov’s dynamic mereotopology is an attempt at the rest: a theory in which neither space points nor time moments are primitive.

The method is to begin with a concrete point-based model, prove enough about it to serve as axioms, and then show that every abstract algebra satisfying those axioms can be represented back in such a model, with space points and time points reconstructed from the algebra itself.

4.2 The snapshot construction

The concrete model formalizes the way a camera records a changing scene. Start with a *time structure* $(T, <)$: a set of moments and a before–after relation. To each moment m assign a contact algebra (B_m, C_m) , the *snapshot* at m . A **dynamic region** is then a choice of one region at each moment:

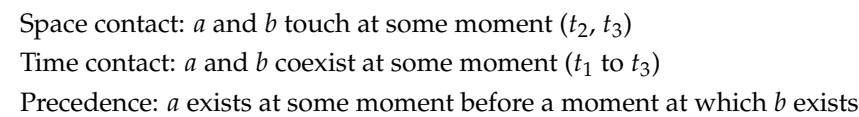
$$a = \langle a_m \rangle_{m \in T}, \quad a_m \in B_m.$$

Vakarelov calls this sequence the *life history* of a . Boolean operations act moment by moment, so $a + b$ is the history whose snapshot at each m is $a_m + b_m$, and $a \leq b$ means a is part of b at every moment. A region *exists* at m when $a_m \neq 0$. A region may exist, cease and exist again, and two different regions may coincide at some moments.

The model is **rich** if it contains, for every set of moments, the region that fills all of space at those moments and is empty otherwise. Richness lets one build the auxiliary regions needed in proofs.

4.3 The relations

The first paper (Vakarelov, 2010) used two relations that do not require time to flow: **stable contact** (a and b touch at every moment) and **unstable contact** (they touch at some moment). It proved representation theorems and gave a decidable quantifier-free logic for them. Later



papers use three relations that do depend on the order of time:

Space contact means the regions touch at some moment. Time contact means they coexist at some moment: a relation of contemporaneity, of the kind Whitehead studied. Precedence means a exists at some moment earlier than a moment at which b exists.

4.4 The abstract algebra

4.5 Time from regions

Properties of time then correspond exactly to axioms about regions. In rich snapshot models, for example:

Property of time	Axiom about regions
Linearity: any two moments are ordered	$a \neq 0$ and $b \neq 0$ imply $a \mathcal{B} b$ or $b \mathcal{B} a$
Density: between two moments lies a third	$a \mathcal{B} b$ implies $a \mathcal{B} p$ or $p^* \mathcal{B} b$, for every p
Transitivity of $\prec$	if not $a \mathcal{B} b$, some c has not $a \mathcal{B} c$ and not $c^* \mathcal{B} b$
No last moment	$a \neq 0$ implies $a \mathcal{B} 1$

This is Leibniz’s relationism made exact: the structure of time is read off relations among the things that exist in it. One property resists: irreflexivity of time. One direction of its correspondence holds, and Vakarelov conjectures the other fails in the abstract setting; it can be recovered in the richer language of Section 4.7.

4.6 Representation

The main theorem states that every DCA embeds into a full snapshot model, preserving whichever time axioms it satisfies. As Vakarelov notes, this is an embedding-type theorem, like representing a Boolean algebra as sets, not a one-to-one correspondence. The 2020 survey adds topological models and a duality theory generalizing Stone duality, which does establish a correspondence between algebras and spaces (Vakarelov, 2020).

4.7 Time representatives and NOW

The basic DCA cannot express past, present and future. Vakarelov (2014–2016) adds *time representatives*: regions that exist at exactly one moment, naming an epoch (“the epoch of Leonardo”). With a distinguished representative **NOW** one can express that a exists now ($a \mathcal{C}^t \mathbf{NOW}$), will exist ($\mathbf{NOW} \mathcal{B} a$), or was in contact with b .

5 Where DCAs sit on SNAP/SPAN

DCAs are abstracted from a snapshot model, so it is natural to classify them as a SNAP formalism. We argue this classification is only half right. The construction is SNAP; the resulting theory is substantially SPAN.

5.1 What is already SPAN-like

The basic entities are histories. An element of a DCA is not a region at a time but a whole life history. Its parts are temporal as well as spatial: the product $a \cdot \mathbf{NOW}$ is the present phase of a , and a region existing only during an interval is a temporal part of any history that contains it there. Having temporal parts is exactly what distinguishes occurrents from continuants. In Grenon and Smith’s terms, the elements of a DCA behave like spatio-temporal regions, the arenas in which SPAN entities unfold.

The primitive relations are relations between histories. Time contact and precedence cannot be evaluated at a single moment; each quantifies over the whole of time. “The flood and the evacuation coexisted” and “the fire preceded the landslide” are expressed directly, without comparing frames. This is what Worboys (2005) asked of an event-oriented GIS.

Time is derived, not assumed. In a SNAP ontology moments are the frame on which everything else hangs. In a DCA they are clusters of the time-contact relation, constructed from how histories overlap in time. A process ontology holds that time is the order of processes, not a container for them, and DCAs realize that view formally.

Temporal structure is a property of the world's regions. Because properties such as density or linearity correspond to axioms about regions, choosing a model of time becomes a claim about which histories exist. A GIS dealing with discrete observation epochs and one modelling continuous processes can share a formalism and differ only in these axioms.

5.2 What remains SNAP

Every moment has a complete spatial algebra. The coordinatewise construction assumes that at each moment there is a whole contact algebra describing all of space. This is the snapshot assumption at its purest. Processes whose spatial extent is only defined over an interval, such as a flow, a trajectory or a diffusion front, must be decomposed into instantaneous states.

Simultaneity is global. Each moment slices all of space at once. This matches classical GIS practice but not physics: relativity denies a global “now”, and even at geographic scales, observation lag, clock differences between sensors, and asynchronous data collection mean that what is “simultaneous” depends on the observer. Vakarelov himself notes that point-free relativistic spacetime remains open ([Vakarelov, 2020](#)).

Identity through time is fixed by indexing. A dynamic region is simply whatever sequence of snapshots one chooses. The algebra contains the history of “the part of the storm north of the river” as readily as the history of the storm. Nothing in it distinguishes a genuine persisting thing from a gerrymandered sequence, and nothing says when a region that splits in two continues as one of the parts, both, or neither.

There are no process primitives. A DCA has regions and relations between them, but no notion of a process as distinct from the region it occupies, no participation of a continuant in a process, and no causal or flow relation. “The flood inundated the parcel” must be paraphrased as a pattern of overlaps between two histories, which loses the asymmetry between the flood as agent and the parcel as patient.

Change need not be continuous. Nothing in the axioms prevents a region from jumping between disconnected positions from one moment to the next. The conceptual neighbourhood constraint of Section 3.5 is not expressible, because the algebra has no notion of adjacent moments or of a transition between them.

5.3 Summary

DCAs sit between the two poles. They offer SPAN-type entities (histories with temporal parts), SPAN-type relations (contemporaneity and precedence) and a SPAN view of time (derived from processes), but they build all of this on a SNAP scaffolding: complete global snapshots, identity by index, and no dynamics beyond the ordering of states. Section 6 turns these observations into requirements.

Table 1: Requirements for a process-oriented GIS, assessed against static contact algebras and DCAs.

Requirement	GIS example	Static CA	DCA	What is needed
R1 Relations between whole histories	“The flood and the evacuation overlapped in time”	No	Yes: time contact, precedence	—
R2 Time derived, not assumed	Moments defined by co-occurring events	No	Yes: moments as clusters	—
R3 Decidable reasoning	Consistency of qualitative constraints	Yes, for RCC-8 and related fragments	Yes, for some quantifier-free logics	Cost of extensions unknown
R4 Continuity of change	A fire cannot jump a river without touching it	Graph statable, not enforceable	No	Transition relation
R5 Processes and participation	“The flood inundated the parcel”	No	Only as overlap of histories	Participation, process sort
R6 Identity under split and merge	A storm cell splits; which part is the storm?	No	Fixed by indexing	Genidentity relation
R7 Local time	Asynchronous sensors; observation lag	No	No: global slices	Causal order
R8 Granularity	Daily vs hourly flood extents; 30 m vs 10 m rasters	Discrete models only	No	Resolution as a second index
R9 Vague extent	Floodplain or fire perimeter with uncertain edge	No	No	Vague or approximate regions

6 Requirements for a process-oriented GIS

DCAs meet three of nine requirements outright and none of the process-specific ones. Table 1 sets each requirement against static contact algebras (CA) and DCAs. This section explains what each requirement asks for in geographic practice and why it is hard; Section 7 gives the formal responses.

6.1 What DCAs already provide (R1, R2)

The first two requirements are where DCAs are strongest. A GIS built on them can store the flood of Section 8 as one history instead of a stack of dated extents, and answer “did the evacuation overlap the flood?” by evaluating time contact once, instead of intersecting timestamps layer by layer. Because moments are derived as clusters of time contact, the temporal resolution of the system is not fixed in advance: adding a new observation that coexists with some histories and not others refines the set of moments rather than requiring a new global time axis. This is the formal content of the claim, common in event-oriented GIS, that time should be read off events rather than imposed on them.

6.2 Decidable reasoning (R3)

The success of RCC-8 in GIS rests partly on its computational profile. Consistency of a set of RCC-8 constraints is NP-complete, and large tractable fragments are known (Renz and Nebel, 1999), so qualitative queries can be checked efficiently in practice. The quantifier-free logics of stable and unstable contact are decidable (Vakarelov, 2010; Nenchev, 2013), which is an encouraging start for DCAs.

Every extension proposed in Section 7 has a computational price that is currently unknown. Participation adds a second sort; genidentity adds a relation with no transitivity to exploit; continuity adds constraints across meeting periods. For discrete time, Theorem 7.8 below reduces continuity to constraints between consecutive moments, which is the shape of a finite transition system, so we expect decidability to survive in that case. For dense time the question is open.

6.3 Continuity (R4)

A GIS reasoning about moving or spreading phenomena needs continuity for three distinct purposes. It rules out impossible histories, for example a fire perimeter that is disjoint from a road at one moment and overlaps it at the next with nothing in between. It supports interpolation, since between two observations a continuous history must pass through the intermediate relations. And it turns apparent violations into evidence: a spot fire, or an outbreak appearing far from any known case, signals an unobserved mechanism.

Three subtleties make continuity harder to formalize than it first appears, and each is addressed in Section 7.4.

Continuity is relative to a contact relation. The pandemic case (Section 8) spreads discontinuously in geographic space and continuously in the network of mobility links. Since adjacency spaces allow any reflexive, symmetric adjacency, the formal remedy is to state continuity relative to the chosen contact, not to build a particular geometry into the axiom.

The allowed transitions depend on the notion of continuity. The standard neighbourhood graph of Figure 2 is appropriate for rigid motion but not for deformation. If a is a fixed disk and b equals a up to time 0 and then expands uniformly, the relation passes directly from EQ to NTPP. Worse, if continuity only means that each region moves continuously in the Hausdorff metric, a region can acquire a small hole that shrinks to nothing, and the pair then passes directly from PO to NTPP (Remark 7.11). The axiom must therefore be parameterized by the transition graph, with the choice of graph justified separately for each class of motion.

Continuity needs finite variability. If the relation between two regions oscillates infinitely often as a moment is approached, no period adjacent to that moment has a constant relation, and any axiom stated in terms of periods says nothing about the change there. Qualitative theories of motion therefore assume that relations change only finitely often in any bounded interval (Galton, 2000). Theorem 7.9 shows that under this assumption the period-based axiom captures exactly the intended condition.

6.4 Processes and participation (R5)

SNAP/SPAN requires two sorts of entity and relations between them. A minimal extension adds a sort of *processes*, each occupying a dynamic region, and a relation of *participation* between continuant histories and processes. Three features of geographic processes show why overlap alone is not enough.

First, participation has roles. “The flood inundated the parcel” and “the parcel inundated the flood” describe the same overlap of histories but different processes; agent and patient are not symmetric. Second, several processes can occupy the same history. A flood, the sediment transport it drives and the contamination it spreads share a region but have different participants and different causal consequences. Third, participation is temporally local: a parcel participates in the flood only while it is inundated, not throughout either history. Worboys’ events have participants in exactly this sense (Worboys, 2005), and Basic Formal Ontology treats participation as the central relation between continuants and occurrents. Section 7.2 proposes a minimal axiomatization.

6.5 Identity (R6)

Hornsby and Egenhofer (2000) catalogue identity-changing operations: creation, destruction, continuation, splitting, merging, and the reappearance of an object after a period of non-existence. In a DCA any sequence of snapshots is a region, so persistence is not a fact about the world but a choice of index. The storm cell that splits, the fire that throws a spot fire across a road, and the viral lineage that branches into variants all require an explicit relation of *genidentity*, “*a* is a later stage of the same thing as *b*”. Section 7.3 argues that this relation cannot be transitive and proposes to treat it as a tolerance relation.

6.6 Local time (R7)

Replacing global moments requires a causal or signal-based order of the kind used in point-based axiomatizations of relativity (Robb, 1914; Goldblatt, 1987). At geographic scales the physical effect is negligible, but an analogous problem is pervasive in data. Temporal databases distinguish the time at which a fact holds from the time at which it was recorded, and surveillance data make the gap visible: a weekly map shows reports, not events, and reporting lags differ between sources. A formalism that assigns every observation to a global moment silently equates the two. A point-free treatment would replace global moments with an order generated by what could have influenced what, and treat simultaneity as a derived and possibly observer-relative relation. We regard this as the most difficult of the requirements and leave it open.

6.7 Granularity (R8)

Geographic data come at a resolution in space and time, and conclusions can change with the resolution and zoning of the data, as the modifiable areal unit problem shows (Openshaw, 1984). A foundation that ignores this forces every dataset to pretend to infinite precision. Discrete contact algebras (Section 3.4) already model rasters and administrative partitions; what has been missing is a principled account of how structures at different resolutions relate to each other and to a continuous theory. Section 7.5 provides one for space, with full proofs: continuous region structures are limits of refinement systems of finite adjacency spaces, and each standard axiom of continuous space corresponds to a precise condition on the refinement.

6.8 Vagueness (R9)

Many geographic regions have indeterminate boundaries: a floodplain, a wildfire’s burning perimeter, the extent of an epidemic wave. Egg-yolk models represent such a region by a pair of crisp regions bounding its possible extent (Cohn and Gotts, 1996). A dynamic version would let both the core and the outer bound change in time, possibly continuously in different senses. We note one connection without developing it: tolerance relations, which

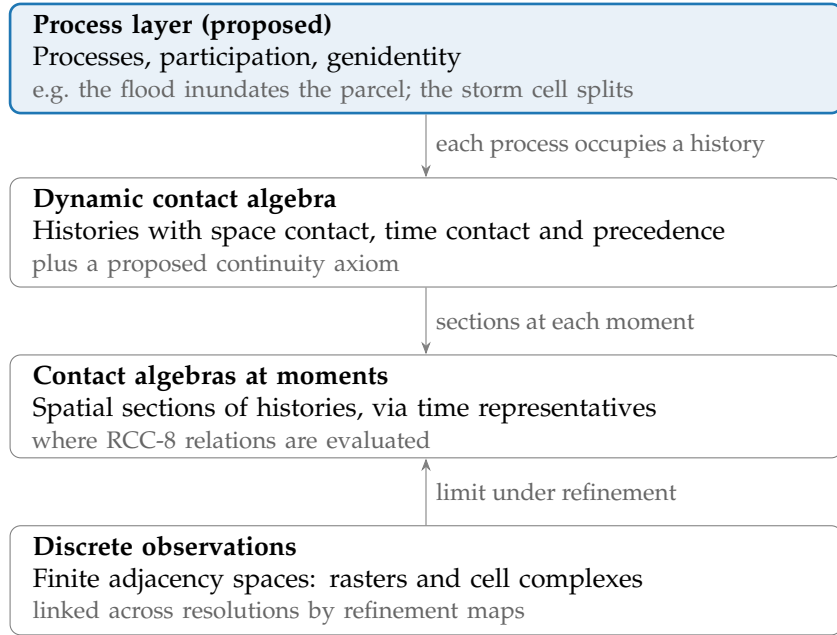


Figure 4: A layered architecture for process-oriented GIS. Processes occupy histories; histories have spatial sections at each moment; and those sections are limits of the discrete cell structures in which data arrive. The highlighted layer is proposed in this paper; the others are existing theory, extended.

appear in this paper both as adjacency and as genidentity, are also the standard formal tool for indiscriminability, and hence a natural starting point for vague extent.

7 Toward a SPAN-first dynamic mereotopology

This section combines proposals with proved results. Sections 7.2 and 7.3 propose new primitives and axioms whose consistency with the existing representation theory is open. Sections 7.4 and 7.5 give precise definitions and full proofs: a point-free formulation of continuity whose meaning is established in discrete and finitely variable dense time, and a set of correspondences between refinement of discrete data and the axioms of continuous space. The aim is a structure in which processes are first-class, histories carry the geometry, snapshots are derived, and discrete data connect to the continuous theory by a principled limit.

Figure 4 summarizes the architecture.

7.1 Snapshots as derived, not primitive

In a DCA with time representatives, the section of a history a at a moment named by a universal time representative u is simply the product $a \cdot u$. Contact between sections is space contact between these products. This lets us state RCC-8 relations at a moment without taking snapshots as the starting point, for example:

$$EC_u(a, b) \iff (a \cdot u) C^s (b \cdot u) \text{ and } (a \cdot u) \cdot (b \cdot u) = 0.$$

The same device generalizes to *periods*: sums of time representatives. Relations can then hold throughout a period, at some point in it, or change within it. The snapshot construction survives as a representation theorem, not as a definition.

7.2 Processes and participation

Definition 7.1 (proposed). A *process-DCA* is a structure $(A, P, \text{occ}, \text{Pt})$ where A is a DCA, P is a set of processes, occ assigns each process p the history $\text{occ}(p)$ that it occupies, and Pt is a participation relation between histories and processes.

Candidate axioms:

(Occ) Every process occupies a nonzero history: $\text{occ}(p) \neq 0$.

(Pt1) A participant is in space contact with what it participates in: $x \text{ Pt } p \Rightarrow x C^s \text{occ}(p)$.

(Pt2) Participation is temporally local: if $x \text{ Pt } p$, there is a period u with $x \cdot u \neq 0$ and $\text{occ}(p) \cdot u \neq 0$ during which x takes part.

The point of separating p from $\text{occ}(p)$ is that distinct processes can occupy the same history. A flood and the sediment transport it drives share a region but are different processes, with different participants. This is exactly the SNAP/SPAN separation of an occurrent from the spatio-temporal region it occupies. Whether (Pt1)–(Pt2) admit a representation theorem extending Vakarelov’s, and what they do to decidability, is open.

7.3 Genidentity, splitting and merging

Let $a G b$ mean that a and b are stages of one persisting thing. The natural first guess is that G is an equivalence relation. It cannot be, if things split. When a storm cell divides into two, each daughter is a later stage of the parent, yet the daughters are not stages of each other. The same failure occurs in reverse for merging.

We therefore propose that genidentity be a **tolerance relation**: reflexive on nonzero histories and symmetric, but not transitive. Splits and merges are then exactly the places where transitivity fails. A history is *unbranched* where G restricted to its stages is transitive. This connects identity through time with the tolerance spaces used below for granularity; in both cases, the failure of transitivity carries the structure.

7.4 Continuity

This subsection gives a precise, point-free formulation of continuity and proves how it behaves. The formulation uses only the algebra of dynamic regions and a sort of *periods*; the proofs take place in rich snapshot models, where the intended meaning of every definition can be checked.

Setting. Throughout, $\mathcal{M}$ is a rich snapshot model (Section 4) over a linear order $(T, <)$, and every snapshot algebra B_m is nondegenerate ($0_m \neq 1_m$). For $S \subseteq T$ let τ_S be the dynamic region with $(\tau_S)_m = 1_m$ for $m \in S$ and 0_m otherwise; richness says these all belong to $\mathcal{M}$. The τ_S are the *periods*, and $\tau_m := \tau_{\{m\}}$ are the *time representatives* of $\mathcal{M}$. We write $S < S'$ when every element of S precedes every element of S' . A set $S \subseteq T$ is *convex* if $x < y < z$ with $x, z \in S$ implies $y \in S$.

Lemma 7.2. For all $S, S' \subseteq T$: $\tau_S \leq \tau_{S'}$ iff $S \subseteq S'$; $\tau_S \cdot \tau_{S'} = \tau_{S \cap S'}$; $\tau_S \neq 0$ iff $S \neq \emptyset$; and $\tau_S \mathcal{B} \tau_{S'}$ iff there are $m \in S$ and $n \in S'$ with $m < n$.

Proof. Boolean operations act coordinatewise, and $1_m \neq 0_m$ in every snapshot, so the first three claims follow at once. For the last, $\tau_S \mathcal{B} \tau_{S'}$ holds iff there are $m < n$ with $(\tau_S)_m \neq 0$ and $(\tau_{S'})_n \neq 0$, that is, with $m \in S$ and $n \in S'$. $\square$

Definition 7.3 (order relations on periods). For periods u, v :

- (i) u wholly precedes v , written $u \triangleleft v$, iff $u \cdot v = 0$ and not $v \mathcal{B} u$;
- (ii) u is convex, written $\text{Cvx}(u)$, iff there is no nonzero period w with $w \cdot u = 0$ such that every nonzero period $w' \leq w$ satisfies both $u \mathcal{B} w'$ and $w' \mathcal{B} u$;
- (iii) u meets v , written $M(u, v)$, iff u and v are nonzero and convex, $u \triangleleft v$, and there is no nonzero period w with $u \triangleleft w$ and $w \triangleleft v$.

Theorem 7.4 (meaning of the order relations). *Let S, S' be nonempty subsets of T . Then:*

- (a) $\tau_S \triangleleft \tau_{S'}$ iff $S < S'$;
- (b) $\text{Cvx}(\tau_S)$ iff S is convex;
- (c) $M(\tau_S, \tau_{S'})$ iff S and S' are convex, $S < S'$, and $S \cup S'$ is convex.

Proof. (a) By Lemma 7.2, “not $\tau_{S'} \mathcal{B} \tau_S$ ” says that there are no $n \in S'$ and $m \in S$ with $n < m$. By linearity this means $m \leq n$ for all $m \in S$ and $n \in S'$. The condition $\tau_S \cdot \tau_{S'} = 0$ says $S \cap S' = \emptyset$, which excludes $m = n$. Together they give $S < S'$, and the converse is immediate.

(b) Suppose S is not convex, witnessed by $x < y < z$ with $x, z \in S$ and $y \notin S$. Take $w = \tau_y$. Then $w \cdot \tau_S = 0$, and the only nonzero period below w is w itself, for which $\tau_S \mathcal{B} w$ (as $x < y$) and $w \mathcal{B} \tau_S$ (as $y < z$). So $\text{Cvx}(\tau_S)$ fails. Conversely, suppose $\text{Cvx}(\tau_S)$ fails, witnessed by $w = \tau_W$ with W nonempty and disjoint from S . Pick $y \in W$ and apply the condition to $w' = \tau_y$: we get $x \in S$ with $x < y$ and $z \in S$ with $y < z$. Since $y \notin S$, S is not convex.

(c) Assume S, S' convex and $S < S'$, which by (a) and (b) is what the first clauses of M require. If some $y \in T$ satisfies $S < \{y\} < S'$, then $w = \tau_y$ satisfies $\tau_S \triangleleft w \triangleleft \tau_{S'}$ by (a), so M fails. Conversely, if some nonzero $w = \tau_W$ has $\tau_S \triangleleft w \triangleleft \tau_{S'}$, then by (a) any $y \in W$ satisfies $S < \{y\} < S'$. Hence $M(\tau_S, \tau_{S'})$ holds iff no element of T lies strictly between S and S' .

It remains to show that, for convex $S < S'$, this is equivalent to convexity of $S \cup S'$. If y lies strictly between S and S' , then $y \notin S \cup S'$ and y lies between an element of S and an element of S' , so $S \cup S'$ is not convex. Conversely, suppose $x < y < z$ with $x, z \in S \cup S'$ and $y \notin S \cup S'$. If x and z both lie in S , or both in S' , convexity of that set gives a contradiction, so $x \in S$ and $z \in S'$. If some $s \in S$ had $y < s$, convexity of S applied to $x < y < s$ would put y in S ; hence $S < \{y\}$. Symmetrically $\{y\} < S'$, so y lies strictly between S and S' . $\square$

In dense time the definition behaves as intended: $[0, 1)$ meets $[1, 2]$, and $[0, 1)$ meets $\{1\}$, which meets $(1, 2]$; but $[0, 1)$ does not meet $(1, 2]$, since the moment 1 lies between them.

Relations at a moment. In any contact algebra, the RCC-8 relations between nonzero x and y are defined from contact, overlap ($x \cdot y \neq 0$), parthood ($x \leq y$) and non-tangential parthood ($x \ll y$, i.e. not $x \mathcal{C} y^*$) in the usual way: DC is non-contact; EC is contact without overlap; PO is overlap with neither a part of the other; EQ is equality; TPP is proper parthood ($x \leq y$, $x \neq y$) with $x \not\ll y$; NTPP is proper parthood with $x \ll y$; and TPPi, NTPPi are the converses. Exactly one of the eight holds for any pair of nonzero regions.

For dynamic regions a, b that both exist at m (that is, $a_m \neq 0 \neq b_m$), write $R_m(a, b)$ when relation R holds between a_m and b_m in B_m .

Lemma 7.5 (instantaneous relations are definable). *Let $x = a \cdot \tau_m$ and $y = b \cdot \tau_m$. Then $a_m \cdot b_m \neq 0$ iff $x \cdot y \neq 0$; $a_m \leq b_m$ iff $x \leq y$; $a_m \mathcal{C}_m b_m$ iff $x \mathcal{C}^s y$; and $a_m \ll b_m$ in B_m iff not $x \mathcal{C}^s (y^* \cdot \tau_m)$. Consequently each $R_m(a, b)$ is expressed by a formula in x, y, τ_m and $\mathcal{C}^s$.*

Proof. The regions x, y and $y^* \cdot \tau_m$ are zero at every moment except m , where they equal a_m, b_m and b_m^* . Overlap and parthood are coordinatewise. Space contact holds iff the coordinates are in contact at some moment, and by axiom (C1) that moment can only be m . Each RCC-8 relation is a Boolean combination of the four basic conditions, and a exists at m iff $x \neq 0$. $\square$

Definition 7.6 (throughout). Relation R holds *throughout* a period u for the pair (a, b) iff for every time representative $\tau_m \leq u$, both a and b exist at m and $R_m(a, b)$.

By Lemma 7.2, the time representatives below τ_S are exactly the τ_m with $m \in S$. Let Γ be a symmetric, irreflexive relation on the eight RCC-8 relations: a *transition graph*.

Axiom 7.7 (Cont_Γ). For all dynamic regions a, b , all periods u, v , and all RCC-8 relations R, R' : if $M(u, v)$, R holds throughout u for (a, b) , and R' holds throughout v for (a, b) , then $R = R'$ or $R \Gamma R'$.

The axiom mentions only regions, periods, precedence and space contact; no moment appears in it. Its two main properties follow.

Theorem 7.8 (discrete time). Let $T = \mathbb{Z}$ with its usual order. Then $\mathcal{M}$ satisfies Cont_Γ iff for all dynamic regions a, b and every $m \in \mathbb{Z}$ such that both exist at m and at $m + 1$, either $R_m(a, b) = R_{m+1}(a, b)$ or $R_m(a, b) \Gamma R_{m+1}(a, b)$.

Proof. ($\Rightarrow$) The singletons $\{m\}$ and $\{m + 1\}$ are convex, $\{m\} < \{m + 1\}$, and their union is convex, so $M(\tau_m, \tau_{m+1})$ by Theorem 7.4(c). The relation $R_m(a, b)$ holds throughout τ_m and $R_{m+1}(a, b)$ throughout τ_{m+1} , so the axiom gives the claim.

($\Leftarrow$) Suppose $M(\tau_S, \tau_{S'})$, with R throughout τ_S and R' throughout $\tau_{S'}$. By Theorem 7.4, S and S' are nonempty and convex with $S < S'$ and nothing between them. S is nonempty and bounded above by any element of S' , so it has a greatest element s ; likewise S' has a least element s' . As nothing lies between S and S' , $s' = s + 1$. Then $R = R_s(a, b)$ and $R' = R_{s+1}(a, b)$, and the hypothesis gives $R = R'$ or $R \Gamma R'$. $\square$

Theorem 7.8 is the case of snapshot data at regular epochs. It says the point-free axiom is exactly the familiar frame-to-frame check. Note what it does and does not mean: a jump from DC to PO between two epochs violates the axiom over $\mathbb{Z}$, but it need not violate continuity of the underlying process, which may have passed through EC between observations. Continuity of the data is continuity at a resolution, which connects this subsection to Section 7.5.

For dense time we need finite variability. Let $J \subseteq T$ be convex and suppose a and b exist at every moment of J . The pair (a, b) has *finite variability on J* if J is the union of finitely many nonempty convex sets $J_1, \dots, J_k$ (the *pieces*), each a maximal convex subset of J on which $m \mapsto R_m(a, b)$ is constant.

Distinct pieces are disjoint, since the union of two overlapping convex sets on which the relation is constant is a larger such set. Disjoint convex sets in a linear order are comparable (if $x < y < x'$ with $x, x' \in J_i$ and $y \in J_j$, convexity puts y in J_i), so we may index the pieces so that $J_1 < \dots < J_k$. Consecutive pieces carry different relations, again by maximality.

Theorem 7.9 (finite variability). Suppose (a, b) has finite variability on a convex set J , with pieces $J_1 < \dots < J_k$ carrying relations $R_1, \dots, R_k$. Then the following are equivalent:

- (i) Cont_Γ holds for (a, b) and all periods $u, v \leq \tau_J$;
- (ii) $R_i \Gamma R_{i+1}$ for every $i < k$.

Proof. (i) $\Rightarrow$ (ii). Fix $i < k$. The pieces J_i and J_{i+1} are convex with $J_i < J_{i+1}$. Their union is convex: an element of T between them lies in J , since J is convex, hence in some piece, which would have to lie strictly between J_i and J_{i+1} , and there is none. By Theorem 7.4(c), $M(\tau_{J_i}, \tau_{J_{i+1}})$. Since R_i holds throughout τ_{J_i} and R_{i+1} throughout $\tau_{J_{i+1}}$, and $R_i \neq R_{i+1}$, the axiom gives $R_i \Gamma R_{i+1}$.

(ii) $\Rightarrow$ (i). Let $u = \tau_S$ and $v = \tau_{S'}$ with $S, S' \subseteq J$, $M(u, v)$, R throughout u and R' throughout v . If $R = R'$ there is nothing to prove, so assume $R \neq R'$.

First, S lies inside a single piece. Suppose instead that S meets two pieces J_i and J_j with $i < j$. Then S also meets J_{i+1} : if $j = i + 1$ this is the assumption, and if $j > i + 1$ convexity of

S puts all of J_{i+1} inside S . Since the relation is constant on S , $R_i = R = R_{i+1}$, contradicting $R_i \neq R_{i+1}$. So $S \subseteq J_p$ for some p , and likewise $S' \subseteq J_q$ for some q . As $R \neq R'$, $p \neq q$, and as $S < S'$, $p < q$.

Second, $q = p + 1$. Otherwise pick $y \in J_{p+1}$ and $x \in S, z \in S'$; then $x < y < z$, and $S \cup S'$ is convex by Theorem 7.4(c), so $y \in S \cup S' \subseteq J_p \cup J_q$, contradicting $y \in J_{p+1}$. Hence $R = R_p$ and $R' = R_{p+1}$, and (ii) gives $R \Gamma R'$. $\square$

For example, over $T = \mathbb{R}$, a history with DC on $[0, 1)$, EC at $\{1\}$ and PO on $(1, 2]$ satisfies Cont_{Γ_0} for the graph Γ_0 of Figure 2; a history with DC on $[0, 1)$ and PO on $[1, 2]$ violates it.

Remark 7.10 (finite variability is needed). Without finite variability the axiom can be vacuous at a moment of change. Let a be a fixed disk and let b be a disk whose centre oscillates as $t \rightarrow 0^+$ so that the relation cycles through DC, EC and PO infinitely often on every interval $(0, \varepsilon)$. Then no period v with $M(\tau_{(-1,0]}, v)$ has a constant relation throughout, and Cont_{Γ} imposes nothing on the transition at 0.

Remark 7.11 (the choice of Γ). Theorems 7.8 and 7.9 hold for every transition graph, but which graph is correct depends on the class of motions. Two examples show this.

Growth. Let a be the closed unit disk in $\mathbb{R}^2$, and let $b_t = a$ for $t \leq 0$ and b_t the closed disk of radius $1 + t$ for $t > 0$. Both regions vary continuously in any reasonable sense, and the relation is EQ on $(-\infty, 0]$ and NTPP on $(0, \infty)$. So a graph for deforming regions must contain the edge EQ–NTPP, and symmetrically EQ–NTPPi.

A vanishing hole. Let a be the closed unit disk and b_0 the closed disk of radius 2. For $t < 0$ let $b_t = b_0 \setminus D_t$, where D_t is the open disk of radius $|t|$ about the origin (for $|t| < 1$). Each b_t is regular closed, and $b_t \rightarrow b_0$ in the Hausdorff metric. For $t < 0$ the relation between a and b_t is PO (they overlap, and $a \not\subseteq b_t$ because of the hole); at $t = 0$ it is NTPP. So Hausdorff continuity of each region alone permits the transition PO–NTPP. The complements do not vary continuously here: a new component of the complement appears at distance nearly 2 from the old one. Requiring continuity of both a region and its complement excludes the example.

Conjecture 7.12. Let Γ_{def} be Γ_0 together with the edges EQ–NTPP and EQ–NTPPi. For histories in which each region and its complement vary continuously in the Hausdorff metric and every pair has finite variability, the transitions that occur are exactly the edges of Γ_{def} ; for rigid motions of regions of equal size they are the edges of Γ_0 not involving TPP, TPPI, NTPP or NTPPi.

Galton’s point-based analysis of continuous change (Galton, 2000) is the natural route to a proof. What the present results establish is the interface: once Γ is fixed for a class of motions, Cont_{Γ} is a correct point-free statement of it in discrete time and in finitely variable dense time.

7.5 Granularity: discrete data as a limit

Data arrive at a resolution. This subsection shows that continuous region structures can be obtained as limits of finite ones under refinement, and that the standard axioms of continuous space fall into two kinds: those that already hold at every finite stage, and those that hold only in the limit, under precise conditions on the refinement. All proofs are given in full. The finite Efremovič characterization (Proposition 7.25) is due to Düntsch and Vakarelov (2007); the other results are, as far as we know, new, though elementary.

Adjacency spaces. An *adjacency space* is a finite nonempty set A of *cells* with a reflexive, symmetric relation $\sim$ (Galton, 1999). Its *contact algebra* $\mathbf{B}(A)$ is the power set of A with

$$X C Y \iff \exists x \in X, \exists y \in Y : x \sim y.$$

For $X \subseteq A$ let $N(X) = \{y : y \sim x \text{ for some } x \in X\}$, and let $d(X, Y)$ be the length of a shortest path in the graph $(A, \sim)$ from a cell of X to a cell of Y (infinite if none exists). Thus $X C Y$ iff $d(X, Y) \leq 1$, and $N(X) \cap N(Y) \neq \emptyset$ iff $d(X, Y) \leq 2$.

Lemma 7.13. $\mathbf{B}(A)$ is a contact algebra. Conversely, every finite contact algebra is isomorphic to $\mathbf{B}(A)$, where A is its set of atoms and $x \sim y$ iff $x C y$.

Proof. For $\mathbf{B}(A)$: (C1) the empty set contains no cell; (C2) enlarging X or Y keeps any witnessing pair; (C3) a witness $y \in Y \cup Z$ lies in Y or in Z ; (C4) follows from symmetry of $\sim$; (C5) if $x \in X \cap Y$ then $x \sim x$ witnesses $X C Y$.

Conversely, let B be a finite contact algebra with atoms A . Every element is the join of the atoms below it, so by (C2) and (C3) (and (C4) for the second argument), $a C b$ iff some atom below a is in contact with some atom below b . The relation $\sim$ on atoms is symmetric by (C4) and reflexive by (C5), since $x \cdot x = x \neq 0$. The map sending a to the set of atoms below it is then an isomorphism $B \cong \mathbf{B}(A)$. $\square$

Refinement. A contact embedding is an injective Boolean homomorphism f with $a C b \iff f(a) C f(b)$. A refinement map $p : A' \rightarrow A$ between adjacency spaces is a surjection satisfying the image condition

$$x \sim y \iff \exists u \in p^{-1}(x), \exists v \in p^{-1}(y) : u \sim' v.$$

Intuitively each fine cell u lies in the coarse cell $p(u)$, and coarse cells are adjacent exactly when some of their parts are.

Proposition 7.14. The contact embeddings $\mathbf{B}(A) \rightarrow \mathbf{B}(A')$ are exactly the maps $X \mapsto p^{-1}(X)$ for refinement maps $p : A' \rightarrow A$.

Proof. Let f be a Boolean embedding. The regions $f(\{x\})$, $x \in A$, are nonzero, pairwise disjoint and have join A' , so each $u \in A'$ lies in exactly one of them; put $p(u) = x$ for that x . Then p is surjective, $f(X) = p^{-1}(X)$, and every Boolean embedding arises this way from a surjection. For such f , contact is preserved and reflected iff $\{x\} C \{y\} \iff p^{-1}(x) C p^{-1}(y)$ for all cells x, y , because contact of arbitrary regions reduces to contact of cells by (C2)–(C3). The right-hand side says some $u \in p^{-1}(x)$ and $v \in p^{-1}(y)$ have $u \sim' v$, so this is exactly the image condition. $\square$

A composite of refinement maps is a refinement map, since composites of contact embeddings are contact embeddings. Two consequences of the image condition are used repeatedly.

Lemma 7.15. Let $p : A' \rightarrow A$ be a refinement map. If $u \sim' v$ then $p(u) \sim p(v)$. Consequently $d'(p^{-1}(X), p^{-1}(Y)) \geq d(X, Y)$ for all $X, Y \subseteq A$.

Proof. The first claim is the right-to-left direction of the image condition, with $x = p(u)$ and $y = p(v)$. Hence p maps a path of length k from $p^{-1}(X)$ to $p^{-1}(Y)$ to a walk of length k from X to Y , which contains a path of length at most k . $\square$

Definition 7.16. A refinement system is a sequence of adjacency spaces $(A_n, \sim_n)_{n \geq 0}$ with refinement maps $p_n : A_{n+1} \rightarrow A_n$. For $n \leq m$ let $p_{nm} : A_m \rightarrow A_n$ be the composite. The limit $\mathbf{B}_\infty$ is the direct limit of the Boolean algebras $\mathbf{B}(A_n)$ along the embeddings $X \mapsto p_n^{-1}(X)$, with $a C b$ iff a and b are in contact in $\mathbf{B}(A_m)$ for some (equivalently, every) stage m at which both are represented.

Concretely, an element of $\mathbf{B}_\infty$ is represented by a set $X \subseteq A_n$ at some stage n , and the same element is represented at every later stage m by $p_{nm}^{-1}(X)$. Contact is well defined because the embeddings preserve and reflect contact.

Theorem 7.17. *The limit of a refinement system is a contact algebra. Conversely, every countable contact algebra is isomorphic to the limit of a refinement system.*

Proof. Each axiom (C1)–(C5) involves finitely many elements. Given finitely many elements of $\mathbf{B}_\infty$, choose a stage at which all are represented; there the axiom holds by Lemma 7.13, and Boolean operations and contact in $\mathbf{B}_\infty$ are computed at that stage.

Conversely, let B be a countable contact algebra, enumerated as $b_0, b_1, \dots$. Let B_n be the Boolean subalgebra generated by $b_0, \dots, b_n$; it is finite because finitely generated Boolean algebras are finite. With the restricted contact relation, B_n satisfies (C1)–(C5), so by Lemma 7.13 it is isomorphic to $\mathbf{B}(A_n)$ with A_n its atoms. The inclusion $B_n \subseteq B_{n+1}$ is a contact embedding, so by Proposition 7.14 it corresponds to a refinement map $A_{n+1} \rightarrow A_n$. The limit of this system is the union of the B_n , which is B . $\square$

We now compare axioms. For $X \subseteq A_n$ we write $\widehat{X}$ for the element of $\mathbf{B}_\infty$ it represents, and $\widehat{x}$ for $\widehat{\{x\}}$.

Proposition 7.18 (atomlessness). *$\mathbf{B}_\infty$ is atomless iff every cell eventually splits: for all n and $x \in A_n$ there is $m > n$ with $|p_{nm}^{-1}(x)| \geq 2$.*

Proof. If some $x \in A_n$ never splits, then at every stage $m \geq n$ the only nonempty subset of $p_{nm}^{-1}(x)$ is the whole fibre, so $\widehat{x}$ is an atom. Conversely, let $a = \widehat{X}$ be nonzero, with $X \subseteq A_n$, and pick $x \in X$. Choose m with $|p_{nm}^{-1}(x)| \geq 2$ and a cell u in that fibre. Then $0 < \widehat{u} < \widehat{x} \leq a$, so a is not an atom. $\square$

Proposition 7.19 (connection). *The connection axiom, $a \neq 0, 1 \Rightarrow a \text{ C } a^*$, holds in $\mathbf{B}_\infty$ iff every graph $(A_n, \sim_n)$ is connected. Moreover, if $(A_m, \sim_m)$ is connected then so is $(A_n, \sim_n)$ for every $n \leq m$.*

Proof. At stage n , a subset X with $\emptyset \neq X \neq A_n$ touches its complement iff some edge of the graph joins X to $A_n \setminus X$. That holds for all such X iff the graph is connected. Every element of $\mathbf{B}_\infty$ is represented at some stage, with complements and contact computed there. For the second claim, by Lemma 7.15 p_{nm} maps paths to walks, and it is surjective, so it maps a connected graph onto a connected graph. $\square$

Connection is therefore a *stagewise* property: it holds in the limit iff it holds at every finite stage. The next two axioms behave differently.

Extensionality. A contact algebra is *extensional* if $a \not\leq b$ implies that some c has $c \text{ C } a$ and not $c \text{ C } b$: regions are determined by what they touch.

Lemma 7.20 (interior form). *A contact algebra is extensional iff it satisfies the interior axiom: every nonzero d has a nonzero c with $c \ll d$.*

Proof. Assume the interior axiom and $a \not\leq b$. Then $d = a \cdot b^* \neq 0$; take $c \neq 0$ with $c \ll d$, that is, not $c \text{ C } d^*$. Since $c \leq d \leq a$ and $c \neq 0$, (C5) and (C2) give $c \text{ C } a$. Since $b \leq a^* + b = d^*$, (C2) gives not $c \text{ C } b$.

Conversely assume extensionality and let $d \neq 0$. If $d = 1$, any nonzero c has $c \ll d$ because $d^* = 0$ and (C1) applies. Otherwise $d \not\leq d^*$, so some c has $c \text{ C } d$ and not $c \text{ C } d^*$. Put $c' = c \cdot d$. By (C2), not $c' \text{ C } d^*$, so $c' \ll d$. And $c' \neq 0$: otherwise $c \leq d^*$, and since $c \neq 0$ (by (C1)) we would have $c \text{ C } c$ by (C5), hence $c \text{ C } d^*$ by (C2). $\square$

Corollary 7.21. *A finite contact algebra $\mathbf{B}(A)$ is extensional iff $\sim$ is the identity relation, that is, iff contact coincides with overlap. In particular, a finite contact algebra that is both extensional and connected has exactly one atom.*

Proof. If $\sim$ is the identity, then $c \ll d$ iff $c \subseteq d$, and $c = d$ witnesses the interior axiom. Conversely, apply the interior axiom to $d = \{x\}$: the only nonempty $c \subseteq \{x\}$ is $\{x\}$, and $\{x\} \ll \{x\}$ says no cell other than x is adjacent to x . For the last claim, a connected graph with the identity as its adjacency has a single vertex. $\square$

So every finite model of a nontrivial connected region structure fails extensionality. The property can only appear in the limit.

Theorem 7.22 (extensionality in the limit). $\mathbf{B}_\infty$ is extensional iff every cell eventually acquires an interior cell: for all n and $x \in A_n$ there are $m \geq n$ and $u \in p_{nm}^{-1}(x)$ such that every $v \in A_m$ with $v \sim_m u$ lies in $p_{nm}^{-1}(x)$.

Proof. By Lemma 7.20 it suffices to check the interior axiom, and it suffices to check it for elements of the form $\hat{x}$: every nonzero element contains some $\hat{x}$, and if $c \ll \hat{x} \leq d$ then $d^* \leq \hat{x}^*$, so not $c \subset d^*$ by (C2), that is, $c \ll d$.

Suppose x acquires an interior cell u at stage m . Then $c = \{\widehat{u}\}$ is nonzero, and no cell adjacent to u lies outside $p_{nm}^{-1}(x)$, so not $c \subset \hat{x}^*$, that is, $c \ll \hat{x}$.

Conversely, suppose $c \neq 0$ and $c \ll \hat{x}$. Represent c at a stage $m \geq n$ as a nonempty set Y . Then $Y \subseteq p_{nm}^{-1}(x)$, since a cell of Y outside the fibre would make c overlap, hence by (C5) touch, $\hat{x}^*$. And no cell of Y is adjacent to a cell outside the fibre. So any $u \in Y$ is an interior cell of x . $\square$

Corollary 7.23. If $\mathbf{B}_\infty$ is connected and extensional and some A_n has at least two cells, then $\mathbf{B}_\infty$ is atomless.

Proof. By Proposition 7.18 we must show that every cell eventually splits. First let $z \in A_m$ with $m \geq n$. Then $|A_m| \geq 2$ by surjectivity, and A_m is connected by Proposition 7.19, so z has a neighbour $w \neq z$. By Theorem 7.22, at some stage $k \geq m$ the fibre $p_{mk}^{-1}(z)$ contains an interior cell u . By the image condition for p_{mk} , some cell of $p_{mk}^{-1}(z)$ is adjacent to a cell of $p_{mk}^{-1}(w)$, which lies outside the fibre of z ; that cell is not interior, so it differs from u , and the fibre of z has at least two cells. If instead $z \in A_m$ with $m < n$, pick any cell of $p_{mn}^{-1}(z)$; by the first case it eventually splits, so the fibre of z eventually has at least two cells. $\square$

Normality. The Efremovič axiom (Section 3) says: if not $a \subset b$, there is c with not $a \subset c$ and not $c^* \subset b$. Thus c is a buffer around b that does not reach a .

Lemma 7.24. In $\mathbf{B}(A)$, let X, Y be nonempty with not $X \subset Y$. A buffer C (with not $X \subset C$ and not $(A \setminus C) \subset Y$) exists iff $d(X, Y) \geq 3$.

Proof. Not $(A \setminus C) \subset Y$ means no cell outside C is adjacent to a cell of Y , that is, $N(Y) \subseteq C$. Not $X \subset C$ means no cell of C is adjacent to a cell of X , that is, $C \cap N(X) = \emptyset$. So a buffer exists iff $N(Y) \cap N(X) = \emptyset$, in which case $C = N(Y)$ works. This is equivalent to $d(X, Y) \geq 3$. $\square$

Proposition 7.25 (Düntsch and Vakarelov, 2007). $\mathbf{B}(A)$ satisfies the Efremovič axiom iff $\sim$ is transitive. In particular, a finite connected contact algebra satisfies it iff every cell is adjacent to every other.

Proof. If $\sim$ is transitive it is an equivalence relation, so $N(X) \cap N(Y) \neq \emptyset$ would give $x \sim z \sim y$, hence $x \sim y$ and $X \subset Y$. By Lemma 7.24, a buffer exists whenever not $X \subset Y$ (the case of an empty set is trivial, taking $C = A$ or $C = \emptyset$). If $\sim$ is not transitive, take $x \sim y \sim z$ with not $x \sim z$; then $X = \{x\}$ and $Y = \{z\}$ are not in contact but $d(X, Y) = 2$, so no buffer exists. For the last claim, a transitive, reflexive, symmetric relation on a connected graph is total. $\square$

Among finite connected algebras, extensionality forces “nothing touches anything else” and the Efremovič axiom forces “everything touches everything”. No nontrivial finite model has both. This is Poincaré’s paradox of the physical continuum in exact form: in perception A is indistinguishable from B and B from C , yet A is distinguishable from C , and the mathematical continuum removes the contradiction.

Theorem 7.26 (normality in the limit). $\mathbf{B}_\infty$ satisfies the Efremovič axiom iff separated cells eventually drift apart: for all n and all $x, y \in A_n$ with $x \not\sim_n y$ there is $m \geq n$ with $d_m(p_{nm}^{-1}(x), p_{nm}^{-1}(y)) \geq 3$.

Proof. ($\Leftarrow$) Let not $a \subset b$, with a, b nonzero and represented at stage n by X, Y . Every pair $(x, y) \in X \times Y$ is non-adjacent, so for each there is a stage m_{xy} at which their fibres are at distance at least 3. By Lemma 7.15, distances never decrease along the system, so at $m = \max m_{xy}$ all pairs are at distance at least 3 simultaneously. Distances between sets are minima over pairs, so $d_m(p_{nm}^{-1}(X), p_{nm}^{-1}(Y)) \geq 3$, and Lemma 7.24 gives a buffer at stage m , which is a buffer in $\mathbf{B}_\infty$.

($\Rightarrow$) Let $x \not\sim_n y$. Then not $\hat{x} \subset \hat{y}$, so the axiom gives a buffer c . Represent c at a stage $m \geq n$. The contact conditions are computed at stage m , so by Lemma 7.24, $d_m(p_{nm}^{-1}(x), p_{nm}^{-1}(y)) \geq 3$. $\square$

The raster paradox. Let G_n be the $2^n \times 2^n$ grid of cells (i, j) , $0 \leq i, j < 2^n$, with the refinement map $p(i, j) = (\lfloor i/2 \rfloor, \lfloor j/2 \rfloor)$, so each cell splits into its four quadrants. Consider two adjacencies: *king* adjacency, $\max(|i - i'|, |j - j'|) \leq 1$ (sharing an edge or a corner), and *rook* adjacency, $|i - i'| + |j - j'| \leq 1$ (sharing an edge). Graph distance is then the Chebyshev distance for king adjacency and the Manhattan distance for rook adjacency. After k refinements, the descendants of (I, J) are the cells (i, j) with $2^k I \leq i < 2^k(I + 1)$ and $2^k J \leq j < 2^k(J + 1)$.

Proposition 7.27. Both grid systems are refinement systems whose limits are connected, extensional and atomless. The king limit satisfies the Efremovič axiom; the rook limit does not.

Proof. *Image condition.* Children of adjacent parents can be chosen adjacent: in each coordinate, if the parents agree take equal children, and if they differ by 1 take the two children on either side of the shared boundary, which differ by 1. Such children are adjacent in both senses, since they differ in the same coordinates as their parents and by at most 1 in each. Conversely, if two cells differ by at most 1 in a coordinate, so do their parents in that coordinate, and equal coordinates have equal parents; so adjacent children have adjacent parents, in both senses.

Connection holds because grid graphs are connected (Proposition 7.19). *Extensionality:* two refinements after stage n , the cell $u = (4I + 1, 4J + 1)$ descends from (I, J) , and all its neighbours in either sense have coordinates in $[4I, 4I + 2] \times [4J, 4J + 2]$, inside the fibre; Theorem 7.22 applies. *Atomlessness* follows from Corollary 7.23.

King, Efremovič. Let $(I, J) \not\sim (I', J')$, so their Chebyshev distance δ is at least 2; say $|I - I'| = \delta$ with $I < I'$. After one refinement the first coordinates of the descendants of (I, J) are at most $2I + 1$ and those of (I', J') at least $2I' \geq 2I + 4$. So the fibres are at Chebyshev distance at least 3, and Theorem 7.26 applies.

Rook, failure. At stage 1, the cells $x = (0, 0)$ and $y = (1, 1)$ are not rook-adjacent. After k refinements, $(2^k - 1, 2^k - 1)$ descends from x and $(2^k, 2^k)$ from y , at Manhattan distance 2. So the fibres never reach distance 3, and by Theorem 7.26 the axiom fails. $\square$

Proposition 7.28. The king limit embeds into the regular closed subsets of $[0, 1]^2$, with contact becoming “sharing a point”.

Proof. Send a set X of stage- n cells to the union $\rho(X)$ of the closed squares $[i2^{-n}, (i + 1)2^{-n}] \times [j2^{-n}, (j + 1)2^{-n}]$ for $(i, j) \in X$. A square is the union of its four quadrants, so ρ is compatible with refinement and well defined on the limit. Distinct sets of cells give distinct unions,

since the squares have nonempty, disjoint interiors. Finite unions of closed squares are regular closed; ρ preserves joins, and the closure of the complement of $\rho(X)$ is $\rho(A_n \setminus X)$, so ρ preserves complements and hence meets. Two closed squares at the same level intersect iff their Chebyshev distance is at most 1, and two unions of squares intersect iff some pair of their squares does, so contact is preserved and reflected. $\square$

The choice between 4- and 8-connectivity, routinely made in raster processing, thus decides whether the continuous theory obtained in the limit is normal. The rook limit is a legitimate contact algebra, extensional and connected, but its contact is “sharing a boundary segment”, and corner contact is exactly what normality needs.

For dynamic data the natural object is doubly indexed, by spatial resolution and by time, and Theorem 7.8 is its temporal counterpart: at a fixed temporal resolution, continuity is a frame-to-frame condition. Whether the limit of a doubly indexed refinement system is a DCA, and which time axioms it satisfies, is the most concrete open problem connecting this framework to real data.

7.6 Open problems

1. A representation theorem for process-DCAs, extending the snapshot representation to processes and participation.
2. Decidability of the quantifier-free fragment with participation and Cont_Γ , first for discrete time, where Theorem 7.8 suggests a transition-system reduction.
3. An abstract axiomatization of the sort of periods, so that Theorem 7.4 becomes part of a representation theorem rather than a statement about rich snapshot models.
4. A proof of Conjecture 7.12, identifying the transition graph for each natural class of continuous motion.
5. Axioms for genidentity as a tolerance relation that characterize split and merge.
6. Conditions under which doubly indexed refinement systems, by resolution and by time, converge to DCAs.
7. A point-free replacement for global simultaneity, suitable for asynchronous observation.

8 Case studies

Across three cases, DCAs handle every question about *when* regions coexist or precede one another, while questions about *how* change happens (continuity, agency, identity) need the extensions of Section 7.

8.1 A river flood

A flood rises over several days, inundates parcels on the floodplain, triggers an evacuation, and recedes. Data arrive as satellite-derived extents at irregular intervals.

The snapshot approach stores each extent as a layer. The DCA approach represents the flood as one history, and relations such as “the evacuation zone coexisted with the flood” or “the levee breach preceded inundation of the town” become direct statements. What DCAs cannot express is the asymmetric claim that the flood *inundated* the parcel, as distinct from the parcel and the flood merely overlapping.

8.2 A wildfire

A fire starts at an ignition point, spreads across fuel types, is stopped at a river on one flank, and crosses a road on the other. Perimeters are mapped every few hours.

The crucial analytic questions concern continuity: did the fire cross the road, or did it spot across it? Spotting means a new ignition that is spatially disconnected from the main front. Distinguishing the two requires knowing whether the fire's relation to the far side passed through EC (continuous spread) or jumped from DC to PO (spotting). This is exactly the continuity axiom above, and it also involves genidentity: is the spot fire part of the same fire?

8.3 A pandemic

A novel respiratory virus emerges in one city. It spreads locally through everyday contact, jumps between continents by air travel, and circulates for several years as successive variants arise and compete. Data arrive as case counts aggregated by administrative unit and week, reported with lags that differ between jurisdictions, alongside genomic sequences that record the virus's lineages.

This case stresses every requirement in Table 1 at once, and it shows that some of them must be interpreted with care.

Histories and first arrival. Treating the epidemic in each region as one history, DCAs express the basic comparative questions directly. "Were the outbreaks in two cities under way at the same time?" is time contact; "Did community transmission in one country begin before it began in another?" is precedence. These are the questions surveillance dashboards answer today by comparing weekly maps.

Continuity relative to the right contact. Spread is not continuous in geographic space: an infected traveller carries the virus from one city to another without it passing through the land between. Read naively, the continuity axiom of Section 7 would rule out such histories. The resolution is that continuity is always relative to a contact relation. An adjacency space need not be planar: cells can be cities, with adjacency given by commuting and flight links as well as shared borders. In the contact algebra of that mobility network, the pandemic's spread *is* continuous, and a jump in geographic space that does not follow a mobility link is evidence of an unobserved route. The same formal constraint thus becomes an analytic tool, provided the choice of contact is made explicit.

Processes and participation. Two variants co-circulating in one city occupy overlapping histories but are different processes with different participants, which is exactly why Section 7 separates a process from the history it occupies. Populations, hospitals and schools participate in transmission in different roles, and questions such as "which districts' populations were infected by the second variant?" are questions about participation, not overlap.

Identity: lineages as genidentity. Viral lineages branch as variants emerge and, occasionally, join through recombination. Genidentity as a tolerance relation captures this: a variant is genidentical with its ancestral lineage, two sister variants are each genidentical with their common ancestor but not with each other, and the failures of transitivity mark the branch points of the phylogeny. Recombination appears as the reverse pattern, a merge.

Local time and granularity. Surveillance data make the snapshot assumption visibly false. A weekly map shows reports, not infections, and reporting lags differ between jurisdictions, so two regions shown as "simultaneous" on a dashboard may have been infected weeks apart.

Aggregation by county and week is a choice of resolution in both space and time. Here the requirements for local time (R7) and granularity (R8) are not refinements but practical necessities, and the doubly indexed refinement systems of Section 7.5 are the natural formal setting.

8.4 Queries classified

Table 2: Typical queries, classified by the formalism needed to express them.

Case	Query	Static CA	DCA	Needs extension
Flood	Did the flood touch the hospital at any time?	Per snapshot only	Yes: space contact	—
Flood	Did the evacuation overlap the flood in time?	No	Yes: time contact	—
Flood	Did the levee breach precede inundation of the town?	No	Yes: precedence	—
Flood	Which parcels did the flood inundate?	No	Only as overlap	Participation
Wildfire	Was the river a barrier throughout on the east flank?	No	Yes: no space contact	—
Wildfire	Did the fire cross the road or spot across it?	No	No	Continuity, genidentity
Wildfire	Is the spot fire part of the same fire?	No	No	Genidentity
Pandemic	Were the outbreaks in two cities under way at the same time?	No	Yes: time contact	—
Pandemic	Did transmission in one country begin before another?	No	Yes: precedence	—
Pandemic	Did the virus reach the island only through its airport?	No	No	Continuity, relative to mobility contact
Pandemic	Which districts were infected by the second variant?	No	Only as overlap	Participation
Pandemic	Is one variant descended from another?	No	No	Genidentity
Pandemic	Were cases reported in the same week actually concurrent?	No	No	Local time, granularity

The pattern in Table 2 is consistent. DCAs already answer the temporal-topological questions that snapshot GIS answers only by comparing layers. The questions they cannot answer are the ones about process: who acted on what, whether change was continuous, and what counts as the same thing.

9 Discussion

9.1 Philosophical grounding

The framework inherits a coherent philosophy of space and time. From Leibniz it takes relationism: space as the order of coexisting things, time as the order of successive ones. From Whitehead it takes the insistence that points and instants be constructed from relations among

extended things, and the demand that space and time be treated together (Whitehead, 1929). DCAs realize the first part of Whitehead’s demand precisely: moments are defined, not assumed.

There is a historical irony in the snapshot construction. Bergson (1907) criticized the “cinematographic” habit of the intellect, which reconstructs becoming from a series of still frames, and Whitehead was sympathetic to that criticism. Vakarelov adopts the cinematographic method explicitly, but as scaffolding: the axioms that emerge no longer mention frames. Our SPAN-first programme can be read as completing that removal, making processes, not frames, the primitive entities.

The proposal also reflects a methodological view about granularity. Continuous space and dense time appear in the theory as limits of refinement, not as given. A raster at a resolution is not a crude copy of a continuous world but a legitimate finite structure; the continuum is the rule that connects such structures, not a hidden original they approximate. Some axioms of continuous space hold only at that limit.

9.2 Related work

Galton (2000) is the closest point-based precursor, with a detailed theory of continuous change of spatial relations. Muller (1998) developed a spatio-temporal mereotopology with histories as primitives, which is SPAN-first in spirit but without the algebraic representation theory of DCAs. Grenon and Smith’s SNAP/SPAN framework, later developed in Basic Formal Ontology, provides the ontological categories but not a region calculus. On the GIS side, Peuquet (1994), Worboys (2005) and Hornsby and Egenhofer (2000) established the representational requirements. Four-dimensional spatio-temporal databases implement histories directly but generally without an axiomatic theory of their relations. Nenchev (2013) studies relational versions of stable and unstable relations.

The contribution of DCAs relative to this work is the combination of point-free time, explicit axioms, representation theorems, and decidable fragments. The contribution of this paper is to place that combination within the GIS debate and to state precisely what it lacks.

9.3 Limitations

Parts of Section 7 remain programmatic. The process layer and genidentity as a tolerance relation are proposals whose consistency with the existing representation theory is unproved. The continuity results are proved for rich snapshot models, not for abstract DCAs, and the correct transition graph for physically continuous motion is conjectured, not proved. The refinement results are proved in full but concern space only; their temporal and doubly indexed counterparts are open. We have not addressed computational cost: adding participation and continuity could make reasoning undecidable or intractable, which would limit practical use. Finally, the case studies are illustrative. A serious evaluation would implement a fragment of the framework and apply it to real flood, fire and epidemiological data.

10 Conclusion

GIScience has long known what it wants from a theory of change: events and processes as first-class entities, relations between histories, and a principled account of identity and continuity. It has lacked a formal foundation with the properties that made static qualitative spatial reasoning succeed.

Dynamic contact algebras supply much of that foundation. Their elements are histories, their primitive relations are contemporaneity and precedence, and their moments are con-

structed from relations among regions, not assumed. Although they are abstracted from a snapshot model, they already carry much of what a SPAN ontology requires.

What they lack can be stated precisely: processes distinct from the regions they occupy, participation, continuity of change, identity under splitting and merging, freedom from global simultaneity, and a link to data at a resolution. We have proposed candidate primitives for each, including genidentity as a tolerance relation whose failures of transitivity mark splits and merges, and a view of discrete data in which the continuum appears as a limit of refinement.

The resulting research agenda is concrete: representation theorems for process-DCAs, decidability of their quantifier-free fragments, a point-free account of periods that meet, and convergence conditions for refinement systems indexed by both resolution and time. Progress on these questions would give process-oriented GIS what RCC-8 gave static GIS: a small, well-understood formal core.

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