

A generalized Gerver sofa for angled corridors

Henrik S. Guttusen

Danish Fundamental Metrology, 2970 Hørsholm, Denmark

henrisro@proton.me

October 4, 2026

The moving sofa problem asks for the maximal area of a two-dimensional object capable of being moved through a right-angled corridor of unit width. J. L. Gerver constructed a sofa of area $2.2195\dots$, which was only recently proven to be optimal by J. Baek. Here, I consider the moving sofa problem in an angled corridor, C_φ , of unit width for turn angles $0 < \varphi \leq \frac{\pi}{2}$. I first generalize Hammersley's arguments to devise simple analytical lower and upper area bounds for all $0 < \varphi \leq \frac{\pi}{2}$. ChatGPT-6 Astra (Codex) is prompted to derive a generalized Gerver sofa parametrized by φ . The generalized Gerver sofa area is non-analytic at the unique root $\varphi_* \in (0, \pi/2)$ of $\exp(\varphi_* \cot \varphi_*) = 1 + 2 \cos \varphi_* - \cos^2 \varphi_*$. For turn angles smaller than φ_* a closed-form expression of the sofa area is obtained. The generalized Gerver sofa is conjectured to be the solution to the moving sofa problem for turn angles $0 < \varphi \leq \frac{\pi}{2}$.

1 Introduction

The moving sofa problem [7, 9] asks for the maximal area of a two-dimensional object capable of being moved through a right-angled corridor of unit width. J. L. Gerver constructed a sofa of area $2.2195\dots$ [3], marginally enhancing the area of J. M. Hammersley's $\frac{\pi}{2} + \frac{2}{\pi} \approx 2.2074\dots$ sofa [4]. Gerver's sofa was recently proven to be optimal by J. Baek [1].

Here, I consider the moving sofa problem in an angled corridor, C_φ with $\varphi \in (0, \frac{\pi}{2}]$, of unit width and devise simple analytical lower and upper bounds for all $\varphi \in (0, \frac{\pi}{2}]$ by generalizing Hammersley's arguments. Codex GPT-6 is used to devise a generalized Gerver sofa for all $\varphi \in (0, \frac{\pi}{2}]$, posing as a conjectured solution to the generalized moving sofa problem.

2 Generalized Hammersley arguments

2.1 Lower bound

For turn angles smaller than $\varphi = \pi/2$, a generalized Hammersley sofa can be constructed, as illustrated in Fig. 1. By virtue of the Hammerlsey design, the constructed

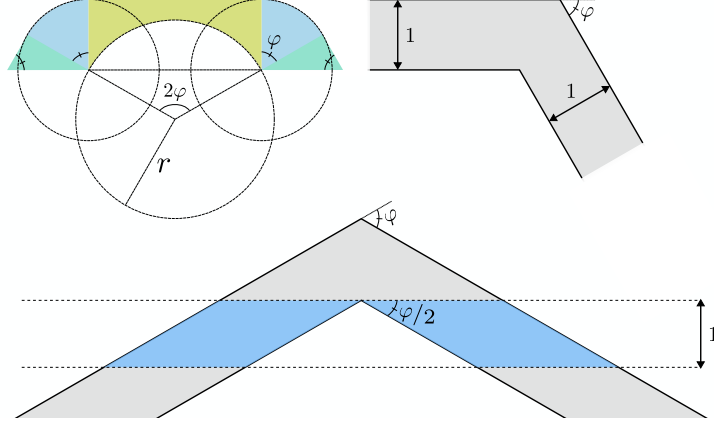


Figure 1: Upper left panel: generalized Hammersley sofa comprising two circular sectors, each connected to a triangle and separated by a rectangle minus a shifted circle. Upper right panel: corridor with turn angle φ , C_φ defining a generalized moving sofa problem. Lower panel: Projection of a strip of unit width onto the half-rotated corridor having maximal area of $A_U(\varphi)$.

sofa will by analogy uniquely translate and rotate through C_φ , always maintaining at least five points of contact throughout its motion.

The area of the generalized Hammersley sofa amounts to¹ $A(r, \varphi) = \varphi + \tan\left(\frac{\pi}{2} - \varphi\right) + 2r \sin(\varphi) - \frac{r^2}{2} (2\varphi - \sin(2\varphi))$.

The radius r that maximizes $A(r, \varphi)$ is $r^*(\varphi) = \frac{2 \sin(\varphi)}{2\varphi - \sin(2\varphi)}$, leading to the optimal area of the generalized Hammersley sofa, leading to the optimal area lower bound

$$A_L(\varphi) = \varphi + \tan\left(\frac{\pi}{2} - \varphi\right) + \frac{2 \sin^2(\varphi)}{2\varphi - \sin(2\varphi)}. \quad (1)$$

The Hammersley case is reproduced at $\varphi = \pi/2$.

2.2 Upper bound

Hammersley's upper bound can also be straightforwardly generalized to arbitrary turn angles. Any feasible sofa candidate must be able to maneuver through the left arm and through the angle when rotated through by half the turn angle, $\varphi/2$. Thus, one may project a strip of unit width onto the half-rotated corridor and calculate the maximal overlap area to obtain an upper bound of the optimal sofa area. The projected area is maximal when the region is about to disconnect into two disjoint pieces, one in each

¹The area of the orange and green regions in Fig. 1 sums to $\varphi + \tan\left(\frac{\pi}{2} - \varphi\right)$, while the area of the blue region is $2r \sin(\varphi) - \frac{r^2}{2} (2\varphi - \sin(2\varphi))$.

arm, at which point the overlap region has an area of (see Fig. 1, cf. Ref. [2])

$$A_U(\varphi) = \frac{2}{\sin(\varphi/2)}, \quad (2)$$

see the lower panel of Fig. 1. Hammersley's upper bound is recovered at $\varphi = \pi/2$: $A_U(\pi/2) = 2\sqrt{2} = 2.8284\dots$. One may also confirm that at small turn angles, $A_L \sim \frac{5}{2\varphi} + \mathcal{O}(\varphi)$, whereas $A_U \sim \frac{4}{\varphi} + \mathcal{O}(\varphi)$, so these bounds have constrained the power-law behaviour of the optimal sofa for small turn angles. At $\varphi = \pi/2$ impressive efforts have previously been made in pushing down the upper bound down by imposing finitely many, carefully selected corridor constraints [6].

3 Generalized Gerver sofa

The generalized Hammersley bounds alongside the generalized Gerver sofa, derived in the present section, are illustrated in Fig. 2.

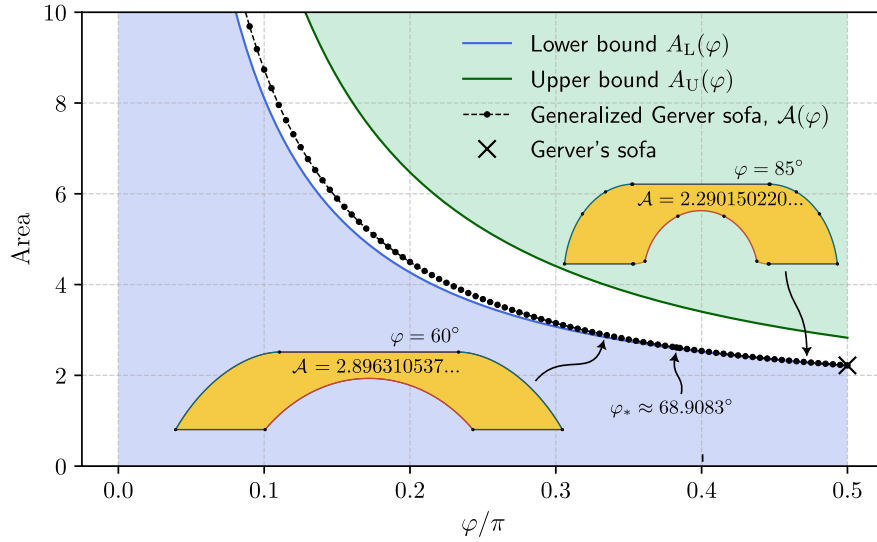


Figure 2: Generalized Hammersley bounds (Eqs. (1) and (2)) for the moving sofa problem in an angled corridor. The area of the generalized Gerver sofa from Sec. 3, which is non-analytic at $\varphi = \varphi_* = 68.908294355\dots^\circ$, is shown with black dots. Examples of the generalized Gerver sofa at $\varphi = 60^\circ$ and $\varphi = 85^\circ$ are illustrated in yellow.

3.1 AI disclaimer

The generalized Gerver sofa construction of the present section was obtained by prompting ChatGPT-6 Astra Medium in Codex. Computations ran for about two hours

and was based on an initial note containing the generalized Hammersley bounds, which were manually derived. The argument provided below was initially generated by ChatGPT-6 Astra but has been moderated by the author.

3.2 Geometry

Adopting the fixed-sofa, rotating-corridor framework [3, 8], let $\mathbf{q}(t) \in \mathbb{R}^2$ denote the inner-corner position at a rotation angle $t \in [0, \varphi]$. In the following, vectors are organized as columns and denoted by bold symbols. Primes denoted differentiation with respect to the scalar argument. Define the wall normals

$$\begin{aligned}\mathbf{n}_1(t) &= (-\sin t, \cos t)^T, \\ \mathbf{n}_2(t) &= (\sin(\varphi - t), \cos(\varphi - t))^T.\end{aligned}\tag{3}$$

Let $R(t)$ be the standard rotation matrix that rotates points by counterclockwise through t , and define $J = R(\frac{\pi}{2})$. For a point in the plane $\mathbf{z} = (x, y)^T$, let $u_i(t, \mathbf{z}) = \mathbf{n}_i(t) \cdot (\mathbf{z} - \mathbf{q}(t))$ be the signed perpendicular from point $\mathbf{z}$ to the i -th inner-wall line for $i = 1, 2$.

The instantaneous corridor H_t and the largest point set compatible with the motion and the entrance/exit strip $S(\mathbf{q})$ are

$$\begin{aligned}H_t &= \{\mathbf{z} : u_1, u_2 \leq 1, \max(u_1, u_2) \geq 0\}, \\ S(\mathbf{q}) &= \{\mathbf{z} : 0 \leq y \leq 1\} \cap \bigcap_{0 \leq t \leq \varphi} H_t.\end{aligned}\tag{4}$$

An optimal candidate sofa boundary uses portions belonging to ∂S of the outer and inner-wall envelopes,

$$\begin{aligned}\mathbf{A}_i &= \mathbf{q} + \mathbf{n}_i + (\mathbf{q}' \cdot \mathbf{n}_i)J\mathbf{n}_i, \\ \mathbf{B}_i &= \mathbf{q} + (\mathbf{q}' \cdot \mathbf{n}_i)J\mathbf{n}_i,\end{aligned}\tag{5}$$

the path of the corner $\mathbf{q}$, and horizontal segments at $y = 0, 1$. Above, all quantities are evaluated at t .

3.3 Contact equations and matching

A contact phase is an interval with fixed exposed boundary types. Let $\beta_i = 1$ when $\mathbf{B}_i$ is exposed and $\chi = 1$ when $\mathbf{q}$ is exposed. Otherwise these indicators are zero. When both outer supports are active, the right-angle contact-balance method [3, 8] may be extended to give

$$\begin{aligned}M &= \sum_{i=1}^2 (1 + \beta_i) \mathbf{n}_i \mathbf{n}_i^T, \\ M\mathbf{q}'' + (\chi J - 2MJ)\mathbf{q}' + \mathbf{n}_1 + \mathbf{n}_2 &= \mathbf{0}.\end{aligned}\tag{6}$$

The above equations for arbitrary angle are derived for the present construction. Each phase becomes a constant-coefficient system. Define the rotating-frame position and

velocity by $\mathbf{p} = R(-t)\mathbf{q}$, $\mathbf{w} = R(-t)\mathbf{q}'$, and $M_0 = \sum_{i=1}^2 (1 + \beta_i)\mathbf{n}_i(0)\mathbf{n}_i(0)^T$. Then

$$\begin{aligned}\mathbf{p}' &= -J\mathbf{p} + \mathbf{w}, \\ \mathbf{w}' &= (J - \chi M_0^{-1}J)\mathbf{w} - M_0^{-1}(\mathbf{n}_1(0) + \mathbf{n}_2(0)),\end{aligned}\tag{7}$$

which may be solved exactly by a matrix exponential. For $\varphi > \varphi_*$, where φ_* is defined in Sec. 3.4, one may use transition angles $0 < a < b < \varphi/2$ and first-half phases according to:

$$\frac{t}{(\beta_1, \beta_2; \chi)} \Big| \begin{array}{ccc} [0, a] & [a, b] & [b, \varphi/2] \\ (1, 0; 0) & (1, 0; 1) & (0, 0; 1) \end{array}\tag{8}$$

Then, reflect the second half: $\mathbf{q}(\varphi - t) = \mathcal{R}\mathbf{q}(t)$, where $\mathcal{R}$ is reflection in the vertical line through $\mathbf{q}(\varphi/2)$. Starting at $\mathbf{q}(0) = \mathbf{0}$, one may then propagate $\mathbf{q}, \mathbf{q}'$ continuously. The two initial velocity components and a, b satisfy

$$[\mathbf{A}_2(0)]_y = 0, \quad [\mathbf{q}'(\varphi/2)]_y = 0, \quad \mathbf{q}(a) = \mathcal{R}\mathbf{B}_1(b),\tag{9}$$

where $[v]_y$ denotes the vertical component of a vector $\mathbf{v}$. From here, one may continue the solution from Gerver's right-angle construction [3], using its five-phase description [8]. In result, the boundary uses $\mathbf{B}_1$ on $[0, b]$, $\mathbf{B}_2$ on $[\varphi - b, \varphi]$, and $\mathbf{q}$ on $[a, \varphi - a]$, together with the outer envelopes and three straight segments. Splitting at changes of analytic formula gives a total of fifteen curved pieces.

3.4 Collapse and reduced branch

For small enough rotation angles, a, b vanish at the unique root $\varphi_* \in (0, \pi/2)$ of

$$e^{\varphi_* \cot \varphi_*} = 1 + 2 \cos \varphi_* - \cos^2 \varphi_*.\tag{10}$$

Numerically, $\varphi_* = 68.908294355 \dots^\circ$. For $0 < \varphi \leq \varphi_*$, only the phase $(\beta_1, \beta_2; \chi) = (0, 0; 1)$ remains. With $\mathbf{q}(0) = \mathbf{0}$, its initial velocity is

$$\begin{aligned}\mathbf{q}'(0) &= \left(-\frac{\cot \varphi + v_0 \cos \varphi}{\sin \varphi}, v_0 \right)^T, \\ v_0 &= \frac{e^{\varphi \cot \varphi} - \sin^2 \varphi}{\sin \varphi \cos \varphi}.\end{aligned}\tag{11}$$

The reduced boundary has three curved pieces $(\mathbf{A}_1, \mathbf{A}_2, \mathbf{q})$ and three straight segments. Its area is

$$\mathcal{A}_{\varphi \leq \varphi_*}(\varphi) = \frac{e^{\varphi \cot \varphi} - \sin^2 \varphi}{\sin \varphi \cos^2 \varphi} - \frac{\varphi}{\cos \varphi}.\tag{12}$$

The five-phase area $\mathcal{A}_{\varphi > \varphi_*}$ is obtained from Green's formula $\mathcal{A}_{\varphi > \varphi_*} = \frac{1}{2} \oint_{\partial S} (x dy - y dx)$, with counterclockwise boundary orientation.

It is noted that near $\varphi = 0$, $\mathcal{A}_{\varphi \leq \varphi_*}(\varphi) \sim \frac{e}{\varphi} + \mathcal{O}(\varphi)$, improving the previous value of $5/2$ from A_L to e for the single-phase construction.

Table 1: Numerical examples of the area of the generalized Gerver sofa for some turn angles. The areas reported in this work agree to at least three decimal places with the reported areas from the numerical approach taken in Ref. [5] for the comparable turn angles.

Turn φ (degrees)	Contact phases	Area
15	1	10.452839295...
30	1	5.333086268...
45	1	3.678598491...
60	1	2.896310537...
68	1	2.639199288...
$\varphi_* = 68.908294...$	1	2.614707332...
69	5	2.612281344...
70	5	2.586367430...
75	5	2.470196808...
80	5	2.372741252...
85	5	2.290150220...
90	5	2.219531669...

3.5 Regularity

Local matching of the two branches, using the positive angular offset $\delta = \varphi - \lim_{\varphi \rightarrow \varphi_*^+} \varphi$, gives

$$\mathcal{A}_{\varphi > \varphi_*}(\varphi_* + \delta) - \mathcal{A}_{\varphi \leq \varphi_*}(\varphi_* + \delta) = -D\delta^3 + \mathcal{O}(\delta^4), \quad (13)$$

where the reduced formula is analytically continued above φ_* and

$$s(\varphi) = \frac{1 + 2 \cos \varphi - \cos^2 \varphi - e^{\varphi \cot \varphi}}{\sin \varphi \cos \varphi}, \quad (14)$$

$$D = \frac{s'(\varphi_*)^3}{6(1 - \frac{\cos \varphi_*}{\sin^2 \varphi_*})} = 0.442500520042 \dots$$

Thus, the constructed area is twice continuously differentiable (C^2), but not three times (C^3) at $\varphi = \varphi_*$. The generalized Gerver sofa area is therefore not an analytic function in this turn angle window.

4 Conclusions

The generalized Gerver sofa construction presented above includes a full-intersection feasibility established for the reduced branch and locally above φ_* . Table 1 reports on computed envelope areas for various turn angles. Global optimality of the generalized family is not established but is conjectured based on the proven optimality of Gerver's sofa at $\varphi = \pi/2$ [1]. Code and documentation of the generalized Gerver sofa may be provided upon request.

A possibly interesting further generalization of the problem is to seek maximal area shapes capable of being moved through a corridor dictated by various more general unit-width-inflated curves.

Acknowledgement

I thank Dan Romik for comments on the draft. I acknowledge use of ChatGPT-6 Astra Medium in Codex to obtain the arguments and the initial presentation of Sec. 3.

References

- [1] Jineon Baek, *Optimality of Gerver's Sofa*, arXiv e-prints (2024), arXiv:2411.19826.
- [2] Hanson Bai and John Bai, *Modeling the moving sofas in circular hallways using geometric methods*, Journal of Emerging Investigators (2025).
- [3] Joseph L. Gerver, *On moving a sofa around a corner*, Geometriae Dedicata **42** (1992), 267–283.
- [4] J. M. Hammersley, *On the enfeeblement of mathematical skills by modern mathematics and by similar soft intellectual trash in schools and universities*, Bulletin of the Institute of Mathematics and Its Applications **4** (1968), 66–85.
- [5] Xingyi He, *A Gas-Driven Algorithm for Variants of the Moving Sofa Problem*, arXiv e-prints (2026), arXiv:2608.11206.
- [6] Yoav Kallus and Dan Romik, *Improved upper bounds in the moving sofa problem*, Advances in Mathematics **340** (2018), 960–982.
- [7] Leo Moser, *Moving Furniture Through a Hallway*, SIAM Review **8** (1966), no. 3, 381.
- [8] Dan Romik, *Differential Equations and Exact Solutions in the Moving Sofa Problem*, Experimental Mathematics **27** (2018), no. 3, 316–330.
- [9] Wikipedia contributors, *Moving sofa problem*, 2026. [Online; accessed 9-January-2026].