

A LEAN FORMALIZATION OF ROBERTS THEOREM ON UNRESTRICTED VALUATIONS

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ABSTRACT. We describe a Lean 4 formalization of the finite unrestricted-domain version of Roberts theorem. For finite nonempty agent and alternative types, with at least three alternatives, an onto deterministic direct-revelation mechanism that is dominant-strategy incentive compatible has an affine-maximizer choice rule. Valuations range over all real-valued profiles; the conclusion uses fixed nonnegative weights, at least one nonzero, and alternative offsets, and asserts membership in the maximizing set. The development follows the modular proof route: taxation and weak monotonicity, perturbation-based tie-breaking to strong monotonicity, no-veto range analysis, affine prices for two agents, and induction through fixed-report slices. We explain the proved tie-set transport lemma, the construction of fresh payments for the tie-broken rule, and the transfer back to arbitrary ties. A version-pinned account distinguishes the proved library from statement-only comparator placeholders and separates source inspection from executable checks. This is an exposition of a classical result and its formal artifact, with no claim of mathematical novelty or formalization priority.

1. SCOPE AND MATHEMATICAL CONTEXT

Roberts theorem characterizes deterministic implementable choice rules on an unrestricted quasilinear domain [6]. Its force comes from allowing every agent to assign arbitrary real values to every alternative. With an onto rule and at least three alternatives, dominant-strategy incentive compatibility forces the rule to maximize a fixed weighted sum of those values plus an alternative-dependent constant. This is a characterization of the choice rule; it is not a claim that every payment rule has one particular formula.

The artifact explained here follows the modular approach of Dobzinski and Nisan [1]. Its weak-to-strong monotonicity reduction is attributed in the source to Lavi, Mu’alem, and Nisan [2]. The contribution of this note is to make the formal model, the proof interfaces, and the verification boundary readable. The mathematical theorem and proof strategy are classical. We do not claim the first formalization of Roberts theorem or a new mechanism-design result.

Implementation claims refer throughout to commit `0c176c66d2afa62301297f443b1e40eda2387ca2` of *roberts-theorem-lean* [5]. The pinned toolchain is Lean `v4.35.0-rc2`; the Mathlib release tag is the same, with manifest revision `065356127b1dc0016f66b7283ce0ce2c4055aa55`. The September 30, 2026 source snapshot includes the proved transport lemma and the rebuilt payment construction. In particular, an earlier description of the transport lemma as a custom axiom does not describe this snapshot.

Date: September 30, 2026.

2020 Mathematics Subject Classification. 91B14, 68V20.

Key words and phrases. Roberts theorem, mechanism design, dominant strategies, affine maximizers, Lean 4, Mathlib.

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2. THE FORMAL ECONOMIC MODEL

Let N be a finite nonempty set of agents and A a finite nonempty set of alternatives. A valuation profile belongs to

$$\mathcal{V} = \{v : N \times A \rightarrow \mathbb{R}\}.$$

The implementation uses the curried function type `Valuation N A := N -> A -> Real`. There is no restriction to nonnegative, normalized, integral, additive, monotone, or bounded valuations. A mechanism $M = (f, p)$ consists of a choice rule $f : \mathcal{V} \rightarrow A$ and payments $p : \mathcal{V} \times N \rightarrow \mathbb{R}$. Payments are subtracted from value, so agent i 's utility at truthful profile v is $v_i(f(v)) - p_i(v)$.

Write $v[i \leftarrow t]$ for the profile obtained by replacing agent i 's valuation with $t : A \rightarrow \mathbb{R}$. Dominant-strategy incentive compatibility (DSIC), as defined by `Roberts.IsDSIC`, is exactly

$$(1) \quad v_i(f(v)) - p_i(v) \geq v_i(f(v[i \leftarrow t])) - p_i(v[i \leftarrow t]) \quad \text{for every } i, v, t.$$

The valuation on both sides is the true valuation v_i ; only the reported profile and its resulting payment change on the right. Surjectivity, or *onto*, means that for every $a \in A$ there exists some profile v with $f(v) = a$.

For fixed $w : N \rightarrow \mathbb{R}$ and $k : A \rightarrow \mathbb{R}$, define

$$(2) \quad S_{w,k}(v, a) = \sum_{i \in N} w_i v_i(a) + k(a).$$

The source definition `Roberts.IsAffineMaximizer` requires $w_i \geq 0$ for every i , some $w_i \neq 0$, and

$$(3) \quad S_{w,k}(v, f(v)) \geq S_{w,k}(v, a) \quad (v \in \mathcal{V}, a \in A).$$

Weights and offsets are chosen once and work for every profile. Some agents may have zero weight. Neither normalization of the weights nor uniqueness of the weights, offsets, or selected alternative is part of the conclusion. In particular, (3) allows an arbitrary selection among tied affine-score maximizers.

Theorem 2.1 (Formalized Roberts theorem). *Suppose N and A are finite and nonempty and $|A| \geq 3$. If $M = (f, p)$ is DSIC and f is onto, then f is an affine maximizer in the sense of (2)–(3).*

The declaration `Roberts.roberts_theorem` appears in `Roberts/Induction.lean`. Its explicit economic hypotheses are DSIC and onto, together with the cardinality condition. The surrounding Lean context supplies `Fintype`, `DecidableEq`, and `Nonempty` instances for both types. Decidable equality is part of this implementation interface; it is not an additional restriction on preferences. Its conclusion concerns M 's choice function and makes no assertion about budget balance, individual rationality, participation constraints, or computation time.

3. TAXATION AND WEAK MONOTONICITY

Fix agent i and the reports of everyone else. The attainable menu is

$$B_i(v) = \{a \in A : \exists t, f(v[i \leftarrow t]) = a\}.$$

The source calls this predicate `Achievable`. Two reports that produce the same alternative must produce the same payment for i : apply DSIC in each direction and cancel the common true value at that alternative. Thus a real menu price $P_i(v, a)$ is well defined on $B_i(v)$ and independent of agent i 's report. The implementation chooses one witnessing report for an attainable alternative; it assigns zero to unattainable alternatives simply to make `price` a total real-valued function. The taxation inequality is used only with an attainability hypothesis.

For every attainable a , the chosen alternative maximizes utility relative to the menu:

$$(4) \quad t(f(v[i \leftarrow t])) - P_i(v, f(v[i \leftarrow t])) \geq t(a) - P_i(v, a).$$

These statements are `taxation_payment`, `price_independent`, and `taxation_ineq` in `Roberts/Taxation.lean`.

Definition 3.1. A choice rule is *weakly monotone* (W-MON) if, whenever $a \neq b$, $f(v[i \leftarrow t]) = a$, and $f(v[i \leftarrow s]) = b$,

$$(5) \quad t(a) - t(b) \geq s(a) - s(b).$$

It is *strongly monotone* (S-MON) if the inequality is strict under the same hypotheses.

Applying (4) first to report t and alternative b , then to report s and alternative a , eliminates the two prices and gives (5). This is `wmon_of_dsic`. The strict variant is a property of changes of outcome; it does not require a strict utility inequality for every deviation. It will be obtained by a new tie-broken rule, rather than assumed of the original mechanism.

4. TIE SETS AND THE STRONG MONOTONICITY REDUCTION

For $x \in A$ and $\varepsilon \in \mathbb{R}$, the simultaneous perturbation is

$$(v^{x,\varepsilon})_j(a) = v_j(a) + \varepsilon \mathbf{1}_{a=x} \quad (j \in N).$$

For a W-MON rule f , define

$$(6) \quad T_f(v) = \{x \in A : \exists \delta > 0 \forall \varepsilon \in (0, \delta), f(v^{x,\varepsilon}) = x\}.$$

This is a perturbational definition, not a reference to weights that have yet to be proved to exist. Raising only the current winner's value for one agent preserves that winner by W-MON. Repeating over the finite agent set shows $f(v) \in T_f(v)$, so the set is nonempty.

Fix a linear order on A and let $g(v) = \max T_f(v)$. The source obtains an order by transport along `Fintype.equivFin` and defines `tieBreak` by `Finset.max`'. This auxiliary order does not impose a tie convention on the original rule.

Lemma 4.1 (Tie-set transport). *Let v, v' differ only in agent i 's report. If $x \in T_f(v)$, $y \in T_f(v')$, and*

$$v'_i(x) - v'_i(y) \geq v_i(x) - v_i(y),$$

then $x \in T_f(v')$.

The declaration `key_lemma` in `Roberts/SMon.lean` proves this statement. Here is the asymmetric perturbation used in its proof. Assume $x \neq y$, take a sufficiently small $\varepsilon > 0$, and put $t = \varepsilon/5$. Set

$$p = v^{x,t}, \quad s = (v')^{y,t}, \quad q_j(a) = v'_j(a) + 4t\mathbf{1}_{a=x} + 2t\mathbf{1}_{a=y}, \quad r = q[i \leftarrow p_i].$$

The tie-set hypotheses give $f(p) = x$ and $f(s) = y$. From p to r , unchanged agent i is left alone and every other agent's x -versus-anything-else gap strictly increases. Sequential W-MON updates imply $f(r) = x$. From s to q , every agent's gap between either element of $\{x, y\}$ and an outside alternative strictly increases; the same update argument gives $f(q) \in \{x, y\}$. If $f(q) = y$, the unilateral transition from r to q would require

$$(v_i(x) - v_i(y)) + t \geq (v'_i(x) - v'_i(y)) + 2t,$$

contradicting the hypothesis and $t > 0$. Hence $f(q) = x$. Moving from q to $(v')^{x,5t}$ strictly increases every agent's x gaps again, so that profile chooses x . The allowed range of ε proves (6) for x at v' .

Proposition 4.2. *The tie-broken rule g is S-MON. If f is onto, then g is onto.*

Proof. Suppose g chooses distinct a, b under two unilateral reports. If the gap inequality were not strict, Lemma 4.1 would place a in the second tie set and, by exchanging the alternatives and profiles, place b in the first. Maximality then gives $a \leq b$ and $b \leq a$, contradicting distinctness. This is `tieBreak_isSMon`.

If $f(v) = a$, every positive simultaneous bump of a has tie set $\{a\}$. Indeed, a further sufficiently small bump of another alternative still leaves all a gaps strictly above their values at v , so W-MON preserves a . Thus $g(v^{a,1}) = a$. Starting from an onto witness for each a proves `tieBreak_onto`. $\square$

4.1. Fresh payments for the auxiliary rule. The proof requires a DSIC mechanism with choice function g . Strong monotonicity implies weak monotonicity, but the original mechanism's prices are not automatically valid for its changed outcomes. The artifact instead proves `wmon_dsic_payments`: every W-MON rule on this full valuation domain admits DSIC payments. This construction is in `Roberts/Induction.lean`.

To see its signs and domain obligations, fix the other reports, write $h(t)$ for the one-agent choice rule, and let $K = h(0)$. Define

$$(7) \quad d(a) = \inf\{t(K) - t(a) : h(t) = K\}.$$

The set is nonempty because the zero report selects K . For each attainable a , W-MON supplies a lower bound using any report selecting a . Thus the infimum used for attainable alternatives is an ordinary finite real infimum. W-MON also gives, whenever $h(t) = a$, $t(K) - t(a) \leq d(a)$, while reports selecting K satisfy $d(a) \leq t(K) - t(a)$ for attainable a . In particular $d(K) = 0$.

The remaining argument shows

$$t(h(t)) + d(h(t)) \geq t(h(s)) + d(h(s)) \quad \text{for all reports } t, s.$$

The source proves this by contradiction: if an attainable alternative had higher utility, raise the anchor K to an intermediate utility level and raise the currently selected alternative slightly. The anchor inequalities rule out both the anchor and the old outcome; W-MON then contradicts that slight raise. Setting the payment at a chosen outcome to $-d(h(t))$ yields DSIC. Values of the total Lean infimum definition at unattainable alternatives are not used to compare actual deviations. Applied to g , this supplies newly constructed payments, which are the ones used in the induction theorem.

5. NO VETO AND AFFINE PRICES

For a fixed report t of agent i , define the range

$$R_i(t) = \{a \in A : \exists v, f(v[i \leftarrow t]) = a\}.$$

This holds i fixed and allows all other agents to vary. It differs from the taxation menu, which holds the other agents fixed and varies i . Agent i has *no veto power* if $R_i(t) = A$ for every t . An alternative a is *dictatable* by i if $R_i(t) = \{a\}$ for some t . The latter describes a report that forces one outcome, not a blanket dictatorship assertion about the entire rule.

Proposition 5.1 (No-veto range theorem). *For an onto S-MON rule with at least three alternatives, there is an agent e such that every $i \neq e$ has no veto power.*

This is `all_but_one_no_veto` in `Roberts/NoVeto.lean`. The proof analyzes single-alternative boosts. Increasing the fixed agent's value of a can add no new range element other than a :

$$R_i(t + d\mathbf{1}_a) \subseteq R_i(t) \cup \{a\} \quad (d > 0).$$

If a is not dictatable, sufficiently large boosts make this range full, with a common finite bound obtained by summing absolute values of finitely many pairwise gap bounds. It follows that an agent with no dictatable alternative has no veto power. Two distinct agents cannot

force distinct alternatives simultaneously. If they both force the same a , that alternative belongs to the first agent's range at every report. An alternative $x \neq a$ is then not dictatable by that agent; boosting it would make the range full while the subset inclusion keeps the range inside $\{a, x\}$. A third alternative gives the contradiction. This accounts for the cardinality assumption in this range argument without strengthening the statement to universal no-veto power.

5.1. The two-agent representation. Suppose there are exactly two distinct agents i_1, i_2 , and i_1 has no veto power. Then every alternative is attainable by i_2 at every profile. The taxation prices of i_2 therefore apply to all of A . For a DSIC, S-MON mechanism the theorem `price_affine_form` proves

$$(8) \quad P_{i_2}(v, a) = H(v) - \alpha v_{i_1}(a) - \beta(a), \quad \alpha \geq 0.$$

Here H may depend on the profile but is common to all alternatives; α and β are fixed. These are menu prices, not a claimed classification of every payment to both agents.

The proof in `Roberts/TwoAgent.lean` first shows that the difference of prices at a, b depends only on $v_{i_1}(a) - v_{i_1}(b)$ and is nonincreasing in that gap. Normalize prices at one reference alternative. The response to a coordinate increase is independent of the alternative and of the base profile; a third alternative supplies the comparison needed for the latter step. The response function $\ell : \mathbb{R} \rightarrow \mathbb{R}$ is additive and nonincreasing. The lemma `additive_mono_linear` proves $\ell(t) = \ell(1)t$, using integer and rational scaling and the Archimedean order structure of the reals. It does not infer real linearity from additivity alone.

Substitution of (8) into taxation shows that the selected alternative maximizes

$$\alpha v_{i_1}(a) + v_{i_2}(a) + \beta(a).$$

Thus `two_agent_affine_maximizer` takes weights $\alpha, 1$. The second weight is positive even if the first is zero. Together with Proposition 5.1, this proves the two-agent base case of the general induction.

6. INDUCTION THROUGH FIXED-REPORT SLICES

The declaration `dsic_smon_affine_aux` proves the DSIC, S-MON, onto theorem by induction on an upper bound for the number of agents. The zero case is excluded by nonemptiness. With one agent, onto makes every alternative attainable, and the taxation menu is independent of the entire profile. Weight one and offsets equal to the negatives of those prices give an affine maximizer. The offsets may depend on the alternative; this base case does not assert unweighted value maximization without offsets.

For $|N| \geq 3$, choose a no-veto agent i_0 distinct from the possibly exceptional agent in Proposition 5.1. The implementation fixes this *no-veto* agent's report t and restricts the remaining agents to the subtype $N \setminus \{i_0\}$. This choice is important: no veto makes each fixed-report slice onto. DSIC and S-MON pass to the slice through explicit extension and restriction identities. The induction hypothesis supplies an affine representation for every slice.

Normalize the slice's nonnegative, nontrivial weights by their positive sum and lift them to N with zero at i_0 . Denote the normalized weights by w^t and the offsets by k^t . They satisfy $\sum_i w_i^t = 1$. The main consistency obligations are:

- (1) The normalized weights away from i_0 do not depend on t . The proof compares slices after a positive coordinate bump and then compares any two reports through a common strict coordinatewise upper report. A finite-dimensional half-space argument shows proportionality of the relevant vectors; normalization converts it to equality.

(2) There is one $\alpha \geq 0$ such that

$$(9) \quad (k^t(a) - \alpha t(a)) - (k^t(b) - \alpha t(b))$$

is independent of t for every a, b .

The source exposes `weights_independent_of_fixed_agent` and `offsets_consistent_of_fixed_agent`; the induction itself also uses their private core lemmas. The offset core retains the explicit nonnegativity of α needed in the final weights.

For offsets, a positive weight on some remaining agent lets that agent force comparisons between alternatives within a slice. Strong monotonicity makes an offset difference depend only on the fixed agent's corresponding value gap. Three-alternative identities give a common additive response, and monotonicity makes it linear. This produces (9), allowing a common profile-dependent additive constant, which is immaterial to maximization.

At a zero reference report define $W_{i_0} = \alpha$, $W_i = w_i^0$ for $i \neq i_0$, and $K(a) = k^0(a)$. The difference between global scores at any a, b equals the difference of the slice scores for $t = v_{i_0}$. That slice chooses $f(v)$, so its maximizing inequality is the desired global one. The remaining weights already contain a positive entry; hence the global weights are nontrivial. This proves `dsic_smon_affine_maximizer`.

7. RETURNING TO THE ORIGINAL RULE

It remains to recover the W-MON rule f , whose ties need not follow the auxiliary order. The declaration `affine_maximizer_transfer` uses the same weights and offsets as for g . Let $x = f(v)$ and $y = g(v)$. If $S_{w,k}(v, x) < S_{w,k}(v, y)$, choose a positive bump η small enough both for the stability of x and for

$$\eta \sum_i w_i < S_{w,k}(v, y) - S_{w,k}(v, x).$$

Bumping x raises its score by exactly $\eta \sum_i w_i$ and leaves y 's score unchanged. But a positive bump of a winner makes the tie-broken rule select that winner, contradicting affine maximization of g at the bumped profile. Therefore the two scores at v are equal, and $f(v)$ is a maximizer as well.

The final assembly has the following dependencies:

- (1) DSIC of M gives W-MON of f by taxation
- (2) Tie-set transport gives S-MON, hence W-MON, of g
- (3) W-MON supplies fresh DSIC payments for g
- (4) Onto is preserved by tie-breaking, and induction makes g an affine maximizer
- (5) The perturbation transfer gives the same affine-maximizer property for f

This is the actual sequence in `Roberts.roberts_theorem`. It neither assumes strong monotonicity of f nor reuses M 's payment rule after changing its choice rule.

8. ARTIFACT INTERFACES AND VERIFICATION BOUNDARY

The library modules form the dependency chain `Defs`, `Taxation`, `SMon`, `NoVeto`, `TwoAgent`, and `Induction`, all in the `Roberts` namespace. The package root `Roberts.lean` and compatibility module `Roberts/Basic.lean` import the development; `Solution.lean` imports the proved package. The separate `Challenge.lean` independently states the model and target for declaration comparison. Its two predicate definitions and target theorem contain three intentional `sorry` placeholders. They are not part of the solution's import chain.

Source inspection on September 30, 2026 found zero `sorry`, `admit`, `axiom`, or `unsafe` tokens in `Roberts/*.lean` and in `Solution.lean`. In particular, `key_lemma` has a proof body, not an axiom declaration. This syntactic inspection is distinct from elaboration and

kernel checking. The project reports a successful local build and an axiom audit whose dependency set for the main theorem is

`{propext, Classical.choice, Quot.sound}`.

The September 27, 2026 packaging commit records successful local declaration, build, placeholder, axiom, comparator, and whitespace checks [4]. A separate Palomar verification run for that same September 27 revision completed successfully at 14:55 UTC that day; its workflow logs identify the submitted revision and the successful comparator and challenge-provenance step [3]. Inspection on September 30 confirmed that the Lean source from that revision to the manuscript pin changes only module headers, public visibility, and definition exposure annotations: after removing those annotations and blank lines, every Lean file is textually identical. The toolchain and dependency manifest are unchanged. This comparison does not constitute a fresh build of the revised module packaging. The pinned repository includes a script to repeat the local checks; its metadata labels review status self-assessed. Neither the recorded mechanical run nor these source checks establish external mathematical peer review.

The verification procedure builds the library, elaborates separate challenge and solution declaration checks, and runs `#print axioms Roberts.roberts_theorem` against the solution. It compares the resulting dependencies with an allowed set rather than treating the allowed list itself as proof of actual dependencies. The comparator targets `IsDSIC`, `IsAffineMaximizer`, and `roberts_theorem`; the valuation abbreviation and mechanism structure remain visible model definitions but are not listed as predicate comparator targets. A reproducible check must use the pinned toolchain and dependency manifest, not merely a recent Lean installation.

The fresh manuscript preparation included source-to-statement inspection, independent source and mathematical review, and a clean LaTeX compilation of an extracted source archive with visual inspection of every PDF page. These are checks on this exposition and its rendering. A fresh Lean execution was not performed in the manuscript’s cloud typesetting workspace, which had no Lean executable. Lean’s kernel checks the propositions supplied to it; it does not by itself validate the correspondence of prose to economic interpretation, historical attribution, or the scope of a repository-status claim.

9. LIMITATIONS AND PROVENANCE

The theorem is confined to finite nonempty agent and alternative types, deterministic direct revelation, unrestricted real valuations, onto choice, and at least three alternatives. It does not formalize a generalization to restricted auction or scheduling domains, randomized mechanisms, infinite sets of agents or alternatives, Bayesian incentive compatibility, or two-alternative rules. The artifact proves the necessary affine-maximizer representation of choice functions; a separate converse characterizing all admissible tie rules or all original payments is not its headline statement.

The weights, order transport, witness prices, and slice representations use classical and noncomputable constructions. The development proves existence and score inequalities; it does not extract an efficient algorithm to recover weights from an arbitrary mechanism. Allowing zero weights and arbitrary affine-score ties is essential to the statement actually encoded.

AI assistance and human understanding. This manuscript was drafted primarily with GPT-6.1 assistance, including exposition, source alignment, and typesetting. The repository’s metadata separately reports GPT-6-Luna assistance for the formalization and packaging. Some human understanding informed the work; no claim is made that every listed author independently understands every formal proof step. The model-assisted review of this manuscript is not independent human peer review, and acceptance by an archive would not establish such review.

Attribution and availability. Roberts is credited for the characterization; Dobzinski and Nisan for the modular proof route; Lavi, Mu’alem, and Nisan for the monotonicity and tie-breaking route; and Lean and Mathlib contributors for the proof infrastructure. The pinned source repository is BSD-3-Clause licensed. This manuscript and its TeX source are separately licensed CC BY 4.0. The TeX archive contains the manuscript and bibliography, while the Lean implementation and verification script remain in the referenced repository.

REFERENCES

1. Shahar Dobzinski and Noam Nisan, *A modular approach to Roberts’ theorem*, Algorithmic Game Theory, SAGT 2009 (Marios Mavronicolas and Vicky G. Papadopoulos, eds.), Lecture Notes in Computer Science, vol. 5814, Springer, 2009, https://doi.org/10.1007/978-3-642-04645-2_3, pp. 14–23.
2. Ron Lavi, Ahuva Mu’alem, and Noam Nisan, *Towards a characterization of truthful combinatorial auctions*, Proceedings of the 44th Annual IEEE Symposium on Foundations of Computer Science, IEEE, 2003, <https://doi.org/10.1109/SFCS.2003.1238230>, pp. 574–583.
3. Palomar Registry, *Mechanical verification of submission ujjeis5lup8v*, <https://github.com/PalomarRegistry/PalomarSubmission/actions/runs/36327496620>, 2026, Completed successfully September 27, 2026; submitted source revision 61bd2b9075588716550bcd3ac9bc68f7392e823f; not mathematical peer review.
4. Arthur Freitas Ramos, *Local verification record in the Roberts packaging commit*, <https://github.com/Arthur742Ramos/roberts-theorem-lean/commit/61bd2b9075588716550bcd3ac9bc68f7392e823f>, 2026, September 27, 2026; project-reported local checks, not external review.
5. ———, *roberts-theorem-lean: a Lean formalization of Roberts theorem*, <https://github.com/Arthur742Ramos/roberts-theorem-lean/tree/0c176c66d2afa62301297f443b1e40eda2387ca2>, 2026, Pinned source inspected September 30, 2026; Lean and Mathlib v4.35.0-rc2.
6. Kevin W. S. Roberts, *The characterization of implementable choice rules*, Aggregation and Revelation of Preferences (Jean-Jacques Laffont, ed.), North-Holland, Amsterdam, 1979, pp. 321–349.