

BREAKING THE FOURTH WALL: THE PLANAR CENTROID BANACH–MAZUR DIAMETER IS STRICTLY LESS THAN 4

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ABSTRACT. We prove that the centroid Banach–Mazur diameter of planar convex bodies is strictly less than 4, (slightly) improving the prior upper bound $69/17 \approx 4.05882$ due to Lassak. The proof combines a sharp covariance-ellipse sandwich, whose equality cases in any single direction occur only for triangles, with a compactness argument: an extremal pair at distance 4 would consist of two triangles, which are linearly equivalent and hence must in fact be at distance 1. The resulting gap below 4 is uniform but nonquantitative.

1. INTRODUCTION

A *convex body* is a compact convex $K \subseteq \mathbb{R}^2$ with nonempty interior; we write $c(K)$ for its centroid and $K_0 = K - c(K)$. The *centroid Banach–Mazur distance* of two planar convex bodies is

$$\delta_{\text{cen}}(C, D) = \inf\{t \geq 1 : C_0 \subseteq AD_0 \subseteq tC_0 \text{ for some } A \in \text{GL}(2)\}.$$

The distance δ_{cen} is symmetric, affine invariant in each argument, and satisfies the multiplicative triangle inequality; each assertion follows by composing the corresponding linear maps and inclusions after centering.

The *planar centroid Banach–Mazur diameter* is the supremum of δ_{cen} over all pairs of planar convex bodies; our main result implies that this diameter is strictly less than 4.

Theorem 1. *There is an absolute constant $\varepsilon_0 > 0$ such that*

$$\delta_{\text{cen}}(C, D) \leq 4 - \varepsilon_0$$

for all planar convex bodies C and D .

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Grünbaum introduced the centroid-constrained version of the Banach–Mazur distance and conjectured that its planar diameter is $5/2$, attained by a parallelogram and a triangle [Grü63, pp. 259–260]. Lassak proved that the distance of that pair is indeed $5/2$ [Las24a], and also established the general upper bound

$$\delta_{\text{cen}}(C, D) \leq \frac{69}{17} \approx 4.05882$$

[Las24b]. To our knowledge, Theorem 1 is the first proof that the diameter is below 4.

The proof is driven by a sharp one-body statement. Associated to every planar convex body with centroid at the origin is a canonical covariance ellipse whose support distance in each direction is proportional to the root-mean-square spread of the uniform mass in that direction. With the normalization used below, the ellipse lies in the body and its double contains the body. Contact with the doubled ellipse in even one direction forces the body to be a triangle. After independently normalizing two bodies so that their covariance ellipses are both the Euclidean unit disk, chaining these inclusions gives the coarse distance bound 4.

The normalized class is compact and δ_{cen} is continuous on it, so the diameter is attained. If that diameter were 4, equality would be required in the one-body estimate for both members of a maximizing pair; both would therefore be triangles. But any two triangles with centroid at the origin are linearly equivalent and have distance 1, giving the desired contradiction.

This also explains where the improvement over Lassak’s bound enters. Lassak’s argument uses affine-regular hexagons supplied by Besicovitch’s theorem [Bes48], together with the sharp $4/21$ bound on the displacement of the centroid from the hexagon center established in [Las24c], leading to $69/17$ [Las24b]. Our covariance ellipses are built from centroid-centered second moments, giving the coarse bound $4 = 2 \cdot 2$. Thus the two sharp one-body factors create the apparent “fourth wall” $2 \cdot 2 = 4$; it is broken not by improving either factor, but because simultaneous equality forces two affinely-equivalent triangles, which break the hypothesized equality.

2. A ONE-DIMENSIONAL MOMENT WINDOW

The geometric argument reduces each direction to a nonnegative concave function with vanishing first moment (Figure 1(a)). We first isolate the resulting one-dimensional estimate.

For a nonzero integrable function $w \geq 0$ on a bounded interval, we write $m_2(w) = \int \tau^2 w(\tau) d\tau / \int w(\tau) d\tau$.

Theorem 2 (Moment window). *Let $a, b > 0$, and let $w : [-a, b] \rightarrow [0, \infty)$ be continuous, concave, and nonzero. If*

$$\int_{-a}^b \tau w(\tau) d\tau = 0,$$

then we have

$$\frac{b^2}{8} \leq m_2(w) \leq \frac{b^2}{2}.$$

The upper and lower equality cases are, respectively, $a = 2b$, $w(\tau) \propto \tau + 2b$, and $a = b/2$, $w(\tau) \propto b - \tau$.

We use the following standard decomposition.

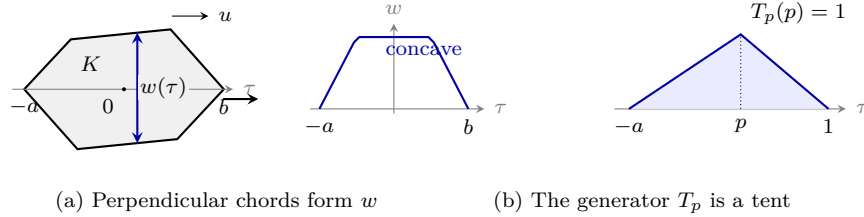
Lemma 3 (Tent decomposition). *Let $a > 0$. Every continuous nonnegative concave function w on $[-a, 1]$ can be written*

$$w(\tau) = \lambda(\tau + a) + \mu(1 - \tau) + \int_{(-a, 1)} T_p(\tau) d\nu(p), \quad \lambda, \mu \geq 0,$$

where ν is a finite positive Borel measure and

$$T_p(\tau) = \min \left\{ \frac{\tau + a}{p + a}, \frac{1 - \tau}{1 - p} \right\}, \quad -a < p < 1.$$

The name is literal: the graph of T_p rises linearly from 0 at $-a$ to height 1 at p , then descends linearly to 0 at 1, so it has the shape of a tent with peak p ; see Figure 1(b). The two affine terms are the one-sided wedges determined by the endpoint values of w . Thus, after removing those wedges, every concave profile is a nonnegative superposition of tents; the form we present makes the equality analysis transparent.



(a) Perpendicular chords form w

(b) The generator T_p is a tent

FIGURE 1. (a) The chord reduction: for fixed u , the section $\{x : \langle x, u \rangle = \tau\}$ is perpendicular to u , and its length is $w(\tau)$; Brunn's theorem makes w concave. (b) The tent generator, drawn with the normalization $b = 1$: the tent T_p vanishes at $-a$ and 1 and has height 1 at p . The two endpoint wedges together with a positive superposition of tents reconstruct w (Lemma 3).

Proof of Lemma 3. Subtract the affine chord joining the endpoint values:

$$\lambda = \frac{w(1)}{a + 1}, \quad \mu = \frac{w(-a)}{a + 1}, \quad g(\tau) = w(\tau) - \lambda(\tau + a) - \mu(1 - \tau).$$

Then g is nonnegative and concave, and vanishes at both endpoints. Its distributional curvature $\sigma = -D^2g$ is a positive locally finite measure on

$(-a, 1)$. Put

$$\gamma(p) = \frac{(p+a)(1-p)}{a+1}, \quad d\nu(p) = \gamma(p) d\sigma(p).$$

Although σ need not be finite, ν is. Indeed, $\gamma'' = -2/(a+1)$, so distributional integration by parts gives

$$\nu((-a, 1)) = \int_{(-a, 1)} \gamma d\sigma = \frac{2}{a+1} \int_{-a}^1 g(\tau) d\tau < \infty.$$

To justify the integration by parts, we may first use cutoffs a positive distance from the endpoints. Concavity and the endpoint values of g imply

$$(\tau+a)|g'_\pm(\tau)| \rightarrow 0 \quad (\tau \downarrow -a), \quad (1-\tau)|g'_\pm(\tau)| \rightarrow 0 \quad (\tau \uparrow 1),$$

where $g'_\pm$ denote the one-sided derivatives. Moreover, $g(\tau) \rightarrow 0$. Since γ vanishes linearly and γ' is bounded, all boundary terms tend to 0.

The Dirichlet Green kernel for $-d^2/d\tau^2$ on $[-a, 1]$ is

$$G(\tau, p) = \begin{cases} \frac{(1-p)(\tau+a)}{a+1}, & \tau \leq p, \\ \frac{(p+a)(1-\tau)}{a+1}, & \tau \geq p, \end{cases} = \gamma(p)T_p(\tau).$$

The function

$$H(\tau) = \int_{(-a, 1)} G(\tau, p) d\sigma(p) = \int_{(-a, 1)} T_p(\tau) d\nu(p)$$

is finite and continuous by dominated convergence, and vanishes at the two endpoints by construction. Testing against a compactly supported smooth function and using Fubini's theorem gives $-D^2 H = \sigma = -D^2 g$. Hence $g - H$ is affine and vanishes at both endpoints, so $g = H$, as required. $\square$

Analytically, Lemma 3 is the one-dimensional Green-kernel representation of a concave function in terms of its distributional curvature. Its use below under a linear moment constraint is closely related to geometric localization and the reduction to affine one-dimensional densities [KLS95, FG04].

Proof of Theorem 2. By scaling if necessary, we may take $b = 1$. We use the decomposition in Lemma 3. The first moments of its three types of generators are

$$\begin{aligned} (1) \quad & \int_{-a}^1 \tau(\tau+a) d\tau = \frac{(2-a)(a+1)^2}{6}, \\ (2) \quad & \int_{-a}^1 \tau(1-\tau) d\tau = -\frac{(2a-1)(a+1)^2}{6}, \\ (3) \quad & \int_{-a}^1 \tau T_p(\tau) d\tau = \frac{(a+1)(1+p-a)}{6}. \end{aligned}$$

If $a > 2$, then all three quantities are negative; if $a < 1/2$, all three are positive. Since the coefficients and the measure in the decomposition are nonnegative and not all zero, the mean-zero condition $\int \tau w(\tau) d\tau = 0$ forces

$$\frac{1}{2} \leq a \leq 2.$$

For functions whose product is integrable, write

$$\mathcal{J}(f, g) = \int_{-a}^1 f(\tau)g(\tau) d\tau.$$

The decomposition may be paired term by term with the polynomials below because ν (as given in Lemma 3) is finite and $0 \leq T_p \leq 1$.

For the upper bound, set

$$\varphi(\tau) = \frac{1}{2} - \tau^2 - \frac{a}{2}\tau.$$

Direct integration gives

$$\begin{aligned} \mathcal{J}(\varphi, \tau + a) &= 0, \\ \mathcal{J}(\varphi, 1 - \tau) &= \frac{(2 - a)(a + 1)^3}{12}, \\ \mathcal{J}(\varphi, T_p) &= \frac{(a + 1)(1 - p)(p + 2)}{12}. \end{aligned}$$

These quantities are nonnegative for $1/2 \leq a \leq 2$ and $-a < p < 1$. Hence

$$0 \leq \mathcal{J}(\varphi, w) = \frac{1}{2} \int_{-a}^1 w - \int_{-a}^1 \tau^2 w,$$

where the first moment of w has been used. This proves that $m_2(w) \leq 1/2$.

For the lower bound, set

$$\psi(\tau) = \tau^2 - \frac{1}{8} + \frac{a}{2}\tau.$$

The corresponding pairings are

$$\begin{aligned} \mathcal{J}(\psi, \tau + a) &= \frac{3(a + 1)^2}{16}, \\ \mathcal{J}(\psi, 1 - \tau) &= \frac{(a + 1)^2(2a - 1)^2}{48}, \\ \mathcal{J}(\psi, T_p) &= \frac{(a + 1)(2p + 1)^2}{48}; \end{aligned}$$

these are all nonnegative, and therefore

$$0 \leq \mathcal{J}(\psi, w) = \int_{-a}^1 \tau^2 w - \frac{1}{8} \int_{-a}^1 w.$$

Thus we see that $m_2(w) \geq 1/8$.

It remains to inspect equality. Equality in the upper bound eliminates the tent measure, since its pairing is strictly positive for every $-a < p < 1$. If $a < 2$, then equality also eliminates the $(1 - \tau)$ -component, after which

the mean-zero condition eliminates the remaining component. Hence $a = 2$; equations (1)–(2) then force $\mu = 0$, so w is proportional to $\tau + 2$.

Equality in the lower bound forces $\lambda = 0$, forces $\mu = 0$ unless $a = 1/2$, and permits tent mass only at $p = -1/2$. If $a > 1/2$, this tent has strictly negative first moment, so the mean-zero condition forces $\nu = 0$, contradicting $w \neq 0$. Hence $a = 1/2$. The point $p = -1/2$ is then an excluded endpoint, so $\nu = 0$, and w is proportional to $1 - \tau$. Rescaling from $b = 1$ proves the theorem. $\square$

3. THE COVARIANCE ELLIPSE AND ITS EQUALITY CASES

Let B^2 denote the Euclidean unit disk and let $S^1 = \partial B^2$. The support function of a compact convex set K is

$$h_K(u) = \max_{x \in K} \langle x, u \rangle \quad (u \in S^1).$$

For a convex body K , put

$$\text{Cov}_K = \frac{1}{|K|} \int_K (x - c(K))(x - c(K))^\top dx,$$

where $|K|$ denotes planar area. Since K has nonempty interior, Cov_K is positive definite. For $u \in S^1$, the scalar $u^\top \text{Cov}_K u$ is the mean squared displacement in direction u of a uniformly random point of K . Define the *covariance ellipse*, with the planar normalization used here, by

$$E_K = \sqrt{2} \text{Cov}_K^{1/2} B^2,$$

so its support distance in direction u is $\sqrt{2u^\top \text{Cov}_K u}$, or $\sqrt{2}$ times the corresponding root-mean-square spread. Up to normalization, E_K is also sometimes called the *inertia ellipse*. Here $\text{Cov}_K^{1/2}$ is the unique positive-definite square root; we use fractional powers for matrix square roots and radicals for scalars.

For $A \in \text{GL}(2)$ and $z \in \mathbb{R}^2$,

$$\text{Cov}_{AK+z} = A \text{Cov}_K A^\top, \quad E_{AK+z} = AE_K,$$

where the second equality is equality of sets: both sides are centered ellipses with shape matrix $2A \text{Cov}_K A^\top$. Thus the construction is translation invariant and linearly equivariant.

For a convex body K with $c(K) = 0$ and $u \in S^1$, let $w_{K,u}(\tau)$ be the length of the chord

$$K \cap \{x : \langle x, u \rangle = \tau\}, \quad -h_K(-u) \leq \tau \leq h_K(u),$$

as in Figure 1(a).

Theorem 4 (Covariance-ellipse sandwich). *Let $K \subset \mathbb{R}^2$ be a convex body with $c(K) = 0$. Then, for every $u \in S^1$,*

$$\sqrt{2u^\top \text{Cov}_K u} \leq h_K(u) \leq \sqrt{8u^\top \text{Cov}_K u}.$$

Equivalently,

$$E_K \subseteq K \subseteq 2E_K.$$

Equality in the lower support bound occurs exactly when $w_{K,u}(\tau) \propto \tau + 2h_K(u)$ on $[-2h_K(u), h_K(u)]$. Equality in the upper support bound occurs exactly when $w_{K,u}(\tau) \propto h_K(u) - \tau$ on $[-h_K(u)/2, h_K(u)]$.

Either equality in one direction forces K to be a triangle. Conversely, the two orientations normal to each side of a triangle realize the two cases.

Proof. Fix $u \in S^1$, and put

$$b = h_K(u), \quad a = h_K(-u).$$

The centroid lies in the interior of K , so $a, b > 0$. Indeed, if a supporting linear functional vanished at the centroid, it would be nonnegative on K and positive on a set of positive area, contrary to its having mean zero.

Write $w = w_{K,u}$. Brunn's theorem makes w concave. It is continuous on the closed support interval; at the endpoints this follows, for example, by writing the upper and lower boundaries as a concave and a convex function and using the closedness of K . Fubini's theorem and the definitions of the centroid and covariance give

$$\int_{-a}^b \tau w(\tau) d\tau = 0, \quad m_2(w) = u^\top \text{Cov}_K u, \quad b = h_K(u).$$

Thus w records the one-dimensional area distribution in direction u , while E_K records its root-mean-square scale. Substituting the preceding identities into Theorem 2 gives

$$\frac{h_K(u)^2}{8} \leq u^\top \text{Cov}_K u \leq \frac{h_K(u)^2}{2}.$$

Solving the right-hand inequality for $h_K(u)$ gives the lower support bound, while solving the left-hand inequality gives the upper support bound:

$$\sqrt{2 u^\top \text{Cov}_K u} \leq h_K(u) \leq \sqrt{8 u^\top \text{Cov}_K u}.$$

They are *directionwise* inequalities because they compare the supports separately for each choice of u .

For a linear map A , the image ellipse AB^2 has support function $h_{AB^2}(u) = \|A^\top u\|$. Since $\text{Cov}_K^{1/2}$ is symmetric, we see that

$$h_{E_K}(u) = \sqrt{2 u^\top \text{Cov}_K u}, \quad h_{2E_K}(u) = \sqrt{8 u^\top \text{Cov}_K u}.$$

For compact convex sets E, F , the standard support-function criterion says that $E \subseteq F$ exactly when $h_E(v) \leq h_F(v)$ for every $v \in S^1$; this follows from their representations as intersections of supporting half-planes. The ellipse inclusions follow.

The one-dimensional equality statements follow directly from Theorem 2. To identify the body attaining equality, we rotate coordinates so that $u = (1, 0)$, and write

$$K = \{(\tau, z) : -a \leq \tau \leq b, \ell(\tau) \leq z \leq r(\tau)\},$$

where ℓ is convex, r is concave, and $w = r - \ell$. In either equality case, w is affine and vanishes at one endpoint. Since $r = \ell + w$ is concave and w is affine, $\ell = r - w$ is also concave. Thus ℓ is both convex and concave, so both ℓ and r are affine. Their graphs meet at the endpoint where $w = 0$, and K is the convex hull of that point and the opposite chord: it is a triangle (see Figure 2). Conversely, the chord profile of a triangle is affine in a direction normal to any side, and reversing that direction interchanges the two equality types. $\square$

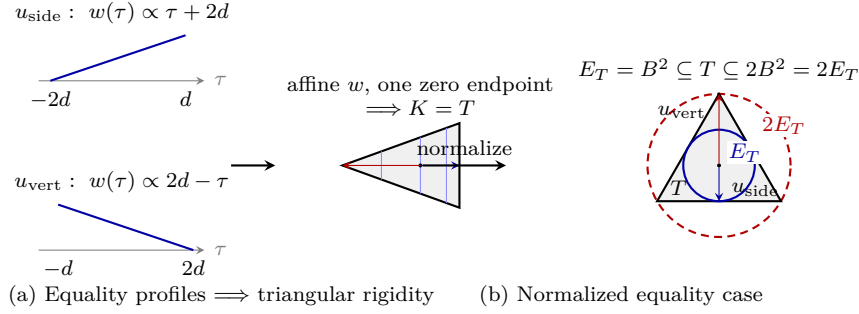


FIGURE 2. The directionwise equality mechanism. Here T is a triangle with centroid 0, d is the support distance from 0 to a fixed side, and u_{side} and $u_{\text{vert}} = -u_{\text{side}}$ are the unit normals pointing toward that side and the opposite vertex. (a) The direction u_{side} gives the increasing equality profile on $[-2d, d]$, and u_{vert} the decreasing one on $[-d, 2d]$; conversely, an affine profile vanishing at one endpoint forces both boundary graphs to be affine, so K is a triangle. (b) After the normalization $E_T = B^2$, the equality triangle is equilateral: B^2 is its incircle and $2B^2$ its circumcircle, with the colored arrows ending at the contact points.

In dimension n , the optimal outer dilation factor in the covariance-ellipsoid sandwich is n , with simplices attaining equality [MP89]; see [BGVV14] for background. Related sharp trace second-moment inequalities, with simplices as the equality cases among polytopes, are proved in [FPS12, Theorem 1.1]. The direct planar proof above supplies the direction-by-direction equality analysis needed in our compactness argument.

Figure 2 illustrates both halves of the equality analysis. Its second panel uses the fact that every centroid-zero triangle is a linear image of a centroid-zero equilateral one; by linear equivariance of the covariance ellipse, the normalization $E_T = B^2$ therefore makes T equilateral up to an orthogonal map.

4. COMPACTNESS AND THE STRICT GAP

Let I_2 denote the 2×2 identity matrix. A body is in *isotropic position* if

$$c(K) = 0, \quad \text{Cov}_K = \frac{1}{2}I_2.$$

Equivalently, $c(K) = 0$ and $E_K = B^2$. Every planar convex body has such an affine normalization, unique up to an orthogonal map: explicitly, we take

$$K \mapsto \frac{1}{\sqrt{2}} \text{Cov}_K^{-1/2}(K - c(K)).$$

We let $\mathcal{I}$ be the class of bodies in isotropic position.

Lemma 5. *The class $\mathcal{I}$ is compact in the Hausdorff topology, and $(C, D) \mapsto \delta_{\text{cen}}(C, D)$ is continuous on $\mathcal{I} \times \mathcal{I}$.*

Proof. Under the map above, Theorem 4 becomes

$$B^2 \subseteq K \subseteq 2B^2 \quad (K \in \mathcal{I}).$$

Blaschke's selection theorem therefore gives sequential compactness of the ambient family. Under Hausdorff convergence inside this fixed sandwich, the indicator functions converge almost everywhere because convex-body boundaries have area zero. Dominated convergence then gives convergence of volumes and of the first and second moments. Hence the conditions $c(K) = 0$ and $\text{Cov}_K = \frac{1}{2}I_2$ are closed, so $\mathcal{I}$ is compact.

For continuity, let $K, L \in \mathcal{I}$ and suppose $d_H(K, L) \leq \eta$, where d_H denotes the Hausdorff distance. Since $B^2 \subseteq K$ and $B^2 \subseteq L$, we have

$$K \subseteq L + \eta B^2 \subseteq (1 + \eta)L, \quad L \subseteq K + \eta B^2 \subseteq (1 + \eta)K.$$

Taking $A = (1 + \eta)I_2$ in the defining infimum for $\delta_{\text{cen}}(K, L)$ gives

$$K \subseteq AL \subseteq (1 + \eta)^2 K,$$

and hence

$$\delta_{\text{cen}}(K, L) \leq (1 + \eta)^2.$$

Now suppose $K_j \rightarrow K$ and $L_j \rightarrow L$ in the Hausdorff topology, and put

$$\eta_j = \max\{d_H(K_j, K), d_H(L_j, L)\}.$$

The multiplicative triangle inequality, applied in both directions, then gives

$$(1 + \eta_j)^{-4} \delta_{\text{cen}}(K, L) \leq \delta_{\text{cen}}(K_j, L_j) \leq (1 + \eta_j)^4 \delta_{\text{cen}}(K, L),$$

which proves continuity on $\mathcal{I} \times \mathcal{I}$. $\square$

Proof of Theorem 1. Independent affine normalizations do not change δ_{cen} . Consequently, the supremum of δ_{cen} over all pairs equals its supremum over $\mathcal{I} \times \mathcal{I}$. By Lemma 5, the latter supremum is attained; call it M .

For $K \in \mathcal{I}$, put

$$q(K) = \max_{u \in S^1} h_K(u).$$

Theorem 4 gives $q(K) \leq 2$. Since $h_{B^2} \equiv 1$, the support-function containment criterion used in the proof of Theorem 4 gives

$$B^2 \subseteq K \subseteq q(K)B^2.$$

Therefore, for $C, D \in \mathcal{I}$, we have

$$C \subseteq q(C)B^2 \subseteq q(C)D \subseteq q(C)q(D)B^2 \subseteq q(C)q(D)C,$$

and hence

$$\delta_{\text{cen}}(C, D) \leq q(C)q(D) \leq 4.$$

Thus $M \leq 4$.

Suppose that $M = 4$, and choose $C, D \in \mathcal{I}$ with $\delta_{\text{cen}}(C, D) = 4$. The preceding inequalities then force $q(C) = q(D) = 2$. The maxima defining $q(C)$ and $q(D)$ are attained, so the equality statement in Theorem 4 implies that both bodies are triangles.

Write $C = \text{conv}\{c_1, c_2, c_3\}$ and $D = \text{conv}\{d_1, d_2, d_3\}$, where $c_1 + c_2 + c_3 = d_1 + d_2 + d_3 = 0$. Because the triangles are nondegenerate, c_1, c_2 and d_1, d_2 are bases of $\mathbb{R}^2$. The linear map taking d_i to c_i for $i = 1, 2$ also takes $d_3 = -d_1 - d_2$ to $c_3 = -c_1 - c_2$, and hence maps D onto C . Thus $\delta_{\text{cen}}(C, D) = 1$, contradicting the choice of the pair.

Thus $M < 4$. Taking $\varepsilon_0 = 4 - M$ completes the proof. $\square$

REFERENCES

- [Bes48] A. S. Besicovitch, *Measure of asymmetry of convex curves*, Journal of the London Mathematical Society **s1-23** (1948), no. 3, 237–240.
- [BGVV14] Silouanos Brazitikos, Apostolos Giannopoulos, Petros Valettas, and Beatrice-Helen Vritsiou, *Geometry of isotropic convex bodies*, Mathematical Surveys and Monographs, vol. 196, American Mathematical Society, Providence, RI, 2014.
- [FG04] Matthieu Fradelizi and Olivier Guédon, *The extreme points of subsets of s -concave probabilities and a geometric localization theorem*, Discrete & Computational Geometry **31** (2004), no. 2, 327–335.
- [FPS12] Matthieu Fradelizi, Grigoris Paouris, and Carsten Schütt, *Simplices in the Euclidean ball*, Canadian Mathematical Bulletin **55** (2012), no. 3, 498–508.
- [Grü63] Branko Grünbaum, *Measures of symmetry for convex sets*, Convexity (Victor Klee, ed.), Proceedings of Symposia in Pure Mathematics, vol. 7, American Mathematical Society, Providence, RI, 1963, pp. 233–270.
- [KLS95] Ravi Kannan, László Lovász, and Miklós Simonovits, *Isoperimetric problems for convex bodies and a localization lemma*, Discrete & Computational Geometry **13** (1995), no. 3–4, 541–559.
- [Las24a] Marek Lassak, *The centroid Banach–Mazur distance between the parallelogram and the triangle*, Journal of Convex Analysis **31** (2024), no. 1, 51–58.
- [Las24b] ———, *Estimation of the centroid Banach–Mazur distance between planar convex bodies*, Ukrainian Mathematical Journal **76** (2024), no. 5, 872–879.
- [Las24c] ———, *Position of the centroid of a planar convex body*, Aequationes Mathematicae **98** (2024), no. 3, 687–695.
- [MP89] V. D. Milman and A. Pajor, *Isotropic position and inertia ellipsoids and zonoids of the unit ball of a normed n -dimensional space*, Geometric Aspects of Functional Analysis: Israel Seminar (GAFA) 1987–88 (J. Lindenstrauss and V. D. Milman, eds.), Lecture Notes in Mathematics, vol. 1376, Springer, Berlin, Heidelberg, 1989, pp. 64–104.

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