

SWAN INDUCTION FOR THE FINITE-LOCAL SPHERE

ABSTRACT. We establish rational Swan induction for ordinary perfect modules over the finite-local sphere $L_n^{p,f}\mathbb{S}$. For a finite abelian ambient group, subgroups of p -rank at most $n + 1$ suffice, and this bound is sharp. The proof combines cyclic homotopy fixed points in telescopic spectra with the isotropy filtration of a chromatic quotient of finite genuine spectra. The result proves Conjecture 7.22 of Clausen–Mathew–Naumann–Noel for Morava E -theory and gives a new proof of their chromatic upper bound for the algebraic K -theory of $L_n^{p,f}\mathbb{S}$ -linear categories. The quotient argument also produces finite complexes realizing the induction relations. We formulate the problem of explicit realizations and give two geometric models at the prime two.

1. INTRODUCTION

Swan induction asks which subgroups suffice to generate the Grothendieck group of representations over a ring spectrum. In chromatic homotopy theory, the answer depends on height. We prove an induction theorem for the finite-local sphere, strengthening the Morava E -theory conjecture of Clausen–Mathew–Naumann–Noel [14, Conjecture 7.22].

Fix a prime p and write

$$R_n = L_n^{p,f}\mathbb{S}, \quad R_{n,(p)} = (R_n)_{(p)} = L_n^f\mathbb{S}_{(p)}.$$

We follow the conventions of [14, 23]: the global finite localization retains $\mathbb{S}[1/p]$, so that $R_0 = \mathbb{S}[1/p]$ and $R_{0,(p)} = H\mathbb{Q}$. For an E_∞ -ring R and a finite group G , let

$$\mathrm{Sw}_R(G) = K_0 \mathrm{Fun}(BG, \mathrm{Perf}(R)) \otimes \mathbb{Q}.$$

Here $\mathrm{Perf}(R)$ denotes ordinary perfect modules, and the actions are homotopy-coherent. This is rational Swan theory [34], [26, Definition 4.11], and [14, Definition 6.1]. A family $\mathcal{F}$ is an *induction family* if

$$[R] \in \sum_{H \in \mathcal{F}} \mathrm{im}(\mathrm{Ind}_H^G : \mathrm{Sw}_R(H) \longrightarrow \mathrm{Sw}_R(G)). \quad (1)$$

The projection formula makes this equivalent to generation of the whole Swan group. Families are closed under conjugacy and passage to subgroups.

For $n \geq 0$, define

$$\begin{aligned} \mathcal{F}_n^p(G) &= \{H \leq G : H \text{ abelian, } d(H_p) \leq n + 1\}, \\ \mathcal{F}_n(G) &= \{H \in \mathcal{F}_n^p(G) : H_{p'} \text{ cyclic}\}, \end{aligned}$$

where H_p and $H_{p'}$ are the p -primary and prime-to- p factors of H , respectively, and $d(H_p)$ is the minimum number of generators of H_p .

Theorem A. *For every $n \geq 0$ and every finite abelian group A , $\mathcal{F}_n^p(A)$ is an induction family for $\mathrm{Perf}(R_n)$, and $\mathcal{F}_n(A)$ is an induction family for $\mathrm{Perf}(R_{n,(p)})$. The bound $n + 1$ on p -rank is optimal in both cases.*

Date: September 27, 2026.

2020 Mathematics Subject Classification. Primary 55P42; Secondary 19A22, 55P91, 55P60, 19D10.

Key words and phrases. Swan induction, chromatic homotopy theory, equivariant stable homotopy theory, finite chromatic localization, algebraic K -theory, Morava E -theory, Burnside rings.

Induction identities persist under extension of scalars. Together with CMNN's complex-oriented induction theorem, Theorem A therefore proves Conjecture 7.22: for a height- n Lubin–Tate theory E_n with $n \geq 1$, the family $\mathcal{F}_n(G)$ gives Swan induction for every finite group G . The case $p = 2$ is [14, Theorem E(3)]; the case $n = 1$ follows from their Theorem 7.13, as noted before Conjecture 7.22. The reduction from arbitrary groups to abelian subgroups uses the complex orientation of E_n ; no such reduction is claimed for R_n .

CMNN's Swan-induction criterion also gives a new proof of the known upper bound

$$L_{T(i)}K(\mathcal{M}) = 0 \quad (i > n + 1) \quad (2)$$

for every small idempotent-complete stable R_n -linear category $\mathcal{M}$ [14, Theorem C, Theorems 4.10 and 6.7]; see also [23, Theorems 1.1 and B]. This proof avoids K -theoretic purity and the inductive descent–vanishing argument, but retains the nilpotence inputs: [23, Lemma 2.3 and Remark 2.4] for the conservative Morava- E probe, and the theorem on May's nilpotence conjecture [27] through CMNN's vanishing criterion.

1.1. Geometric realizations. Write $V_r = (C_p)^r$. The quotient argument also produces finite genuine V_{n+2} -cell spectra whose underlying spectra are annihilated by $L_n^{p,f}$ and whose full geometric fixed points have nonzero Euler characteristic. Representation suspension gives finite based complexes. The construction uses exactness on K_0 , however, and does not specify their attaching maps. This leads to the following question.

Question 1.1. For every prime p and $n \geq 0$, give a uniformly explicit finite geometric or combinatorial construction of a based V_{n+2} -CW complex A satisfying

$$L_n^{p,f}\Sigma^\infty A = 0, \quad \tilde{\chi}(A^{V_{n+2}}) \neq 0.$$

Can such a construction avoid telescopic Swan theory and the existential lifting of a Grothendieck class?

The question concerns explicit models, not finite existence. Section 7 gives two examples at $p = 2$: a coordinate-sign action on $\mathbb{RP}^2$ for $n = 0$, and a four-letter partition-spectrum construction for $n = 1$. The rank $n + 2$ is minimal by chromatic Smith theory and the cyclic Lefschetz formula (Lemma 5.3).

1.2. Proof strategy. Let $e_r \in A(V_r)_\mathbb{Q}$ be the Burnside idempotent supported at the full group, and set $\beta_r = p^r \lambda(e_r)$ in a Swan group. Proper-subgroup induction is equivalent to $\beta_r = 0$. In telescopic spectra, cyclic homotopy fixed points give

$$\mathrm{Fix}_{C_p}(\beta_{r+1}) = (p[C^*(BC_p; \mathbf{1})] - p^r)\beta_r.$$

A decomposition complex supplies $\beta_{2n+2} = 0$. The tensor-rank estimate of Section 3 makes the multiplier invertible for $r > n + 1$, so descent gives $\beta_{n+2} = 0$. For Morava E_n -coefficients, the cochain class is already the scalar p^n , and this estimate is unnecessary.

Section 5 passes to ordinary perfect modules by induction on height. Quotient finite genuine G -spectra by those of underlying type at least $m + 1$. For $G = V_r$, the height- m kernel has a filtration by tensor ideals, indexed by isotropy rank. Its factor at a rank- h subgroup H is the category of compact $T(m - h)$ -local representations of G/H . The full Burnside idempotent maps to the full idempotent of this quotient group. Since height and group rank decrease by the same amount, local Swan induction makes its action zero on every factor. The resulting idempotent action is nilpotent, hence zero, and induction proves vanishing in the genuine quotient. The underlying-spectrum functor then gives the ordinary-perfect relation. A finite p -primary tensor factor handles global coefficients. Finite Euler witnesses follow from exactness on K_0 ; the final two sections give the applications and explicit models.

1.3. Conventions. Categories are stable ∞ -categories. Intermediate chromatic categories are p -local; R_n and the final geometric witnesses are integral. A finite G -cell spectrum is built by finitely many attachments of orbit cells $G/H_+ \wedge S^a$, not merely a compact retract. Write uX for its underlying coherent G -spectrum and $\Phi^H X$ for geometric fixed points. Type forgets the action and is the least j detected by $K(j)$, with $K(0) = H\mathbb{Q}$ and $\text{type}(0) = \infty$. Euler characteristics are reduced. Tensor products are derived, $\widehat{\otimes}$ denotes local tensor product, and ω denotes compactness. We take small skeleta in forming Grothendieck groups. Algebraic K -theory is nonconnective; its positive-height localizations agree with the connective version [14, Conventions].

2. BURNSIDE CLASSES AND CYCLIC FIXED POINTS

For a stable symmetric monoidal category $\mathcal{D}$ with exact tensor product, write

$$\text{Sw}_{\mathcal{D}}(G) = K_0 \text{Fun}(BG, \mathcal{D})_{\mathbb{Q}}.$$

Permutation objects define a ring homomorphism

$$\lambda_{\mathcal{D}} : A(G)_{\mathbb{Q}} \longrightarrow \text{Sw}_{\mathcal{D}}(G), \quad [S] \longmapsto [\mathbf{1}[S]].$$

It commutes with induction and restriction. The mark $\phi^H([S]) = |S^H|$ identifies $A(G)_{\mathbb{Q}}$ with a product of copies of $\mathbb{Q}$: on transitive sets, its matrix is triangular with nonzero diagonal. For a finite G -cell spectrum X , cellular additivity gives

$$\phi^H(\chi^G(X)) = \chi(\Phi^H X), \quad [\mathbf{1} \otimes uX] = \lambda_{\mathcal{D}}(\chi^G(X)). \quad (3)$$

The second identity follows by taking the alternating sum of the finite orbit-cell filtration; it extends the space-level calculation of [14, Example 6.3] to finite cell spectra. It is used over p -local coefficients for p -local cell spectra, and over arbitrary coefficients for integral cell spectra.

2.1. The induction obstruction. For $V_r = (C_p)^r$, let e_r have mark one at V_r and zero at all proper subgroups. Its integral normalization is

$$b_r = p^r e_r = \sum_{U \leq V_r} w_{r,c} [V_r/U], \quad w_{r,c} = (-1)^c p^{r-c+\binom{c}{2}}, \quad c = \text{codim } U. \quad (4)$$

Indeed, at a subgroup of codimension t the right side has mark

$$p^r \sum_{c=0}^t \begin{bmatrix} t \\ c \end{bmatrix}_p (-1)^c p^{\binom{c}{2}} = p^r \prod_{j=0}^{t-1} (1 - p^j).$$

This is p^r for $t = 0$ and zero otherwise. The identity follows by expanding $\prod_{j=0}^{t-1} (1 + p^j z)$ and setting $z = -1$. Put $\beta_r = \lambda_{\mathcal{D}}(b_r)$, with $b_0 = \beta_0 = 1$. Then

$$\beta_r = 0 \iff V_r \text{ admits proper-subgroup Swan induction in } \mathcal{D}. \quad (5)$$

In fact, e_r restricts to zero on each proper subgroup, so its linearization annihilates every proper induction by the projection formula. Conversely, $1 - e_r$ is a rational combination of proper transitive sets. These observations apply to arbitrary inducing classes, not only permutation classes.

2.2. Cyclic descent. Fix $n \geq 1$. Let $\mathcal{C}$ be $\text{Sp}_{T(n)}$, $\text{Sp}_{K(n)}$, or the category of $K(n)$ -local E_n -modules, and let $\mathcal{D} = \mathcal{C}^{\text{dual}}$. In the last case $\mathcal{D} = \text{Perf}(E_n)$ [15, Example 3.11.3]. All operations in $\mathcal{C}$ are localized.

Lemma 2.1. *Finite-group homotopy fixed points in $\mathcal{C}$ are exact and preserve dualizable objects. They commute with colimit-preserving strong symmetric monoidal functors between these local categories.*

Proof. The local norm from homotopy orbits to homotopy fixed points is an equivalence. The $K(n)$ -local result is due to Greenlees–Sadofsky and Hovey–Sadofsky [16, 18], and the telescopic result to Kuhn [21, Theorem 1.5]; we use [13, Theorem 1.1 and Corollary 2.7]. For local E_n -modules, the underlying norm is the same equivalence. For $M : BG \rightarrow \mathcal{D}$, let DM be its dual diagram. For any $Z \in \mathcal{C}$,

$$\underline{\mathrm{Hom}}_{BG}(\mathrm{colim}_{BG} M, Z) \simeq \lim_{BG} (DM \hat{\otimes} Z) \simeq (\mathrm{colim}_{BG} DM) \hat{\otimes} Z.$$

Thus the colimit is dualizable, as is the limit. Stability gives exactness. The stated functors preserve the norm and colimits, hence also these limits. Naturality retains additional commuting group actions. $\square$

Let $B = K_0(\mathcal{D})_{\mathbb{Q}}$. The cochain object

$$C^*(BC_p; \mathbf{1}) = \mathbf{1}^{hC_p}$$

is the internal dual of $\mathbf{1} \otimes \Sigma_+^\infty BC_p$. We write its Grothendieck class as $[C^*(BC_p; \mathbf{1})]$, and let Fix_H denote the additive map on K_0 induced by homotopy H -fixed points.

Lemma 2.2 (Rank descent). *Taking fixed points in the last factor gives a B -linear map $\mathrm{Fix}_{C_p} : \mathrm{Sw}_{\mathcal{D}}(V_{r+1}) \rightarrow \mathrm{Sw}_{\mathcal{D}}(V_r)$ satisfying*

$$\mathrm{Fix}_{C_p}(\beta_{r+1}) = (p[C^*(BC_p; \mathbf{1})] - p^r)\beta_r. \quad (6)$$

Consequently,

$$\mathrm{Fix}_{V_r}(\beta_r) = \prod_{j=0}^{r-1} (p[C^*(BC_p; \mathbf{1})] - p^j). \quad (7)$$

Proof. Lemma 2.1 defines the map. Tensoring with a dualizable coefficient object of trivial action commutes with limits, giving B -linearity. Write $V_{r+1} = V_r \oplus C_p$. For a subgroup $H \leq V_r$ of codimension d , the subgroups projecting onto H are $H \oplus C_p$ and the p^{r-d} graphs of maps $H \rightarrow C_p$. Their permutation C_p -actions are respectively trivial and free. The first contributes $[C^*(BC_p; \mathbf{1})]$; the second contributes one. In the free case, the homotopy quotient maps to the ordinary quotient by a V_r -equivariant underlying equivalence. The norm comparison therefore preserves the residual coherent action. The coefficient of $[\mathbf{1}[V_r/H]]$ is

$$[C^*(BC_p; \mathbf{1})] w_{r+1,d} + p^{r-d} w_{r+1,d+1} = (p[C^*(BC_p; \mathbf{1})] - p^r) w_{r,d},$$

since $w_{r+1,d} = p w_{r,d}$ and $w_{r+1,d+1} = -p^d w_{r,d}$. Iteration proves (7). $\square$

2.3. Decomposition spectra. Decomposition spectra supply the initial local relation. Their height-one member will also allow passage from p -local to global coefficients in Section 5.

Lemma 2.3. *For each $k \geq 1$ there is an integral finite genuine V_{2k} -cell spectrum $Z_{p,k}$ whose underlying spectrum is p -primary of type k and whose full Euler mark is nonzero. For $k = 1$ the full Euler mark is $-p$.*

Proof. Let $\mathcal{L}_{p^k}$ be the topological poset of proper unordered orthogonal decompositions of $\mathbb{C}^{p^k}$, and set

$$Z_{p,k} = \Sigma_{V_{2k}}^\infty \mathcal{L}_{p^k}^\diamond.$$

The diamond is unreduced suspension based at a suspension vertex; no disjoint basepoint is added. By [3, Proposition 9.1(1),(3)], its underlying spectrum is rationally acyclic, p -local, and of type k . We use the decomposition-space statement of ABS, who derive its chromatic assertion from [2, Theorem 2.2].

To describe the action, index a basis by $a \in \mathbb{F}_p^k$ and choose a primitive complex p th root ζ . The translations of this basis, the diagonal operators $e_a \mapsto \zeta^{b \cdot a} e_a$, and the scalar circle generate the group $\Gamma_k \subset U(p^k)$ of [4, Section 2]. Scalars act trivially on decompositions, so the action factors through

$\Gamma_k/S^1 = V_{2k}$. The full fixed decomposition space is the symplectic building of $\mathbb{F}_p^{2k}$ [4, Theorem 1.2] (Theorem 1.1 in the published version). Solomon–Tits [1, Theorem 4.127] gives its nonzero reduced Euler characteristic, and hence a nonzero full Euler mark after suspension. When $k = 1$ the building consists of $p + 1$ points, so that mark is $-p$.

For equivariant finiteness, use the finite cube in the proof of [3, Proposition 9.1]. Its vertices are spaces of strict chains of decompositions with prescribed numbers of components. The group preserves these numbers, and the face maps forget decompositions, so the cube and its nerve-realization comparison are equivariant. More explicitly, use iterated equivariant mapping cylinders to form its total homotopy cofiber. For every subgroup $H \leq V_{2k}$, taking H -fixed points gives the same construction on H -fixed chains: the simplex and cylinder coordinates have trivial action. This is the nerve construction for the fixed subposet, followed by unreduced suspension. Thus the comparison with $\mathcal{L}_{p^k}^\diamond$ is an equivalence on all fixed points. Each cube vertex is a finite union of compact smooth unitary homogeneous manifolds, hence has a finite equivariant triangulation [19]. Cellular approximation and mapping cylinders now give a finite V_{2k} -cell model. Rational acyclicity and p -locality imply that its underlying spectrum is p -primary. $\square$

Corollary 2.4. *In each coefficient category above, $\beta_{2n+2} = 0$.*

Proof. Take $k = n + 1$ in Lemma 2.3, and write $a = \chi^{V_{2k}}(Z_{p,k})$ and $c = \phi^{V_{2k}}(a) \neq 0$. The underlying type implies $\lambda(a) = [\mathbf{1} \otimes uZ_{p,k}] = 0$. Marks give $e_{2k}a = ce_{2k}$, so $c\beta_{2k} = 0$. Cancel c rationally. $\square$

3. A RANK ESTIMATE FOR TENSOR CATEGORIES

The following estimate will make the descent multipliers invertible. Its hypothesis is a splitting of an object after a conservative tensor functor, not merely an equality of numerical ranks.

Proposition 3.1. *Let $F : \mathcal{D} \rightarrow \mathcal{E}$ be a conservative exact strong symmetric monoidal $\mathbb{S}_p^\wedge$ -linear functor between small idempotent-complete stable symmetric monoidal categories with exact tensor products. Suppose $F(Y) \simeq \mathbf{1}_{\mathcal{E}}^{\oplus D}$, where $D \geq 1$, and $[Y] \in K_0(\mathcal{D})_{\mathbb{Q}}$ is algebraic over $\mathbb{Q}$. Then $[Y] - c$ is a unit for every $c \in \mathbb{Q} \setminus \{-D, -D + 1, \dots, D\}$.*

Write $B = K_0(\mathcal{D})_{\mathbb{Q}}$ and

$$\Lambda^{(p')} = \mathbb{Q}[\mathbf{p}_m : p \nmid m], \quad \sigma_D : \Lambda^{(p')} \longrightarrow \mathbb{Q}[z_1, \dots, z_D]^{\Sigma_D}, \quad \mathbf{p}_m \longmapsto \sum_i z_i^m.$$

The rank identities realize prime-to- p Adams values as traces in dimension at most D . Algebraicity makes these traces finite-valued, so connected monodromy can be removed without changing them. Symmetric-group projectors in chromatic rank arguments go back to Jeff Smith [32, Appendix C]; see also [22, Section 6]. We verify the operations needed here directly; general polynomial functoriality of stable K_0 is established in [9, Proposition 3.7].

3.1. Projective tensor operations. Using split Grothendieck groups for ordinary projective modules, Brauer’s projective-character theory gives an isomorphism of induction algebras

$$\mathcal{P}_p = \bigoplus_{d \geq 0} K_0(\text{proj } \mathbb{Z}_p[\Sigma_d])_{\mathbb{Q}} \simeq \Lambda^{(p')}. \quad (8)$$

Every field is a splitting field for Σ_d [20, Theorem 11.5], and projectives lift over $\mathbb{Z}_p$. The ordinary characters of indecomposable projectives are independent and span the class functions supported on p -regular classes [33, Part III, Chapters 14–16]; the integral lattice statement is also recalled in [24, Section 1]. The Frobenius characteristic

$$\text{ch}(\chi) = \sum_{\mu \vdash d} \frac{\chi(\mu)}{z_\mu} \mathbf{p}_\mu, \quad z_\mu = \prod_j j^{m_j} m_j!,$$

where m_j is the multiplicity of j in μ , identifies these characters with the span of partitions all of whose parts are prime to p . The induction formula proves (8). In particular, $\mathbf{p}_m$ is the rational virtual projective character that takes value m on m -cycles and zero elsewhere.

A finite projective right $\mathbb{Z}_p[\Sigma_d]$ -module P lifts to a finite projective $\mathbb{S}_p^\wedge[\Sigma_d]$ -module $\tilde{P}$ [25, Corollary 7.2.2.19 and Remark 7.2.2.20]. For an actual object X , define its projective tensor construction by

$$\mathsf{T}_P(X) = \tilde{P} \otimes_{\mathbb{S}_p^\wedge[\Sigma_d]} X^{\otimes d}.$$

The relative tensor can be computed in $\mathrm{Ind}(\mathcal{D})$; projectivity makes the result a retract of a finite sum of tensor powers, so it belongs to $\mathcal{D}$. Additivity in P and induction give an algebra map $\mathrm{ev}_X : \mathcal{P}_p \rightarrow B$.

Lemma 3.2. *For $p \nmid m$, the rule $[X] \mapsto \mathrm{ev}_X(\mathbf{p}_m)$ defines a unital $\mathbb{Q}$ -algebra endomorphism ψ^m of B , and $\psi^1 = \mathrm{id}$.*

Proof. For a cofiber sequence $X' \rightarrow X \rightarrow X''$, the tensor-cube filtration of $X^{\otimes m}$ has graded terms

$$\mathrm{Ind}_{\Sigma_i \times \Sigma_{m-i}}^{\Sigma_m} ((X'')^{\otimes i} \otimes (X')^{\otimes (m-i)}), \quad 0 \leq i \leq m.$$

The tensor powers carry the symmetry actions of the stable tensor category, including the Koszul signs under graded realizations. The full-cycle character restricts to zero on every proper two-block Young subgroup. Injectivity of projective characters makes each mixed term vanish after virtual projective evaluation. This proves additivity on cofiber sequences, including suspension.

For multiplicativity, use

$$\mathrm{Ind}_{\mathrm{diag} \Sigma_m}^{\Sigma_m \times \Sigma_m} \mathbf{p}_m = \mathbf{p}_m \boxtimes \mathbf{p}_m.$$

Both characters are m^2 on a pair of m -cycles and zero elsewhere. Evaluating on $X^{\otimes m} \otimes Z^{\otimes m}$ gives the assertion. For an actual projective P , its coinvariant sphere module is finite projective, with π_0 equal to the ordinary $\mathbb{Z}_p$ -coinvariants. These are free, of rank the multiplicity of the trivial ordinary character. The resulting object is a sum of that many units, by the projective-module comparison in [25, Corollary 7.2.2.19]. For $\mathbf{p}_m$ this multiplicity is one. This proves unitality, and $\mathbf{p}_1$ acts as the identity. $\square$

3.2. The specialization kernel.

Lemma 3.3. *If $F(Y) \simeq \mathbf{1}_{\mathcal{E}}^{\oplus D}$, then $\ker \sigma_D \subseteq \ker \mathrm{ev}_Y$.*

Proof. Fix a degree d . A partition is p -regular if no part occurs p or more times. For each p -regular partition $\mu \vdash d$, let P_μ be the indecomposable projective $\mathbb{Z}_p[\Sigma_d]$ -module lifting the projective cover of the simple module indexed by μ . James's unitriangularity theorem [20, Chapters 11–12] and Brauer reciprocity [33, Part III, Chapters 14–16] give

$$\mathrm{ch}(P_\mu) = s_\mu + \sum_{\lambda \triangleleft \mu} d_{\lambda\mu} s_\lambda, \quad d_{\lambda\mu} \in \mathbb{Z}_{\geq 0},$$

where s_λ is a Schur function and $\lambda \triangleleft \mu$ means that μ strictly dominates λ .

Write $\ell(\mu)$ for the number of rows of μ . Dominance implies $\ell(\lambda) \geq \ell(\mu)$ in this sum. Since $s_\lambda(z_1, \dots, z_D) = 0$ exactly when $\ell(\lambda) > D$, the specialization of $\mathrm{ch}(P_\mu)$ is zero for $\ell(\mu) > D$. For $\ell(\mu) \leq D$, the surviving expressions are still unitriangular in the Schur basis and hence linearly independent. Consequently

$$(\ker \sigma_D)_d = \mathrm{span}_{\mathbb{Q}} \{ \mathrm{ch}(P_\mu) : \mu \text{ is } p\text{-regular, } \ell(\mu) > D \}.$$

For each projective on the right, the module

$$P_\mu \otimes_{\mathbb{Z}_p[\Sigma_d]} (\mathbb{Z}_p^D)^{\otimes d} \tag{9}$$

is $\mathbb{Z}_p$ -free, and its generic fiber, viewed as a polynomial representation of GL_D , has character $\sigma_D(\mathrm{ch}(P_\mu)) = 0$, by the usual Frobenius character formula. It is therefore zero. The corresponding

finite projective sphere module $\mathbb{T}_{P_\mu}((\mathbb{S}_p^\wedge)^{\oplus D})$ has zero π_0 , so is zero [25, Corollary 7.2.2.19]. Extending to $\mathcal{E}$ and using the specified splitting of $F(Y)$ gives $F(\mathbb{T}_{P_\mu}(Y)) = 0$. Conservativity gives $\mathbb{T}_{P_\mu}(Y) = 0$, and the displayed spanning formula proves the lemma. $\square$

Only the additive operations ψ^m act on virtual classes. The map ev_Y is evaluated on the fixed object Y , and conservativity is applied to actual zero objects.

3.3. Finite-valued power sums.

Lemma 3.4. *Let k be an algebraically closed field of characteristic zero. Suppose that $(a_m)_{m \geq 1, p \nmid m}$ takes finitely many values in k and that evaluation $\mathfrak{p}_m \mapsto a_m$ annihilates $\ker \sigma_D$. There are a vector space W with $\dim W \leq D$ and a finite-order automorphism h of W such that*

$$a_m = \text{tr}(h^m | W) \quad (p \nmid m).$$

In particular, if $a_1 \in \mathbb{Q}$, then $a_1 \in \mathbb{Z}$ and $|a_1| \leq D$.

Proof. Choose a primitive p th root $\zeta \in k$ and form the formal series

$$H(t) = \exp \left(\sum_{p \nmid m} (\zeta^m - 1) a_m \frac{t^m}{m} \right) \in 1 + tk[[t]].$$

We first construct a nonzero polynomial P , of degree at most D , with

$$P(t) = H(t)P(\zeta t). \tag{10}$$

For universal power sums, $P(t) = \prod_i (1 - z_i t)$ is a solution. Thus every $(D+1)$ -minor of the coefficient matrix of $P \mapsto P - HP(\zeta t)$ vanishes universally. Each minor is a polynomial in finitely many power sums; the specialization hypothesis, after extension of scalars to k , makes it vanish here as well. There are only $D+1$ columns, so their independence would be detected by finitely many rows. The specialized system therefore has a nonzero solution. If its order at zero is j , equation (10) forces $\zeta^j = 1$. Dividing by t^j preserves the equation, after which we normalize $P(0) = 1$.

Choose a semisimple $g \in \text{GL}(V)$ with $\det(1 - tg) = P(t)$ and $\dim V = \deg P \leq D$, allowing $V = 0$ when $P = 1$. Formal logarithms of (10) give $a_m = \text{tr}(g^m | V)$ for $p \nmid m$. Let $\mathcal{G} = \overline{\langle g \rangle}^{\text{Zar}}$ and $T = \mathcal{G}^\circ$. The group $\mathcal{G}$ is diagonalizable, T is a torus, and $\mathcal{G}/T$ is finite [29, Remark 12.5, Theorem 12.9(c), and Section 12e]. Choose N divisible by p and by $|\mathcal{G}/T|$. The N th-power map has image T , so the integer powers of g^N are dense in T . The same holds for nonnegative powers. Indeed, if $u = g^N$ and Z_+ is their closure, the descending chain $Z_+ \supseteq uZ_+ \supseteq u^2Z_+ \supseteq \cdots$ stabilizes by Noetherianity. Translating back gives $uZ_+ = Z_+$, so Z_+ contains all integer powers of u .

For $p \nmid m$, the regular function $t \mapsto \text{tr}(g^m t | V)$ on T takes only the finitely many values a_{m+N_a} on this dense set. It is therefore constant, since T is irreducible. As g commutes with T , the T -weight decomposition identifies its constant coefficient with $\text{tr}(g^m | V^T)$. Set $W = V^T$ and $h = g|_W$. Evaluation at $t = 1$ gives the required trace identity, and $h^N = 1$ because $g^N \in T$. Finally, a_1 is a sum of at most D roots of unity. If it is rational, it is an integer; embedding their cyclotomic field into $\mathbb{C}$ gives $|a_1| \leq D$. $\square$

Proof of Proposition 3.1. If $[Y] - c$ were not a unit, a maximal ideal would give a homomorphism $\chi : B \rightarrow k$ to an algebraically closed field with $\chi([Y]) = c$. Choose a nonzero $f \in \mathbb{Q}[t]$ with $f([Y]) = 0$. By Lemma 3.2, each $a_m = \chi(\psi^m([Y]))$ is a root of f , so these values form a finite set. Lemma 3.3 gives the rank- D power-sum identities. Lemma 3.4 now implies $c = a_1 \in \{-D, -D+1, \dots, D\}$, a contradiction. $\square$

4. THE TELESCOPIC EULER RELATION

Let $\mathcal{D}_n = (\mathrm{Sp}_{T(n)})^{\mathrm{dual}}$, and let λ_n be its permutation linearization.

Proposition 4.1. *For every $n \geq 1$, one has*

$$\lambda_n(e_{n+2}) = 0 \quad \text{in } \mathrm{Sw}_{\mathcal{D}_n}(V_{n+2}).$$

Proof. Localized scalar extension gives an exact strong symmetric monoidal functor

$$F : \mathcal{D}_n \longrightarrow \mathrm{Perf}(E_n), \quad X \longmapsto L_{K(n)}(E_n \otimes X). \quad (11)$$

The target is ordinary perfect by [15, Example 3.11.3]. This functor is conservative. If $F(X) = 0$, base change to a residue Morava K -theory gives $K(n)_*X = 0$. The internal endomorphism algebra $X \hat{\otimes} DX$ is then a $T(n)$ -local, $K(n)$ -acyclic unital ring spectrum; its ordinary ring structure is supplied by the lax monoidal forgetful functor. By [23, Lemma 2.3 and Remark 2.4], it is $T(n)$ -acyclic, hence zero. In particular $\mathrm{id}_X = 0$. The local categories are p -complete, since p -inverted spectra are acyclic [23, Lemma 2.2(v)–(vi)]; their unit maps give the required $\mathbb{S}_p^\wedge$ -linear structures.

The cyclic-cochain calculation [14, Lemma 4.2] is

$$C^*(BC_p; E_n) \simeq E_n^{\oplus p^n}. \quad (12)$$

Set $Y = C^*(BC_p; \mathbf{1})$. Lemma 2.1 gives $F(Y) \simeq E_n^{\oplus p^n}$. Applying (7) to $\beta_{2n+2} = 0$ from Corollary 2.4, and dividing by p^{2n+2} , gives

$$\prod_{j=-1}^{2n} ([C^*(BC_p; \mathbf{1})] - p^j) = 0.$$

Thus $[Y]$ is algebraic. Proposition 3.1, applied with $D = p^n$, makes the factors at p^{-1} and at $p^{n+1}, \dots, p^{2n}$ units. Canceling these factors gives the cochain relation

$$\prod_{j=0}^n ([C^*(BC_p; \mathbf{1})] - p^j) = 0 \quad \text{in } K_0(\mathcal{D}_n)\mathbb{Q}. \quad (13)$$

In particular, $p[C^*(BC_p; \mathbf{1})] - p^r$ is a unit for $r > n + 1$. Starting again from $\beta_{2n+2} = 0$, successive applications of (6) give $\beta_{n+2} = 0$. Division by p^{n+2} proves the proposition. $\square$

Remark 4.2. For $\mathrm{Perf}(E_n)$, equation (12) makes the multiplier in (6) equal to $p^{n+1} - p^r$. Thus Corollary 2.4 and cyclic descent alone prove $\lambda_{\mathrm{Perf}(E_n)}(e_{n+2}) = 0$, without the tensor-rank estimate. The extra argument above is needed for the finite-local sphere.

Example 4.3 (An annihilating representation at height two). At $p = 2$, the height-two cochain relation can be derived from a single projective representation of Σ_6 . This gives a direct calculation using Lemma 3.2 and the geometric seed, without the specialization or finite-monodromy lemmas. Work in $\mathcal{D}_2$ and let F be the conservative functor (11).

A rank-two cofiber. Constant cochains and the universal double-cover transfer define maps $\eta, \tau : \mathbf{1} \rightarrow C^*(BC_2; \mathbf{1})$. The second map is adjoint to the stable transfer $\Sigma_+^\infty BC_2 \rightarrow \Sigma_+^\infty EC_2 \simeq \mathbb{S}$, after extension to the local unit. Set

$$C_{\mathrm{tr}} = \mathrm{cofib}(\mathbf{1}^{\oplus 2} \xrightarrow{(\eta, \tau)} C^*(BC_2; \mathbf{1})). \quad (14)$$

For the Honda height-two theory, choose a periodic coordinate t whose reduction is the standard Honda 2-typical coordinate, so $[2](t) \equiv t^4 \pmod{\mathfrak{m}}$, where $\mathfrak{m} = (2, u_1) \subset E_2^0 = W(\mathbb{F}_4)[[u_1]]$. The cyclic-cohomology calculation and Quillen's transfer formula [31] give

$$E_2^0(BC_2) = E_2^0[[t]]/([2](t)), \quad \tau \longmapsto [2](t)/t;$$

see [14, Lemma 4.2] and [6, Section 2, equation (1)]. Modulo $\mathfrak{m}$, the two columns of $F(\eta, \tau)$ are 1 and t^3 in $\mathbb{F}_4[t]/(t^4)$. They extend to a basis, so

$$F(C_{\mathrm{tr}}) \simeq E_2^{\oplus 2}, \quad [C_{\mathrm{tr}}] = [C^*(BC_2; \mathbf{1})] - 2.$$

Only the cofiber is used; no splitting into two units is asserted in $\mathcal{D}_2$.

The annihilator. Use the tableau and its row and column groups

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & \\ 6 & & \end{array}, \quad R = \Sigma_{\{1,2,3\}} \times \Sigma_{\{4,5\}}, \quad C = \Sigma_{\{1,4,6\}} \times \Sigma_{\{2,5\}}.$$

The hook lengths are 5, 3, 1, 3, 1, 1, with product 45. Thus

$$e_{321} = \frac{1}{45} \left(\sum_{r \in R} r \right) \left(\sum_{c \in C} \text{sgn}(c) c \right) \in \mathbb{Z}_2[\Sigma_6] \quad (15)$$

is a Young idempotent [22, Proposition 6.4 with $d = 3$, and Section 6.5, equation (6.5)]. Its right ideal $P_{321} = e_{321}\mathbb{Z}_2[\Sigma_6]$ is projective and, by the hook-length formula, has $\mathbb{Z}_2$ -rank $6!/45 = 16$. This rank can also be read directly from the idempotent: its identity coefficient is $1/45$, so its trace on the regular representation is $6!/45$. The first column alternates three tensor positions. It annihilates $(E_2^{\oplus 2})^{\otimes 6}$: in each basis tensor, two of those three entries agree, and terms cancel in pairs. Consequently the projective tensor construction satisfies

$$F(\mathbb{T}_{P_{321}}(C_{\text{tr}})) = 0, \quad \mathbb{T}_{P_{321}}(C_{\text{tr}}) = 0, \quad (16)$$

where the second equality uses conservativity. This is an actual zero object, not a numerical rank calculation.

The ordinary character of P_{321} is zero on classes with an even cycle and has the following remaining values:

cycle type	1^6	$3\ 1^3$	3^2	$5\ 1$
$\chi_{P_{321}}$	16	-2	-2	1
z_μ	720	18	18	5

They may be checked directly from (15) by computing $\text{tr}(L_{e_{321}} R_g \mid \mathbb{Q}_2[\Sigma_6])$. Injectivity of projective characters also gives the identity

$$4[P_{321}] = [\mathbb{Z}_2[C_5 \setminus \Sigma_6]] - [\mathbb{Z}_2[(C_3 \times C_3) \setminus \Sigma_6]],$$

where $C_5 = \langle (12345) \rangle$ and $C_3 \times C_3 = \langle (123), (456) \rangle$. The two permutation characters have values $(144, 0, 0, 4)$ and $(80, 8, 8, 0)$ on the four listed classes. Their modules are projective because the subgroup orders are odd. Thus the Frobenius characteristic is

$$\text{ch}(P_{321}) = \frac{\mathbf{p}_1^6 - 5\mathbf{p}_1^3\mathbf{p}_3 - 5\mathbf{p}_3^2 + 9\mathbf{p}_1\mathbf{p}_5}{45}.$$

Evaluating (16) gives the explicit identity

$$[C_{\text{tr}}]^6 - 5[C_{\text{tr}}]^3\psi^3([C_{\text{tr}}]) - 5\psi^3([C_{\text{tr}}])^2 + 9[C_{\text{tr}}]\psi^5([C_{\text{tr}}]) = 0. \quad (17)$$

Here ψ^3, ψ^5 are the Grothendieck-ring operations of Lemma 3.2.

Eliminating the extraneous factors. Independently, the seed $\beta_6 = 0$ and (7) give

$$f([C_{\text{tr}}]) = 0, \quad f(T) = (T + \frac{3}{2})(T + 1)T(T - 2)(T - 6)(T - 14). \quad (18)$$

Under a residue-field character, let a, b, c be the values of $[C_{\text{tr}}]$ and its third and fifth Adams operations. All three lie in $\{-\frac{3}{2}, -1, 0, 2, 6, 14\}$, since the operations are unital $\mathbb{Q}$ -algebra endomorphisms. Equation (17) becomes

$$a^6 - 5a^3b - 5b^2 + 9ac = 0.$$

If $a = -\frac{3}{2}$, the first term has denominator 64 in lowest terms, whereas the other terms have denominators dividing 16, a contradiction. If $a = 6$ or 14 , then $|b|, |c| \leq 14$ would imply

$$a^6 \leq 70a^3 + 980 + 126a,$$

which fails at both values. Thus $[C_{\text{tr}}] + \frac{3}{2}$, $[C_{\text{tr}}] - 6$, and $[C_{\text{tr}}] - 14$ belong to no maximal ideal and are units. Canceling them in (18) yields $[C_{\text{tr}}]([C_{\text{tr}}] + 1)([C_{\text{tr}}] - 2) = 0$, or

$$([C^*(BC_2; \mathbf{1})] - 1)([C^*(BC_2; \mathbf{1})] - 2)([C^*(BC_2; \mathbf{1})] - 4) = 0. \quad (19)$$

This proof has not assumed that the Adams operations fix the cochain class or that the Grothendieck ring is reduced. By Lemma 2.1 and the projection formula, tensor powers of cochains are cochains on products. Expanding (19) therefore gives

$$[C^*(B(C_2)^3; \mathbf{1})] + 14[C^*(BC_2; \mathbf{1})] = 7[C^*(B(C_2)^2; \mathbf{1})] + 8[\mathbf{1}]. \quad (20)$$

The same calculation works in dualizable $K(2)$ -local spectra.

Remark 4.4. In $K(2)$ -local spectra at $p = 2$, the cochain object is not a direct sum of four units. Here $\mathbf{1} = L_{K(2)}\mathbb{S}$. Choose E_2 for the Honda formal group $\mathbb{H}_2$ over $\mathbb{F}_4$, with $E_2^0 = W(\mathbb{F}_4)[[u_1]]$ and $\mathfrak{m} = (2, u_1)$. The cyclic cohomology calculation gives

$$E_2^0(BC_2)/\mathfrak{m} \cong \mathbb{F}_4[t]/(t^4).$$

The unextended Morava stabilizer $\mathbb{S}_2 = \text{Aut}_{\mathbb{F}_4}(\mathbb{H}_2)$ contains an order-three automorphism given by $t \mapsto \zeta t$ in the Honda coordinate, where $\zeta \in \mathbb{F}_4$ is a primitive cube root [10, Section 1.1]. Choosing a generator or its inverse to match the action convention, its action on the displayed cohomology has eigenvalues $1, \zeta, \zeta^2, 1$. The comparison $L_{K(2)}(E_2 \otimes C^*(BC_2; \mathbf{1})) \simeq C^*(BC_2; E_2)$ is natural in coefficient automorphisms. A splitting into four units would therefore make this representation trivial: $\mathbb{S}_2$ fixes the residue field of E_2^0 , and each unit summand supplies an invariant basis vector. This contradicts the displayed eigenvalues. This distinguishes objects only; it does not imply that their classes differ in rational K_0 or that all three roots in (19) occur.

5. CHROMATIC QUOTIENTS AND EULER REALIZATION

We pass from the local relation to the finite-local sphere by induction on height in a quotient of genuine spectra. The height layers have a second filtration, by isotropy; its factors are compact telescopic representations of smaller quotient groups.

5.1. The quotient and its layers. For a finite group G , put

$$\mathcal{S}(G) = (\text{Sp}_{(p)}^G)^\omega, \quad \mathcal{I}_a(G) = \{X \in \mathcal{S}(G) : \text{type}(uX) \geq a\}, \quad \mathcal{I}_0(G) = \mathcal{S}(G).$$

Each $\mathcal{I}_a(G)$ is a thick tensor ideal of $\mathcal{S}(G)$: smashing with a finite spectrum cannot decrease the underlying type. Write

$$\mathcal{Q}_m(G) = (\mathcal{S}(G)/\mathcal{I}_{m+1}(G))^{\text{idem}}, \quad \mathcal{Q}_{-1}(G) = 0. \quad (21)$$

This is the compact symmetric monoidal category of the smashing finite localization $\mathcal{L}_{m,G}$ away from $\mathcal{I}_{m+1}(G)$. The kernel of $\mathcal{Q}_m(G) \rightarrow \mathcal{Q}_{m-1}(G)$ is the thick tensor ideal

$$\mathcal{J}_m(G) = (\mathcal{I}_m(G)/\mathcal{I}_{m+1}(G))^{\text{idem}}.$$

Compact localization and cofinality give exactness at the middle term of

$$K_0(\mathcal{J}_m(G))_{\mathbb{Q}} \longrightarrow K_0(\mathcal{Q}_m(G))_{\mathbb{Q}} \longrightarrow K_0(\mathcal{Q}_{m-1}(G))_{\mathbb{Q}}; \quad (22)$$

see [30, Theorem 2.1 and Lemma 0.6]. Here the triangle relations give $K_0(\mathcal{A}/\mathcal{B}) = K_0(\mathcal{A})/\text{im } K_0(\mathcal{B})$ for a Verdier quotient, and passage to the idempotent completion is injective on K_0 by cofinality. These facts give the asserted middle exactness without assuming surjectivity onto the completed quotient. All these categories carry the action of finite genuine permutation spectra. Let $\rho_{m,G} : A(G)_{\mathbb{Q}} \rightarrow K_0(\mathcal{Q}_m(G))_{\mathbb{Q}}$ be permutation linearization. This is a map of Grothendieck rings. Throughout the argument, vanishing of a Burnside class refers to this map, not to its action as an endomorphism of the localized sphere.

We use $T(0) = H\mathbb{Q}$, and set $\mathrm{Sp}_{L_s^f} = 0$ for $s < 0$. For elementary abelian G , rank gives a convenient isotropy filtration.

Lemma 5.1 (The isotropy factors of a height layer). *Let $G = V_r$. The category $\mathcal{J}_m(G)$ has a finite filtration by thick tensor ideals of $\mathcal{Q}_m(G)$, with idempotent-completed successive quotients*

$$\coprod_{\substack{H \leq G \\ \mathrm{rk}(H)=h}} \mathrm{Fun}(B(G/H), \mathrm{Sp}_{T(m-h)})^\omega, \quad 0 \leq h \leq \min(m, r). \quad (23)$$

The action of a finite G -set S on the H -factor is tensoring with the local permutation object $\mathbf{1}[S^H]$, with its G/H -action.

Proof. Put $A_m = \mathcal{L}_{m,G}\mathbb{S}_G$. We first identify its geometric fixed points. Chromatic Smith theory and type-function realization give

$$\min_{X \in \mathcal{I}_a(G)} \mathrm{type}(\Phi^H X) = \max\{a - \mathrm{rk}(H), 0\}; \quad (24)$$

The lower bound follows by applying [7, Theorem 2.1] to $\mathrm{Res}_H^G X$. For realization, put $f_a(H) = \max\{a - \mathrm{rk}(H), 0\}$. If $K \leq H$, then

$$f_a(K) \leq f_a(H) + \mathrm{rk}(H/K),$$

since $\mathrm{rk}(H/K) = \mathrm{rk}(H) - \mathrm{rk}(K)$. Thus f_a satisfies the inequalities of [7, Theorem 1.3], which realizes it. Applying Φ^H to the local unit gives the finite localization associated to the thick tensor ideal generated by $\Phi^H \mathcal{I}_{m+1}(G)$. To verify this, its unit fiber lies in the localizing subcategory generated by those images, while its tensor product with each image and its dual vanishes. The latter condition is locality. The thick-subcategory theorem [17, Theorem 7] and (24) therefore give

$$\Phi^H A_m \simeq L_{m-h}^f \mathbb{S}_{(p)}, \quad h = \mathrm{rk}(H). \quad (25)$$

Equation (25) is an equivalence of ordinary spectra. It lifts to coherent G/H -spectra with trivial action, since the unit map factors through pointwise finite localization; it is not an identification of genuine residual spectra.

In the presentable localized category, let $\mathcal{T}_m$ be the kernel of localization to height $m - 1$. Its colocalization is tensoring with $\mathrm{fib}(A_m \rightarrow A_{m-1})$, where $A_{-1} = 0$. Isotropy localization and colocalization are also tensor functors [8, Theorem 8.6], so they commute with this colocalization and with the height localizations. Consequently, taking an isotropy quotient of $\mathcal{T}_m$ gives the height kernel in the corresponding isotropy quotient.

Let $\mathcal{F}_h^{\mathrm{iso}}$ be the family of subgroups of rank at most h , and let $\mathcal{T}_{m, \leq h}$ be the intersection of $\mathcal{T}_m$ with $\mathcal{F}_h^{\mathrm{iso}}$ -isotropy objects. These are localizing tensor ideals of the ambient localized category: both the height-kernel condition and $(E\mathcal{F}_h^{\mathrm{iso}})_+ \otimes Z \simeq Z$ are preserved by tensoring with any object. Each intersection is generated by the ambient compact objects

$$\mathcal{L}_{m,G}(X \otimes G/K_+), \quad X \in \mathcal{I}_m(G), \quad K \in \mathcal{F}_h^{\mathrm{iso}}.$$

Indeed, applying $(E\mathcal{F}_h^{\mathrm{iso}})_+ \otimes -$ to the compact generators of $\mathcal{T}_m$ generates this intersection, and its cellular filtration gives the displayed generators. Thus the inclusions preserve compact objects, and $\mathcal{J}_{m, \leq h} = \mathcal{T}_{m, \leq h} \cap \mathcal{Q}_m(G)$ gives the claimed filtration by thick tensor ideals. The usual isotropy quotient at H is the category of coherent G/H -spectra, with geometric fixed points as quotient functor [5, Theorem 5.1.27]. In the quotient at a subgroup H of rank h , a displayed generator indexed by K of rank at most h vanishes unless $K = H$. For $K = H$, its image is the free G/H -object on the underlying spectrum of $\Phi^H X$, after finite localization.

By (25), the ambient coefficient ring at H is $R_{s,(p)}$, where $s = m - h \geq 0$, and the height colocalization becomes tensoring with $\mathrm{fib}(R_{s,(p)} \rightarrow R_{s-1,(p)})$. Here $R_{-1,(p)} = 0$. The resulting factor is therefore the category of $R_{s-1,(p)}$ -acyclic $R_{s,(p)}$ -modules with G/H -action. Equation (24) with

$a = m$, together with the thick-subcategory theorem, shows that the underlying spectra $\Phi^H X$ for $X \in \mathcal{I}_m(G)$ generate precisely the thick subcategory of finite spectra of type at least s . Hence the factor is generated by the free G/H -object on $R_{s,(p)} \otimes F_s$, for a finite type- s spectrum F_s . Torsion-completion duality identifies this category with $\text{Fun}(B(G/H), \text{Sp}_{T(s)})$ [8, Theorem 2.21(3) and Proposition 2.34]; for $s = 0$ this is just rational spectra. The completion is $T(s)$ -localization, since among L_s^f -local spectra the F_s -acyclic and $T(s)$ -acyclic objects are exactly the L_{s-1}^f -local objects. There is no factor for $h > m$, by (25). Taking compact objects gives (23), with idempotent completion as in [30, Theorem 2.1]. Distinct subgroups of the same rank are incomparable, which gives the displayed finite product. Finally, $\Phi^H(\Sigma_G^\infty S_+) = \Sigma^\infty(S^H)_+$, and completion commutes with tensoring by this finite permutation object. This proves the assertion about the action. $\square$

The product in (23) describes a successive quotient, not a splitting of $\mathcal{J}_m(G)$. The stages are ideals in the ambient tensor category; they need not contain its unit. Their images in $K_0(\mathcal{Q}_m(G))_{\mathbb{Q}}$ are therefore ideals and are preserved by the Burnside action. No injectivity of the maps from the stages' Grothendieck groups is needed. Compactness follows from the generators above, not from compactness of arbitrary isotropy pieces.

5.2. Idempotents and induction on height. We use a simple consequence of exactness on K_0 . If an idempotent operator e preserves a finite filtration of a stable category and acts as zero on the rational K_0 of each completed quotient, then it acts as zero on the rational K_0 of the whole category. Indeed, exactness makes e lower the induced filtration on K_0 ; hence some power e^N is zero. Since $e^N = e$, the assertion follows. Only exactness at the middle term is needed, not surjectivity onto the K_0 of an idempotent-completed quotient.

Proposition 5.2. *For $m \geq 0$ and $r \geq m + 2$, the full-group idempotent satisfies*

$$\rho_{m,V_r}(e_r) = 0.$$

Proof. Fix $G = V_r$ and induct on m , starting with $\mathcal{Q}_{-1}(G) = 0$. On the H -factor of (23), the action of e_G is that of $e_{G/H}$. In fact the map

$$A(G)_{\mathbb{Q}} \longrightarrow A(G/H)_{\mathbb{Q}}, \quad [S] \longmapsto [S^H],$$

sends e_G to $e_{G/H}$, as its marks at K/H are the original marks at $K \geq H$. Put $s = m - \text{rk}(H)$. Then

$$\text{rk}(G/H) = r - \text{rk}(H) \geq s + 2.$$

Proposition 4.1 makes the permutation class of $e_{G/H}$ zero in $K_0 \text{Fun}(B(G/H), \mathcal{D}_s)_{\mathbb{Q}}$ when $s \geq 1$. The assertion at larger ranks follows by inflating the critical-rank unit formula and using (5). For $s = 0$, it is ordinary rational cyclic induction [14, Theorem 7.6].

Pointwise dualizable local representations act on compact local representations: tensoring preserves compactness because its right adjoint is tensoring with the dual and preserves colimits. Thus the vanishing local Swan class annihilates the K_0 of the H -factor. The preceding filtration argument shows that e_G annihilates $K_0(\mathcal{J}_m(G))_{\mathbb{Q}}$.

Set $\varepsilon = \rho_{m,G}(e_G)$. Its image at height $m - 1$ is zero by induction, so (22) gives $\varepsilon = i_*(z)$ for some $z \in K_0(\mathcal{J}_m(G))_{\mathbb{Q}}$. The maps preserve the permutation action, and consequently

$$\varepsilon = \varepsilon^2 = i_*(e_G z) = 0. \tag{26}$$

This completes the induction. $\square$

Thus both the height and the residual group rank decrease by $\text{rk}(H)$ on an isotropy factor. At the critical rank $r = m + 2$, every factor is again in the critical range for the local Swan relation.

5.3. Ordinary perfect coefficients.

Proof of Theorem A. Fix $n \geq 0$ and $G = V_{n+2}$. The exact functor $X \mapsto R_{n,(p)} \otimes uX$ on $\mathcal{S}(G)$ annihilates $\mathcal{I}_{n+1}(G)$ and takes values in $\text{Fun}(BG, \text{Perf}(R_{n,(p)}))$. It therefore factors through $\mathcal{Q}_n(G)$. Proposition 5.2 gives $\lambda_{\text{Perf}(R_{n,(p)})}(e_G) = 0$.

To obtain the global relation, choose $G \twoheadrightarrow V_2$ and let $B_G = \text{Inf}_{V_2}^G Z_{p,1}$, using Lemma 2.3. Its underlying spectrum is integral finite and p -primary, and its full Euler mark is $-p$. The exact functor

$$X \longmapsto R_n \otimes uB_G \otimes uX$$

takes $\mathcal{S}(G)$ to $\text{Fun}(BG, \text{Perf}(R_n))$. Indeed, any finite p -local spectrum is a retract of the p -localization of a finite integral spectrum [30, Theorem 2.1]; smashing with the finite p -primary uB_G makes that retract integral finite. The functor annihilates $\mathcal{I}_{n+1}(G)$, since its values there are p -primary and of type at least $n+1$ before extension to R_n . It too factors through $\mathcal{Q}_n(G)$. On permutation classes its induced map is multiplication by $[R_n \otimes uB_G]$. Writing $a = \chi^G(B_G)$, cellular additivity and $e_G a = -pe_G$ give

$$0 = \lambda_{\text{Perf}(R_n)}(e_G)[R_n \otimes uB_G] = \lambda_{\text{Perf}(R_n)}(e_G a) = -p\lambda_{\text{Perf}(R_n)}(e_G). \quad (27)$$

Cancel $-p$ rationally and use (5). Any finite abelian group whose p -part requires at least $n+2$ generators surjects onto V_{n+2} . Inflation of the unit formula gives proper-subgroup induction for it. Repeated substitution strictly decreases subgroup order and gives $\mathcal{F}_n^p(A)$. The projection formula and transitivity justify substitution for arbitrary classes [14, Remark 7.1].

After extension to $R_{n,(p)}$, prime-to- p group orders are units. Cyclic induction for the prime-to- p factor [14, Theorem 7.6], combined with the same substitution, gives $\mathcal{F}_n(A)$. Extension of scalars preserves these identities.

For sharpness when $n \geq 1$, extend coefficients to E_n . Full homotopy fixed points in $\text{Perf}(E_n)$ are exact by Lemma 2.1. Compose them with generic even-minus-odd rank: for $k = \text{Frac}(\pi_0 E_n)$ this is

$$\dim_k(\pi_0 M \otimes k) - \dim_k(\pi_1 M \otimes k).$$

It is additive by the periodic long exact sequence. Equations (7) and (12) send β_r to

$$\prod_{j=0}^{r-1} (p^{n+1} - p^j),$$

which is nonzero for $r \leq n+1$. Thus proper-subgroup induction fails for V_{n+1} . For $n=0$, extend to $H\mathbb{Q}$: the trivial representation of C_p is not a rational multiple of the regular representation. $\square$

In particular, for $r = n+2$ the rational Swan ring of R_n satisfies

$$p^r[R_n] = \sum_{c=1}^r (-1)^{c+1} p^{r-c+} \binom{c}{2} \sum_{\substack{U \leq V_r \\ \text{codim } U = c}} [R_n[V_r/U]]. \quad (28)$$

Its displayed coefficients are integral, but the equality is asserted rationally. Multiplication and the projection formula give the corresponding identity for every Swan class.

5.4. Finite Euler witnesses. The quotient vanishing also gives finite geometric witnesses. Their existence is a consequence of the argument, not an input to the ordinary-perfect induction theorem.

Lemma 5.3. *If X is finite V_r -cell and $\text{type}(uX) \geq d \geq 1$, then $\chi(\Phi^H X) = 0$ whenever $\text{rk}(H) \leq d$.*

Proof. For $H \neq 1$, choose a hyperplane $J < H$. Chromatic Smith theory, applied to $\text{Res}_J^{V_r} X$ [7, Theorem 2.1], gives $\text{type}(\Phi^J X) \geq d - \text{rk}(J) \geq 1$. The cyclic Lefschetz formula identifies $\chi(\Phi^H X)$ with the alternating trace of a generator of H/J on this zero rational homology. The formula holds on orbit cells by counting fixed points and extends through finite cofibers. For $H = 1$, use rational acyclicity directly. $\square$

Proposition 5.4. *For every $m \geq 1$ there is an integral finite genuine V_{m+1} -cell spectrum X_m^{int} with p -primary underlying spectrum of type m and*

$$\chi^{V_{m+1}}(X_m^{\text{int}}) = c_m^{\text{int}} e_{m+1}, \quad c_m^{\text{int}} \neq 0.$$

In particular, $R_{m-1} \otimes uX_m^{\text{int}} = 0$.

Proof. Put $G = V_{m+1}$, and choose a finite G -cell spectrum W with $\chi^G(W) = p^{m+1}e_G$, using (4); negative coefficients are realized by suspensions. Proposition 5.2 gives $[\mathcal{L}_{m-1,G}W] = 0$ rationally. Exactness for compact localization and cofinality [30, Theorem 2.1 and Lemma 0.6] give a compact $X \in \mathcal{I}_m(G)$ with $[X] = N[W]$ in $K_0(\mathcal{S}(G))$ for some $N > 0$. A difference of classes in the kernel is represented by a sum with a suspension. The full subcategory of finite p -local G -cell spectra is triangulated and dense in $\mathcal{S}(G)$. Since $[X] = N[W]$ and W belongs to this subcategory, the dense-subcategory criterion [30, Corollary 0.10] implies that X also has a finite cell model. The additive functions $\chi\Phi^H$ yield $\chi^G(X) = Np^{m+1}e_G$. Lemma 5.3 then forces $\text{type}(uX) = m$.

Clear the finitely many prime-to- p denominators in its cell attachments to obtain an integral finite cell spectrum X' with $X'_{(p)} \simeq X$. Multiplying the source of an attachment by its denominator does not change its p -local cofiber. Choose a quotient $G \twoheadrightarrow V_2$, and set

$$X_m^{\text{int}} = X' \wedge \text{Inf}_{V_2}^G Z_{p,1}.$$

The extra factor from Lemma 2.3 is p -primary underneath, of type one, with full Euler mark $-p$. It makes the underlying spectrum p -primary without changing its type at p , and multiplies its full Euler mark by $-p$. Thus the result is annihilated by the global R_{m-1} and has the asserted Euler class. $\square$

Thus finite Euler realization follows from vanishing in the chromatic quotient. The construction determines the Euler class but not the attaching maps, as in Question 1.1.

6. MORAVA E -THEORY AND CHROMATIC VANISHING

We give the deductions stated in the introduction. First, E_n receives R_n , and p is its only noninvertible prime. Theorem A therefore gives $\mathcal{F}_n(A)$ -induction for every finite abelian A . A complex orientation $MU \rightarrow E_n$ refines to an E_2 -map by [12, Theorem 1.2], since E_n is even and E_∞ . Its underlying E_1 -map permits CMNN's abelian-subgroup induction theorem [14, Theorem 7.4]. Substitution of the abelian formulas proves Conjecture 7.22 for arbitrary finite ambient groups. Remark 4.2 also gives the shorter proof specific to E_n .

For (2), CMNN's criterion [14, Theorem 6.7] says that proper-subgroup Swan induction for V_r over an E_∞ -ring R implies $L_{T(i)}K(R) = 0$ for $i \geq r$. Apply it to R_n with $r = n + 2$; this includes $n = 0$. The criterion uses [14, Proposition 6.6], whose proof passes to a Morava- E algebra at the height being tested and invokes [27, Theorem 2.1]. Thus this deduction retains May nilpotence, though not K -theoretic purity. For an R_n -linear category $\mathcal{M}$, lax symmetric monoidality of algebraic K -theory [11, Theorem 1.13] makes $K(\mathcal{M})$ a unital $K(R_n)$ -module. After localization it is a module over the zero ring, and is therefore zero. This proves (2), including the cases $\text{Perf}(R_n)$ and $\text{Perf}(E_n)$.

7. EXPLICIT MODELS AT THE PRIME TWO

The models below are independent of Euler-class lifting. The first is a finite space at height zero; the second is a specified finite geometric spectrum at positive height.

7.1. The projective plane. Let $G = C_2^2$ act on $A = \mathbb{RP}^2$ by

$$a[x : y : z] = [-x : y : z], \quad b[x : y : z] = [x : -y : z],$$

with basepoint $[0 : 0 : 1]$. The full fixed set consists of the three coordinate points, so $\tilde{\chi}(A^G) = 2$. Each subgroup of order two fixes a projective line and an isolated point. All proper reduced Euler marks consequently vanish, including the trivial mark. The usual cell attachment gives

$$\Sigma^\infty A \simeq \Sigma \mathbb{S}/2$$

underneath. The smooth action has a finite equivariant triangulation [19], and its Euler class is

$$\chi^G(\Sigma_G^\infty A) = 2[G/G] - \sum_{|H|=2} [G/H] + [G/1] = 2e_G. \quad (29)$$

Since $\mathbb{S}[1/2]$ annihilates the underlying spectrum, (29) realizes the height-zero induction relation.

7.2. A positive-height model from four letters. Let $\Delta = \langle (12)(34), (13)(24) \rangle \leq \Sigma_4$, and let

$$\rho = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \sum x_i = 0\}$$

be the reduced real permutation representation. Its complexification embeds Σ_4 in $U(3)$. The proper partition poset $\mathcal{P}_4$ has an order complex with thirteen vertices and eighteen edges. Set

$$Y = D\Sigma^\infty \mathcal{P}_4^\diamond \wedge S^\rho,$$

where D is genuine Spanier–Whitehead duality and the diamond is unreduced suspension based at a suspension vertex. Identify $U(3)$ with $U(\rho \otimes_{\mathbb{R}} \mathbb{C})$ and let κ be conjugation with respect to the real form ρ . It fixes the permutation matrices pointwise. Put

$$\mathcal{U} = U(3) \rtimes \langle \kappa \rangle, \quad J = \Sigma_4 \times \langle \kappa \rangle, \quad G = \Delta \times \langle \kappa \rangle \cong C_2^3.$$

Inflate Y to J with trivial additional involution and define

$$X = \Sigma \operatorname{Res}_G^{\mathcal{U}} \operatorname{Ind}_J^{\mathcal{U}} \operatorname{Inf}_{\Sigma_4}^J Y. \quad (30)$$

The sphere S^ρ uses the real representation; the unitary embedding uses its complexification.

Proposition 7.1. *The spectrum (30) has an integral finite G -cell model, two-local underlying type two, and*

$$\Phi^G X \simeq \bigvee^{16} \mathbb{S}.$$

Its proper Euler marks vanish. If P is the projective-plane spectrum inflated along $G \rightarrow \Delta$, then $X^{\text{int}} = X \wedge P$ has 2-primary underlying spectrum of type two and

$$R_1 \otimes uX^{\text{int}} = 0, \quad \Phi^G X^{\text{int}} \simeq \bigvee^{32} \mathbb{S}.$$

Here $p = 2$, so $R_1 = L_1^{2,f} \mathbb{S}$.

Proof. Duality and representation spheres preserve finite cell spectra for finite groups. Each orbit cell induced from J to $\mathcal{U}$, after restriction to G , is a suspension of a compact homogeneous space $\mathcal{U}/H$ with $H \leq J$ finite. Equivariant triangulation and cellular approximation give a finite G -cell model [19]. After forgetting κ and localizing at 2, the construction, apart from the outer suspension, is $F(2)$ as defined in [7, Section 4]. Its underlying two-local type is two by their proof of Theorem 2.2, which applies [2, Theorem 0.4]. The sphere S^ρ uses the reduced real standard representation: this is the odd-sphere case of Arone’s theorem, not the sphere of the complexification.

For the full fixed points, write $C = C_{U(3)}(\Delta)$. The three nontrivial characters of Δ occur once each, so $C \cong U(1)^3$ and $C \cap \Sigma_4 = \Delta$. The transporter of Δ into Σ_4 is $C\Sigma_4$: an intertwiner identifies the reduced regular character with a transported permutation character, and adding back the trivial line shows that the action is regular. A permutation then conjugates it to the original action. This is the transporter calculation in [4, Section 5], recalled in the proof of [7, Proposition 4.2]. For a based J -space B , it gives a homeomorphism

$$(C/\Delta)_+ \wedge B^\Delta \longrightarrow (U(3)_+ \wedge_{\Sigma_4} B)^\Delta, \quad (c\Delta, b) \longmapsto [c, b],$$

which commutes with conjugation on the first factor and the κ -action on the second.

This comparison also holds in genuine residual C_2 -spectra. Suspend the finite J -spectrum by the restriction of a sufficiently large $\mathcal{U}$ -representation V until it is a suspension spectrum [28, Chapter XII]. Such restrictions contain every finite J -representation: by Peter–Weyl and Frobenius reciprocity, each irreducible J -representation occurs in the restriction of some $\mathcal{U}$ -representation, and finite direct sums give the required multiplicities. Apply the space-level comparison and desuspend using the induction projection formula. Since c centralizes Δ , it preserves V^Δ . The trivialization $(c, v) \mapsto (c, cv)$ of the fixed representation bundle is well defined over C/Δ and is conjugation-equivariant, since $\kappa(cv) = \bar{c}\kappa(v)$. Thus, with the actual residual actions retained, the comparison is

$$\Phi^\Delta(\mathrm{Res}_G^\mathcal{U} \mathrm{Ind}_J^\mathcal{U} \mathrm{Inf} Y) \simeq \Sigma_{C_2}^\infty(C/\Delta)_+ \wedge \mathrm{Inf}_1^{C_2} \Phi^\Delta Y.$$

The characters of Δ are real. In a real Fourier basis, conjugation on the quotient three-torus C/Δ is inversion, with eight fixed points. The poset $\mathcal{P}_4^\Delta$ consists of the three coset partitions of order-two subgroups, and $\rho^\Delta = 0$. Hence $\Phi^\Delta Y \simeq \bigvee^2 S^{-1}$, with trivial extra involution. Taking the remaining geometric fixed points and the outer suspension gives sixteen degree-zero spheres. Lemma 5.3 gives zero proper Euler marks.

The projective-plane factor P is 2-primary underneath, of type one, with two full fixed spheres. Tensoring retains type two, kills the underlying spectrum after inverting 2, and doubles the full sphere count. This proves the assertions for X^{int} . $\square$

Cellular additivity for X^{int} gives the following relation in $K_0 \mathrm{Fun}(BG, \mathrm{Perf}(R_1))$, before rationalization:

$$32[R_1] - 16 \sum_{\dim H=2} [R_1[G/H]] + 16 \sum_{\dim L=1} [R_1[G/L]] - 32[R_1[G]] = 0.$$

After division by sixteen, its rational form is

$$2[R_1] = \sum_{\dim H=2} [R_1[G/H]] - \sum_{\dim L=1} [R_1[G/L]] + 2[R_1[G]], \quad G = C_2^3.$$

Each sum has seven terms. A sufficiently large even multiple of the real regular representation suspends the finite cell spectrum to the suspension spectrum of a finite based G -CW complex, without changing any Euler mark [28, Chapter XII]. The same observation applies to the integral witnesses of Proposition 5.4. The difference in Question 1.1 is that (30) specifies the geometric operations, whereas general Euler-class lifting does not determine the attaching maps.

Numbered citations to versioned arXiv references refer to the indicated versions.

REFERENCES

- [1] Peter Abramenko and Kenneth S. Brown, *Buildings: Theory and applications*, Graduate Texts in Mathematics, vol. 248, Springer, New York, 2008. doi:10.1007/978-0-387-78835-7.
- [2] Gregory Arone, *Iterates of the suspension map and Mitchell’s finite spectra with A_k -free cohomology*, Math. Res. Lett. **5** (1998), no. 4, 485–496.
- [3] Gregory Arone, Ilan Barnea, and Tomer M. Schlank, *Suspension spectra of matrix algebras, the rank filtration, and rational noncommutative CW-spectra*, Math. Ann. **386** (2023), 35–84. arXiv:2101.09778v2.
- [4] Gregory Arone and Kathryn Lesh, *Fixed points of coisotropic subgroups of Γ_k on decomposition spaces*, Homology Homotopy Appl. **22** (2020), no. 1, 77–96. doi:10.4310/HHA.2020.v22.n1.a6; arXiv:1701.06070v2. Theorem 1.2 of this arXiv version is Theorem 1.1 of the published paper.
- [5] David Ayala, Aaron Mazel-Gee, and Nick Rozenblyum, *Stratified noncommutative geometry*, manuscript, 2023. arXiv:1910.14602v5.
- [6] Malkhaz Bakuradze and Stewart Priddy, *Transfer and complex oriented cohomology rings*, Algebr. Geom. Topol. **3** (2003), 473–509. doi:10.2140/agt.2003.3.473; arXiv:math/0307182.
- [7] Tobias Barthel, Markus Hausmann, Niko Naumann, Thomas Nikolaus, Justin Noel, and Nathaniel Stapleton, *The Balmer spectrum of the equivariant homotopy category of a finite abelian group*, Invent. Math. **216** (2019), 215–240. doi:10.1007/s00222-018-0846-5; arXiv:1709.04828v3.

- [8] Tobias Barthel, Drew Heard, and Gabriel Valenzuela, *Local duality in algebra and topology*, Adv. Math. **335** (2018), 563–663. doi:10.1016/j.aim.2018.07.017; arXiv:1511.03526v3.
- [9] Clark Barwick, Saul Glasman, Akhil Mathew, and Thomas Nikolaus, *K-theory and polynomial functors*, preprint, 2021. arXiv:2102.00936v2.
- [10] Agnès Beaudry, *Towards the homotopy of the $K(2)$ -local Moore spectrum at $p = 2$* , Adv. Math. **306** (2017), 722–788. doi:10.1016/j.aim.2016.10.020; arXiv:1501.06082v2.
- [11] Andrew J. Blumberg, David Gepner, and Gonçalo Tabuada, *Uniqueness of the multiplicative cyclotomic trace*, Adv. Math. **260** (2014), 191–232. doi:10.1016/j.aim.2014.02.004; arXiv:1103.3923v3.
- [12] Steven Greg Chadwick and Michael A. Mandell, *E_n genera*, Geom. Topol. **19** (2015), no. 6, 3193–3232. doi:10.2140/gt.2015.19.3193; arXiv:1310.3336.
- [13] Dustin Clausen and Akhil Mathew, *A short proof of telescopic Tate vanishing*, Proc. Amer. Math. Soc. **145** (2017), no. 12, 5413–5417. arXiv:1608.02063v2.
- [14] Dustin Clausen, Akhil Mathew, Niko Naumann, and Justin Noel, *Descent and vanishing in chromatic algebraic K-theory via group actions*, Ann. Sci. Éc. Norm. Supér. (4) **57** (2024), no. 4, 1135–1190. arXiv:2011.08233v2.
- [15] Tony Feng, David Treumann, and Allen Yuan, *G-spectra of cyclic defect*, preprint, 2023. arXiv:2301.03184v1.
- [16] J. P. C. Greenlees and Hal Sadofsky, *The Tate spectrum of v_n -periodic complex oriented theories*, Math. Z. **222** (1996), no. 3, 391–405.
- [17] Michael J. Hopkins and Jeffrey H. Smith, *Nilpotence and stable homotopy theory II*, Ann. of Math. (2) **148** (1998), no. 1, 1–49. doi:10.2307/120991.
- [18] Mark Hovey and Hal Sadofsky, *Tate cohomology lowers chromatic Bousfield classes*, Proc. Amer. Math. Soc. **124** (1996), no. 11, 3579–3585.
- [19] Sören Illman, *Smooth equivariant triangulations of G -manifolds for G a finite group*, Math. Ann. **233** (1978), no. 3, 199–220. doi:10.1007/BF01405351.
- [20] Gordon D. James, *The representation theory of the symmetric groups*, Lecture Notes in Mathematics, vol. 682, Springer, Berlin–New York, 1978. doi:10.1007/BFb0067708.
- [21] Nicholas J. Kuhn, *Tate cohomology and periodic localization of polynomial functors*, Invent. Math. **157** (2004), no. 2, 345–370. arXiv:math/0307173.
- [22] Nicholas J. Kuhn and Christopher J. R. Lloyd, *Chromatic fixed point theory and the Balmer spectrum for extraspecial 2-groups*, preprint, 2022. arXiv:2008.00330v2.
- [23] Markus Land, Akhil Mathew, Lennart Meier, and Georg Tamme, *Purity in chromatically localized algebraic K-theory*, J. Amer. Math. Soc. **37** (2024), no. 4, 1011–1040. doi:10.1090/jams/1043; arXiv:2001.10425v5.
- [24] Yanjun Liu and Wolfgang Willems, *Quasi-projective Brauer characters*, J. Algebra **499** (2018), 506–515. doi:10.1016/j.jalgebra.2017.12.016.
- [25] Jacob Lurie, *Higher algebra*, manuscript, September 2017. <https://www.math.ias.edu/~lurie/papers/HA.pdf>.
- [26] Cary Malkiewich, *Coassembly and the K-theory of finite groups*, Adv. Math. **307** (2017), 100–146. doi:10.1016/j.aim.2016.11.017; arXiv:1503.06504v4.
- [27] Akhil Mathew, Niko Naumann, and Justin Noel, *On a nilpotence conjecture of J. P. May*, J. Topol. **8** (2015), no. 4, 917–932. arXiv:1403.2023v3.
- [28] J. Peter May, with contributions by M. Cole, G. Comezana, S. Costenoble, A. D. Elmendorf, J. P. C. Greenlees, L. G. Lewis, Jr., R. J. Piacenza, G. Triantafyllou, and S. Waner, *Equivariant homotopy and cohomology theory*, CBMS Regional Conference Series in Mathematics, vol. 91, American Mathematical Society, Providence, RI, 1996. doi:10.1090/cbms/091.
- [29] J. S. Milne, *Algebraic groups: The theory of group schemes of finite type over a field*, Cambridge Studies in Advanced Mathematics, vol. 170, Cambridge University Press, Cambridge, 2017. doi:10.1017/9781316711736.
- [30] Amnon Neeman, *The connection between the K-theory localization theorem of Thomason, Trobaugh and Yao and the smashing subcategories of Bousfield and Ravenel*, Ann. Sci. Éc. Norm. Supér. (4) **25** (1992), no. 5, 547–566. doi:10.24033/asens.1659.
- [31] Daniel Quillen, *Elementary proofs of some results of cobordism theory using Steenrod operations*, Adv. Math. **7** (1971), 29–56.
- [32] Douglas C. Ravenel, *Nilpotence and periodicity in stable homotopy theory*, Annals of Mathematics Studies, vol. 128, Princeton University Press, 1992.
- [33] Jean-Pierre Serre, *Linear representations of finite groups*, Graduate Texts in Mathematics, vol. 42, Springer, New York–Heidelberg, 1977. doi:10.1007/978-1-4684-9458-7.
- [34] Richard G. Swan, *Induced representations and projective modules*, Ann. of Math. (2) **71** (1960), no. 3, 552–578. doi:10.2307/1969944.